MRI Systems III: Gradients

M219 - Principles and Applications of MRI

Kyung Sung, Ph.D.

1/24/2022

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- Course schedule
  - https://mrrl.ucla.edu/pages/m219_2022
- Assignments
  - Homework #1 due on 1/26 by 5pm
  - Homework #2 will be out on 1/26

Course Overview

- Office Hours
  - TA (Ran Yan) - Tuesday 4-5pm
    https://uclahs.zoom.us/j/96870184581?pwd=VkczL0lyRkxsQ3FHcnIxQ1M2U3hPdz09

    Password: 900645

  - Instructor (Kyung Sung) - Friday 2-3pm
    https://uclahs.zoom.us/j/94058312815?pwd=TkI3ajhkamdGTnhqOVNnbk5RMnJGQT09

    Password: 888767

Bloch Equations with Relaxation

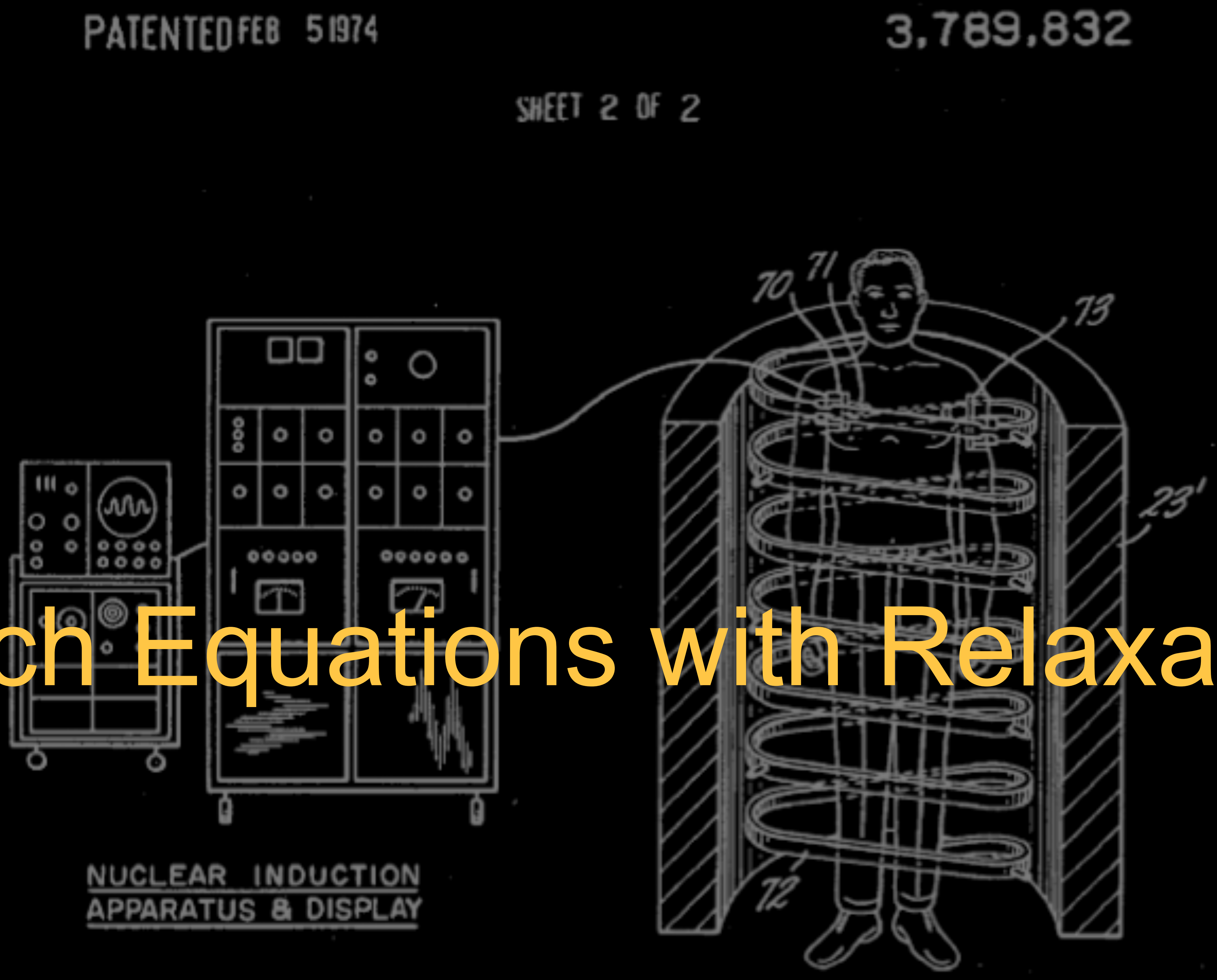


FIG. 2

David Geffen
School of Medicine

UCLA
Radiology

Bloch Equations with Relaxation

$$\frac{d\vec{\mathrm{M}}}{dt} = \vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}} - \frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2} - \frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}$$

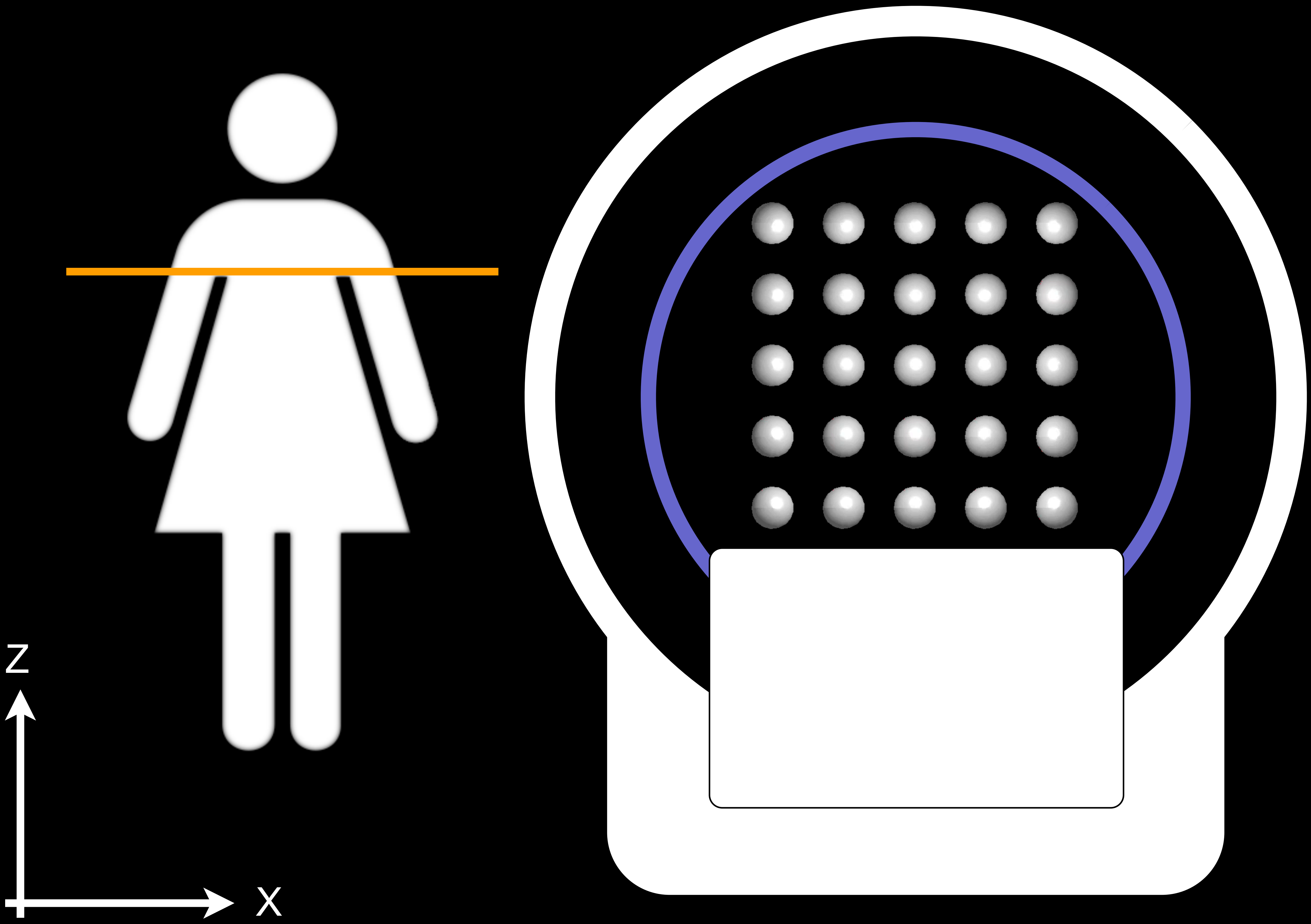
- **Differential Equation**
  - Ordinary, Coupled, Non-linear
- **No analytic solution, in general.**
  - Analytic solutions for simple cases.
  - Numerical solutions for all cases.
- **Phenomenological**
  - Exponential behavior is an approximation.

Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2}}_{\substack{\text{Transverse} \\ \text{Relaxation}}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\substack{\text{Longitudinal} \\ \text{Relaxation}}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO
- Diffusion
  - Spins are thermodynamically driven to exchange positions.
  - Bloch-Torrey Equations

Excitation and Relaxation



The magnetization relaxes after excitation (forced precession).

Bloch Equations – Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \underbrace{\gamma \vec{M}_{rot} \times \vec{B}_{eff}}_{\text{“Precession”}} - \underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that M experiences in the rotating frame

Fictitious field created by the rotating frame that demodulates the apparent effect of B_0

The applied B_0 and B_1 field in the rotating frame

Free Precession
in the Rotating Frame
with Relaxation

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{\mathrm{B}}_{rot} = \mathrm{B}_0 \hat{k}$$

$$\vec{B}_{eff} = \vec{0}$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

Solution:

$$M_{z'}(t) = M_z^0 e^{-t/T_1} + M_0\left(1 - e^{-t/T_1}\right)$$

$$M_{x'y'}(t) = M_{x'y'}(0_+)e^{-t/T_2}$$

The precessional term drops out in the rotating frame.

Forced Precession
in the Rotating Frame
with Relaxation

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{B}_{rot} = B_0 \hat{k} + B_1^e(t)\hat{i}'$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

The precessional term ***does not*** drop out in the rotating frame.

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

- **B1 induced nutation**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, coupled PDEs!**
- **When does this equation apply?**

The precessional term ***does not*** drop out in the rotating frame.

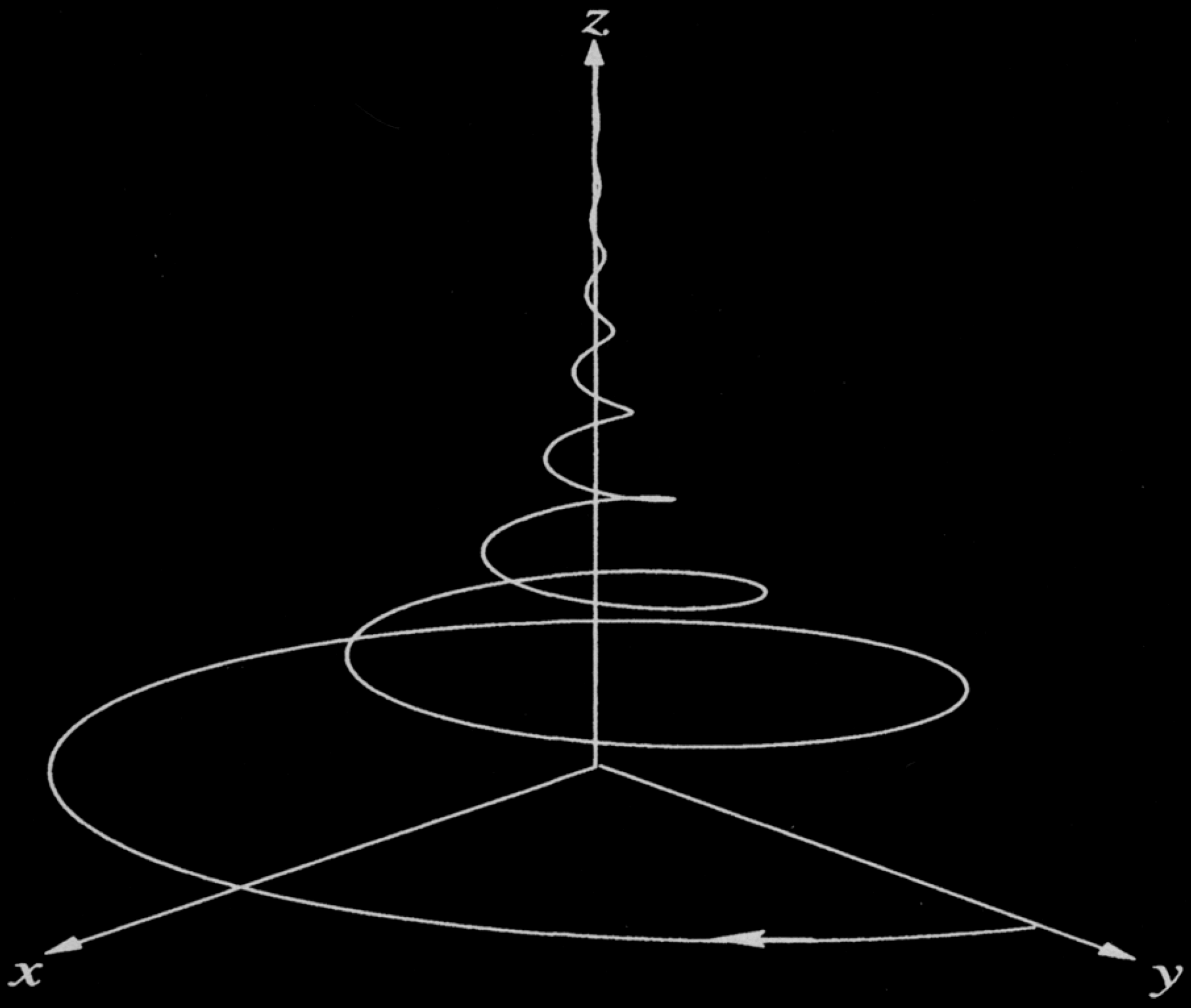
Forced Precession in the Rotating Frame with Relaxation

- RF pulses are short
  - 100μs to 5ms
- Relaxation time constants are long
  - T_1 O(100s) ms
  - T_2 O(10s) ms
- Complicated Coupling
- Best suited for simulation

Free? Forced? Relaxation?

- We've considered all combinations of:
  - Free and forced precession
  - With and without relaxation
  - Laboratory and rotating frames
- Which one's concern M219 the most?
  - Free precession in the rotating frame with relaxation
  - Forced precession in the rotating frame without relaxation.
- We can, in fact, simulate all of them…

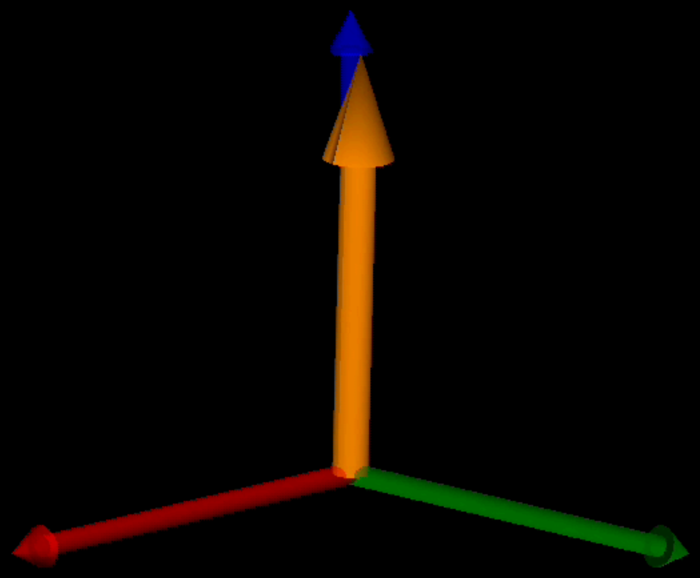
Spin Gymnastics - Lab Frame



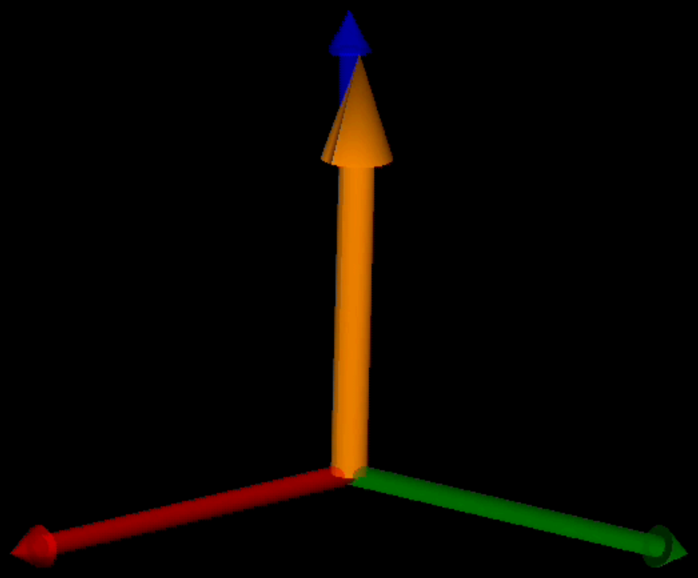
Spin Gymnastics - Rotating Frame

$$M_Z\left(t\right) = M_Z^0 e^{-\frac{t}{T_1}} + M_0\left(1 - e^{-\frac{t}{T_1}}\right)$$

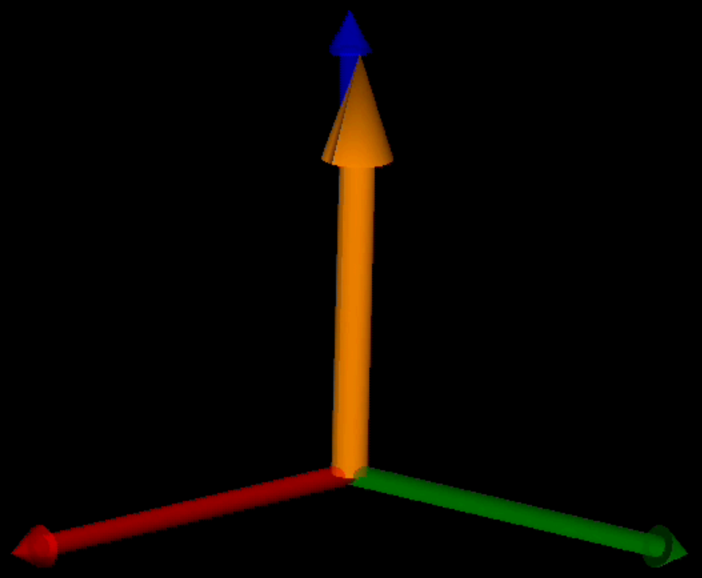
$$M_{xy}\left(t\right) = M_{xy}^0 e^{-t/T_2}$$



90° RF



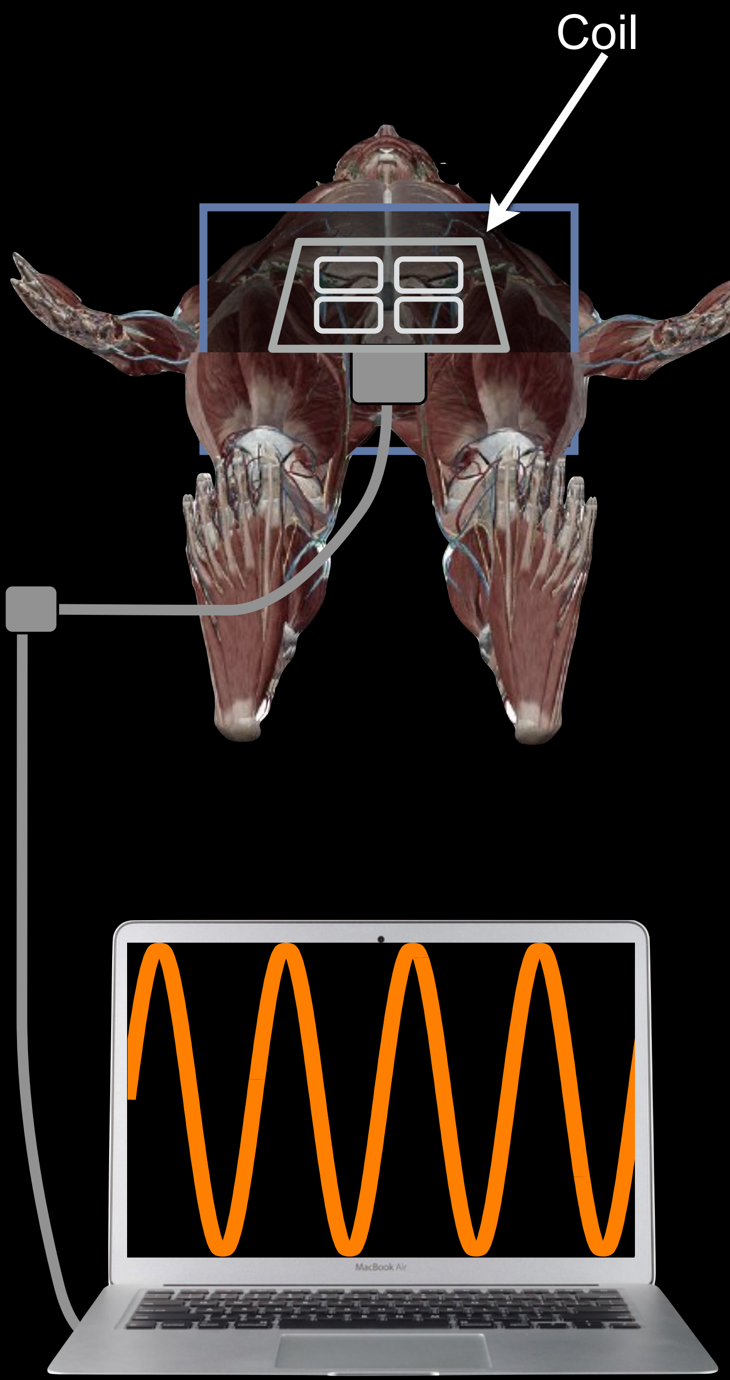
135° RF

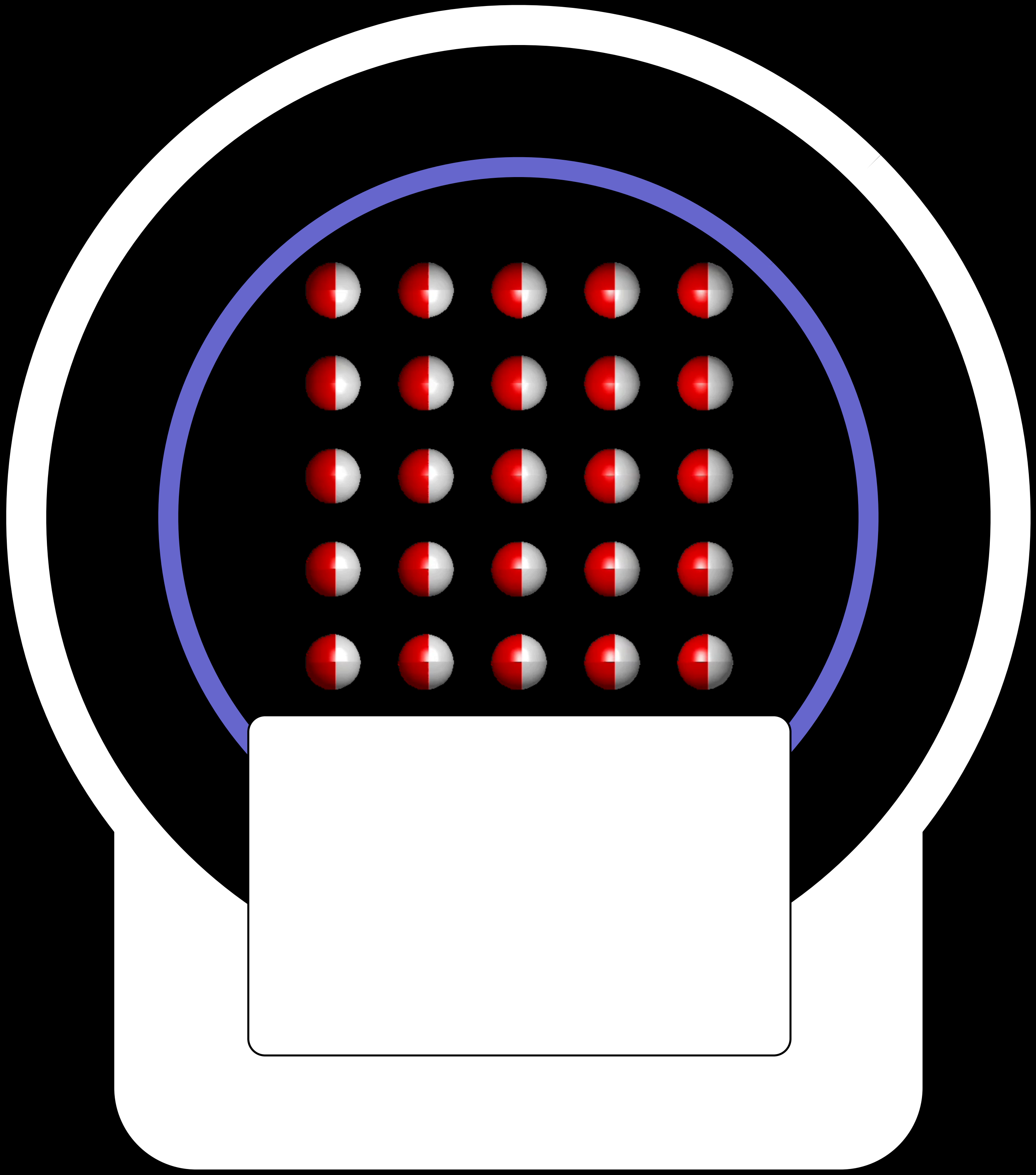


180° RF

How do we measure M_{xy}?

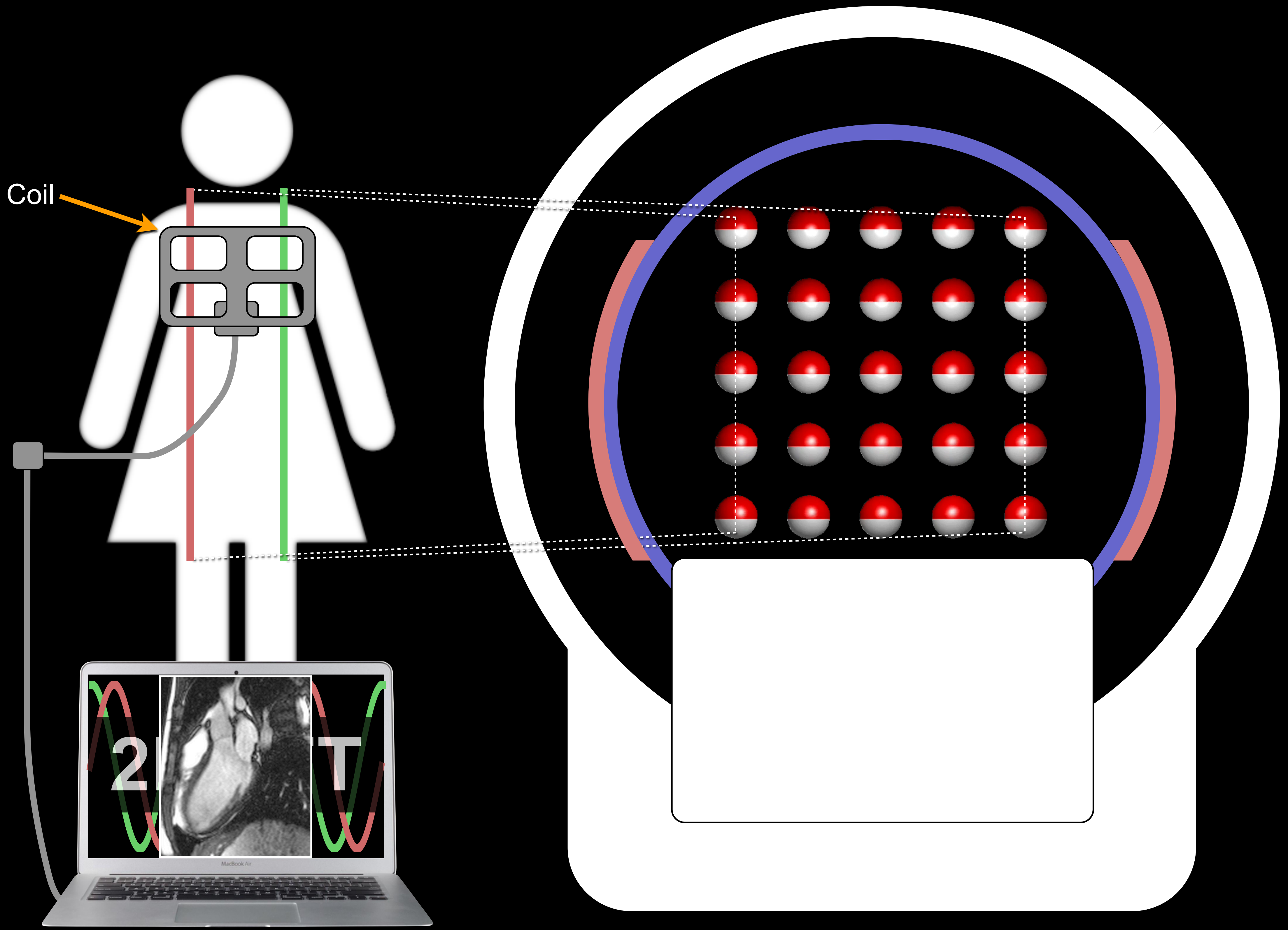
Faraday’s Law of Induction





Precessing spins *induce* a current in a nearby coil.

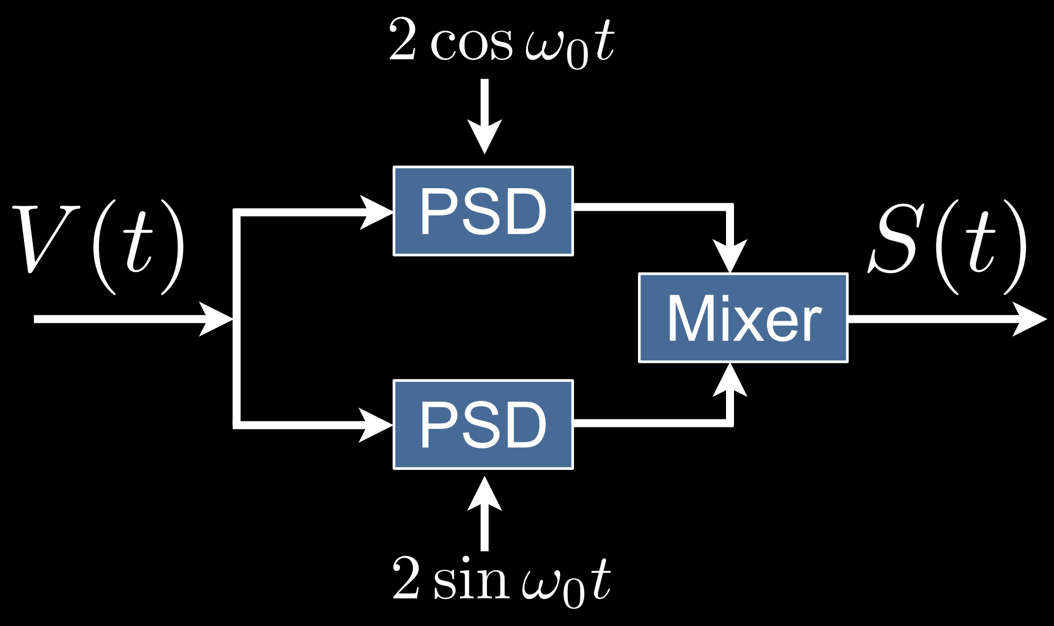
Faraday’s Law of Induction



The trick is to encode spatial information and image contrast in the echo.

Signals in MRI

$$V(t)$$



$2\cos\omega_0 t$

$V(t)$ → PSD → Mixer → $S(t)$

PSD

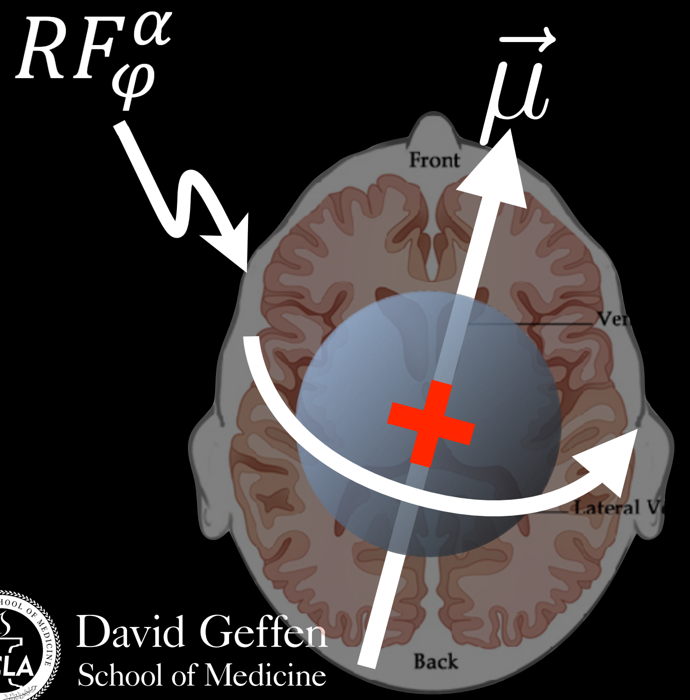
$2\sin\omega_0 t$

$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}t \\ \bar{\gamma}\,\vec{G}_{fe}\left(t - TE\right) \end{cases}$$

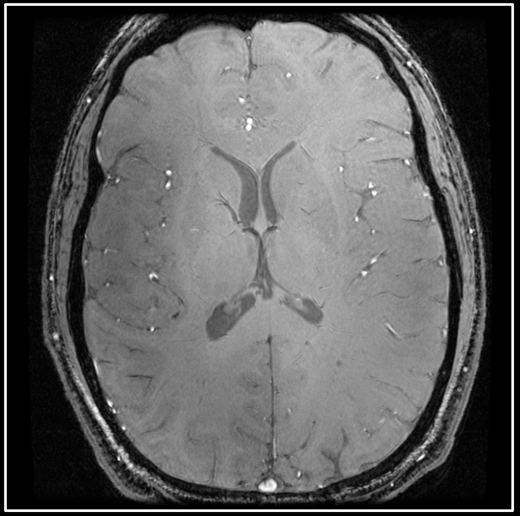
$$S(\vec{k})$$

We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?

$$M_{xy}\left(\vec{r}, t\right)$$

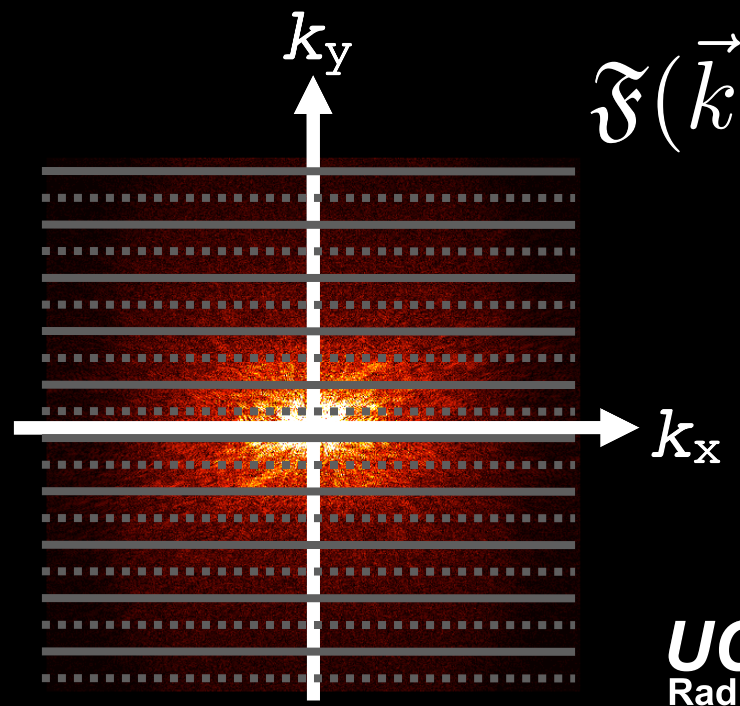


$$I\left(\vec{r}\right)$$

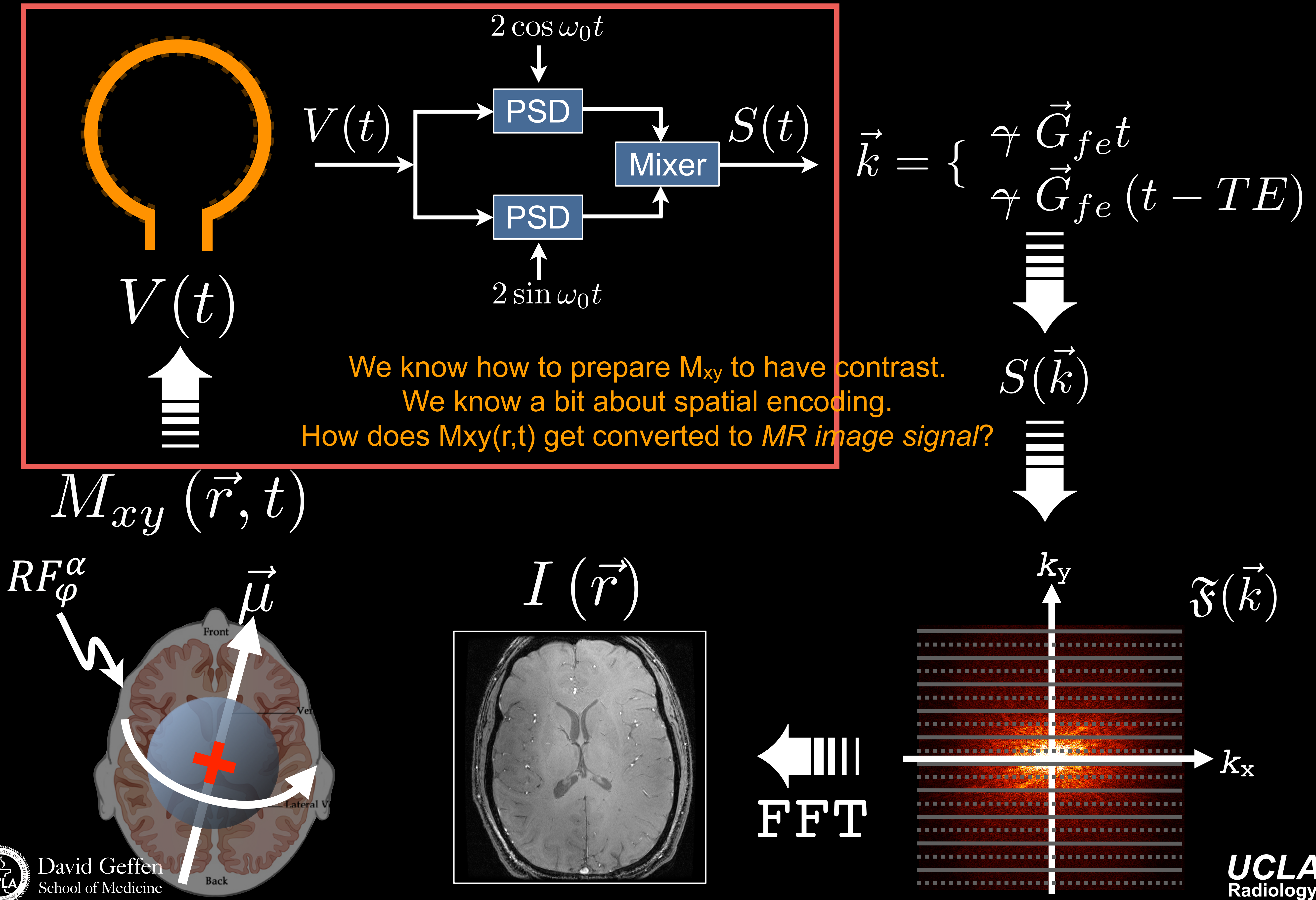


FFT

$$\mathfrak{F}(\vec{k})$$



Signals in MRI



$2\cos\omega_0 t$

$V(t)$ → PSD, PSD → Mixer → $S(t)$

$2\sin\omega_0 t$

$V(t)$

$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}\,t \\ \bar{\gamma}\,\vec{G}_{fe}\,(t - TE) \end{cases}$$

$S(\vec{k})$

We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?

$M_{xy}(\vec{r}, t)$

RF_{φ}^{α} $\vec{\mu}$

$I(\vec{r})$

FFT

k_y $\mathfrak{F}(\vec{k})$ k_x

David Geffen School of Medicine

UCLA Radiology

Basic Detection Principles

Magnetic Flux Through The Coil – *Reciprocity*

$$\Phi\left(t\right) = \int_{object} \vec{B}_r\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$

Eqn. 3.126

Magnetic Flux

Coil Sensitivity

Bulk Magnetization

What happens if the coil has poor sensitivity?

What happens if the coil's sensitivity is perpendicular to the bulk magnetization? How would that happen?

Basic Detection Principles

We get here

$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r}, 0) e^{-i\gamma \Delta B(\mathbf{r}) t} d\mathbf{r}$$

From Here

$$V(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int_{object} \vec{B}(\vec{r}) \cdot \vec{M}(\vec{r}, t)\, d\vec{r}$$

with 25 pages of Math!

Basic Detection Principles

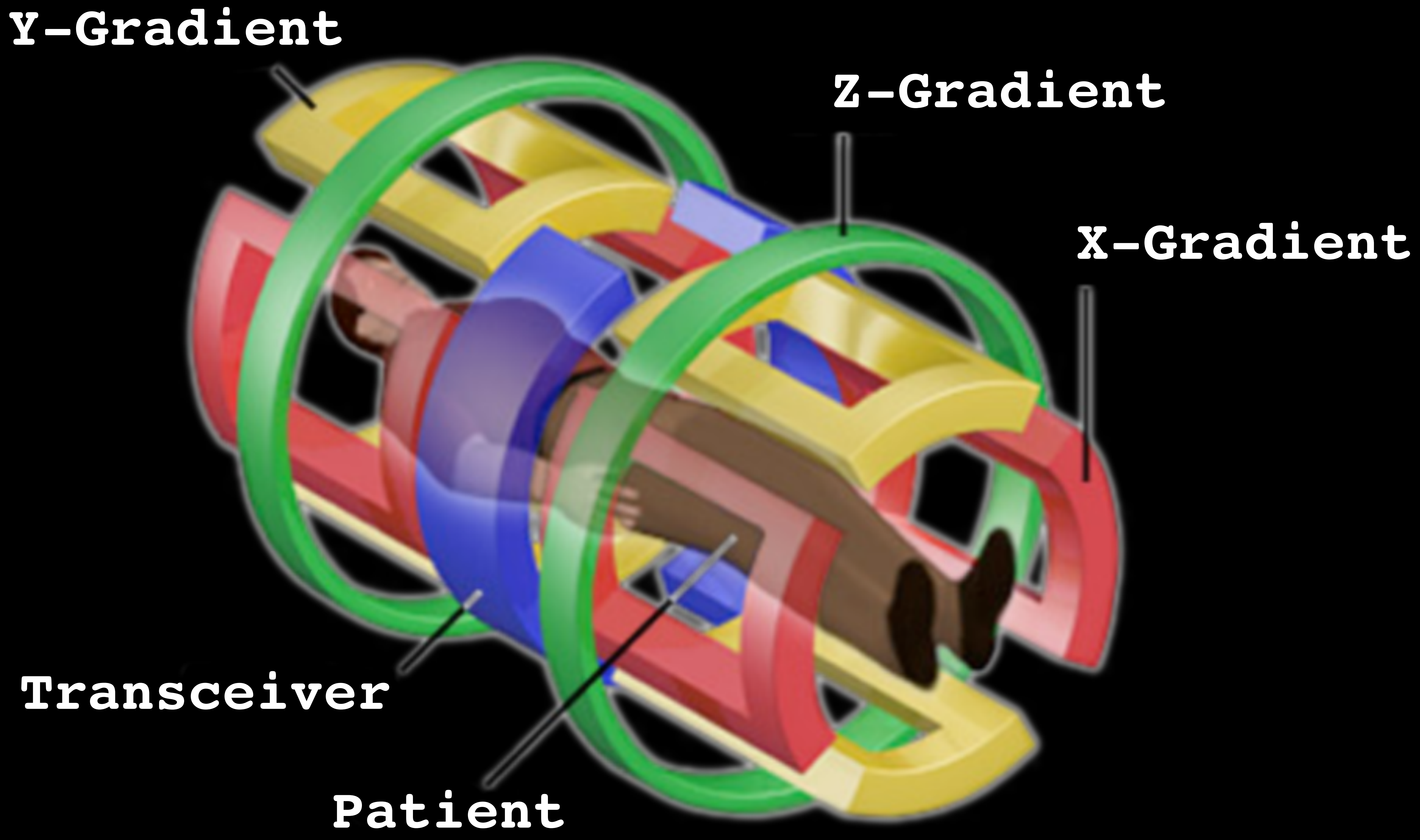
$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r},0)e^{-i\gamma\Delta B(\mathbf{r})t}d\mathbf{r}$$

Observations

Detected signal is the vector sum of all transverse magnetizations in the "rotating frame" within the imaging volume.

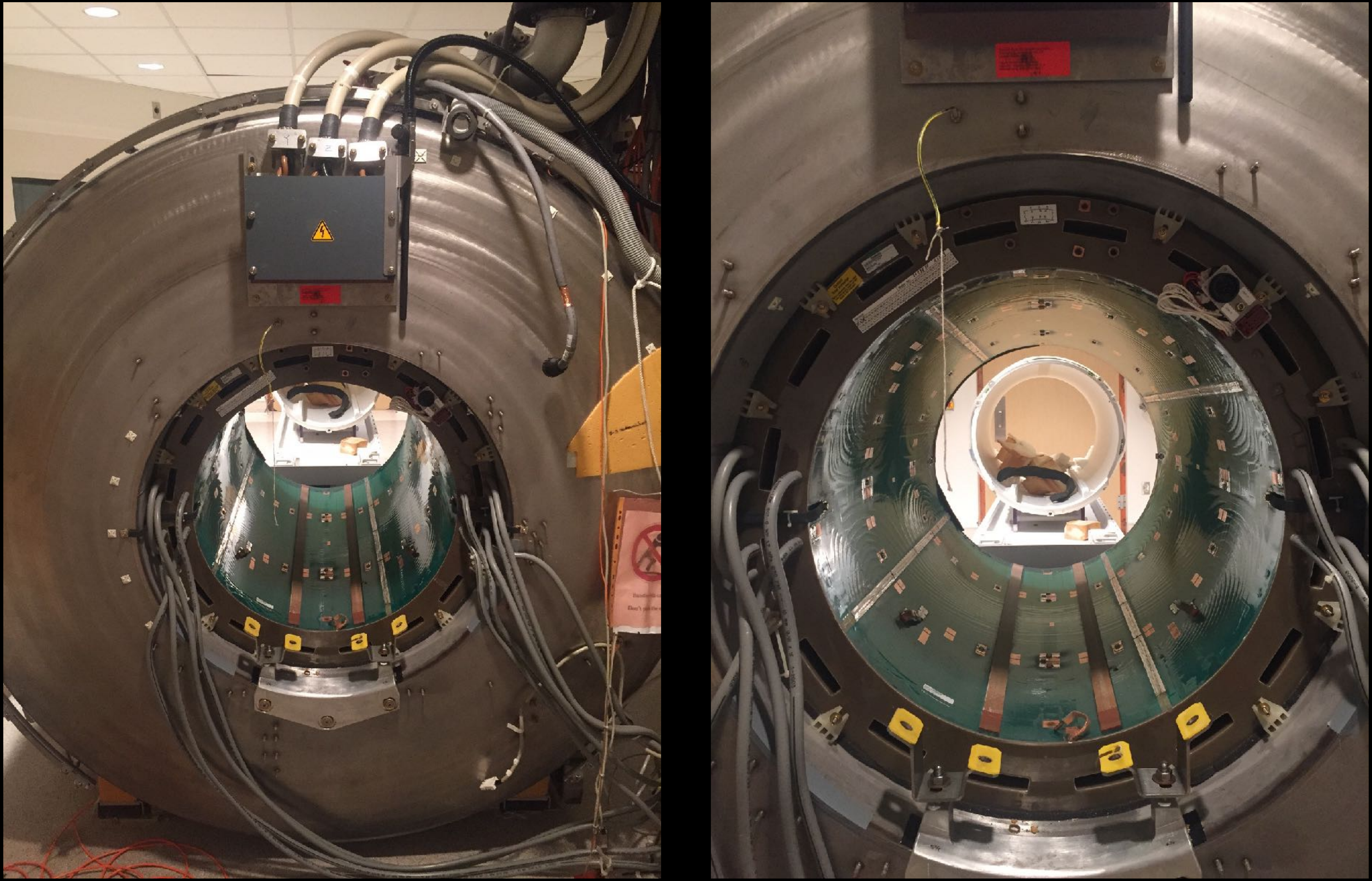
The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging

Gradient Hardware

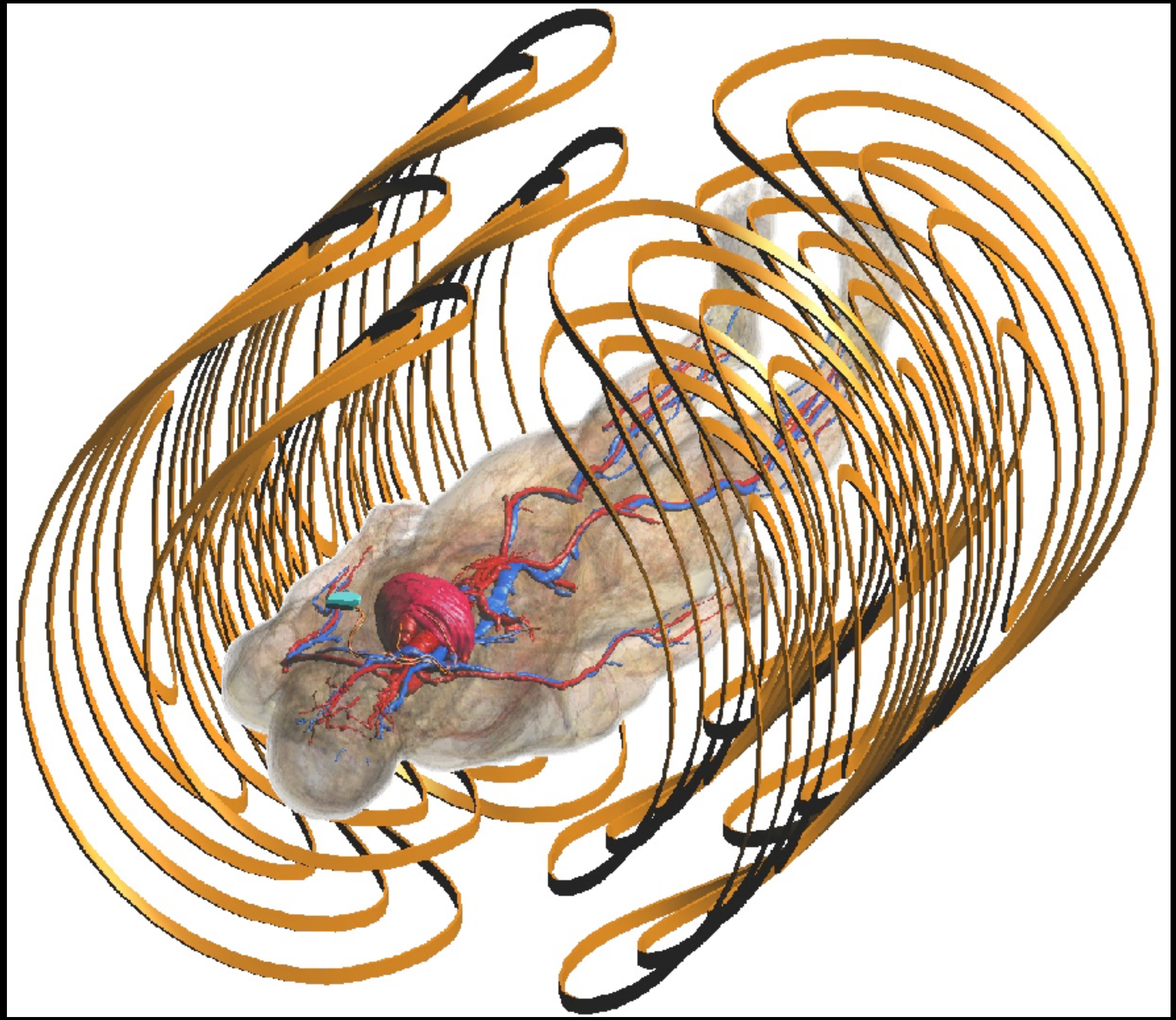


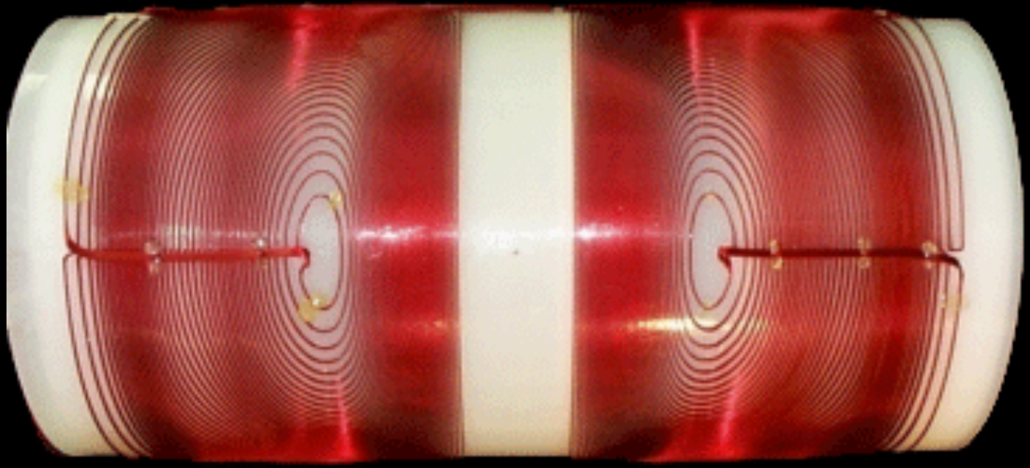
http://www.magnet.fsu.edu

Gradient Hardware

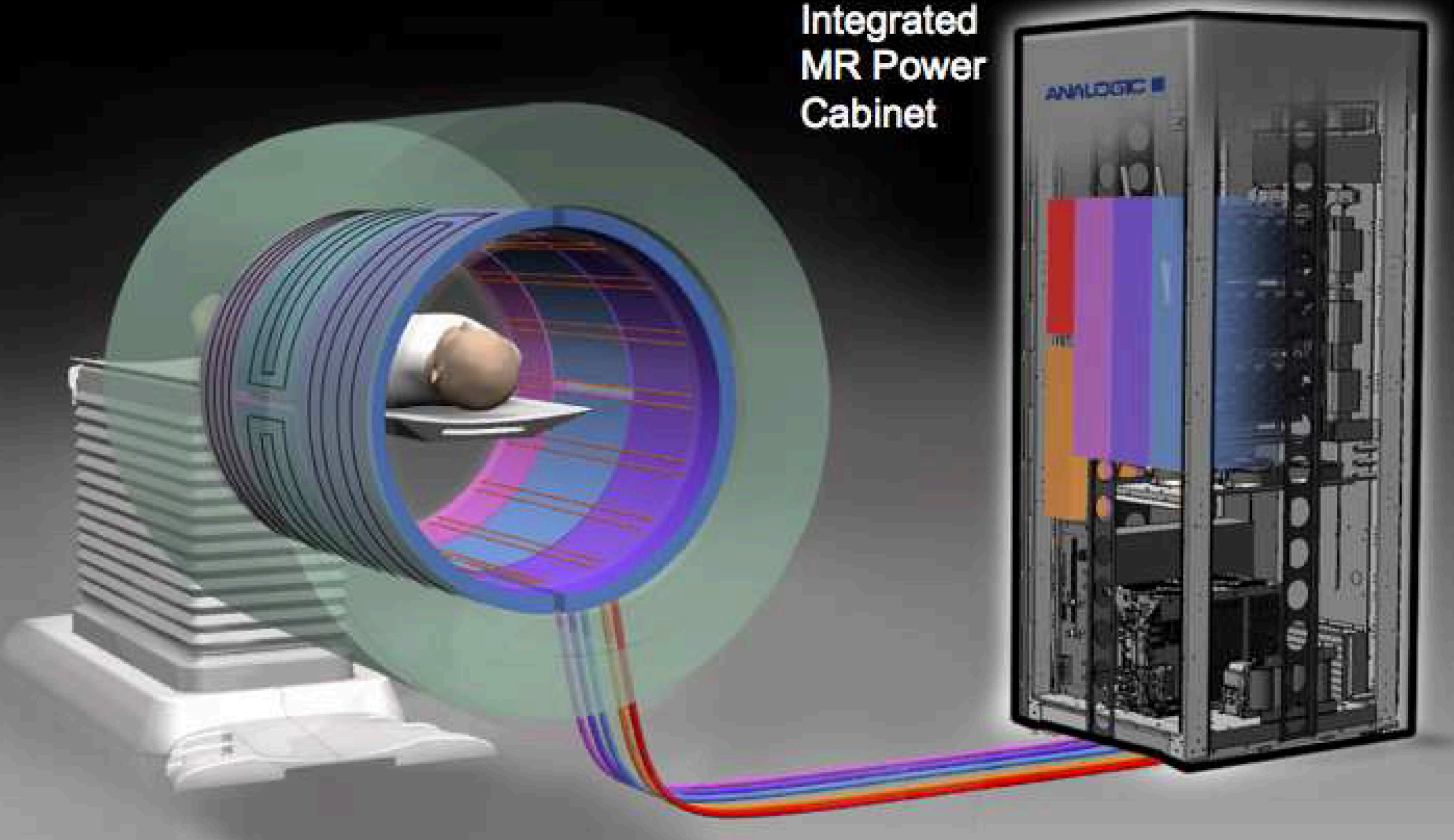


Gradient Hardware





Gradient Hardware



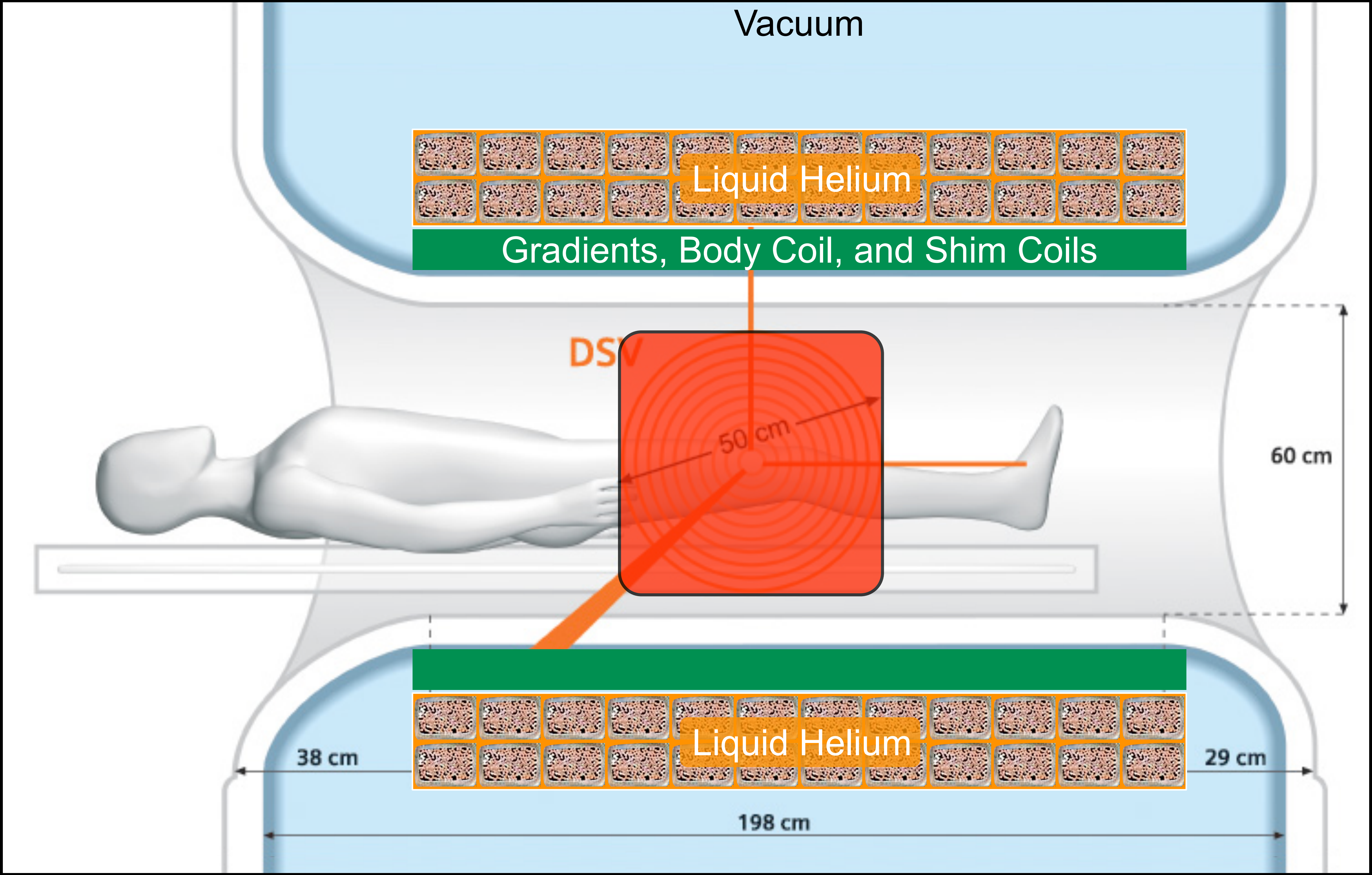
Gradients

- Primary function
  - Encode spatial information
    - Slice selection
    - Phase encoding
    - Frequency encoding
- Secondary functions
  - Sensitize/de-sensitize images to motion
  - Minimize artifacts (crushers & spoilers)
  - Magnetization re-phasing in slice selection
  - Magnetization de-phasing during readout

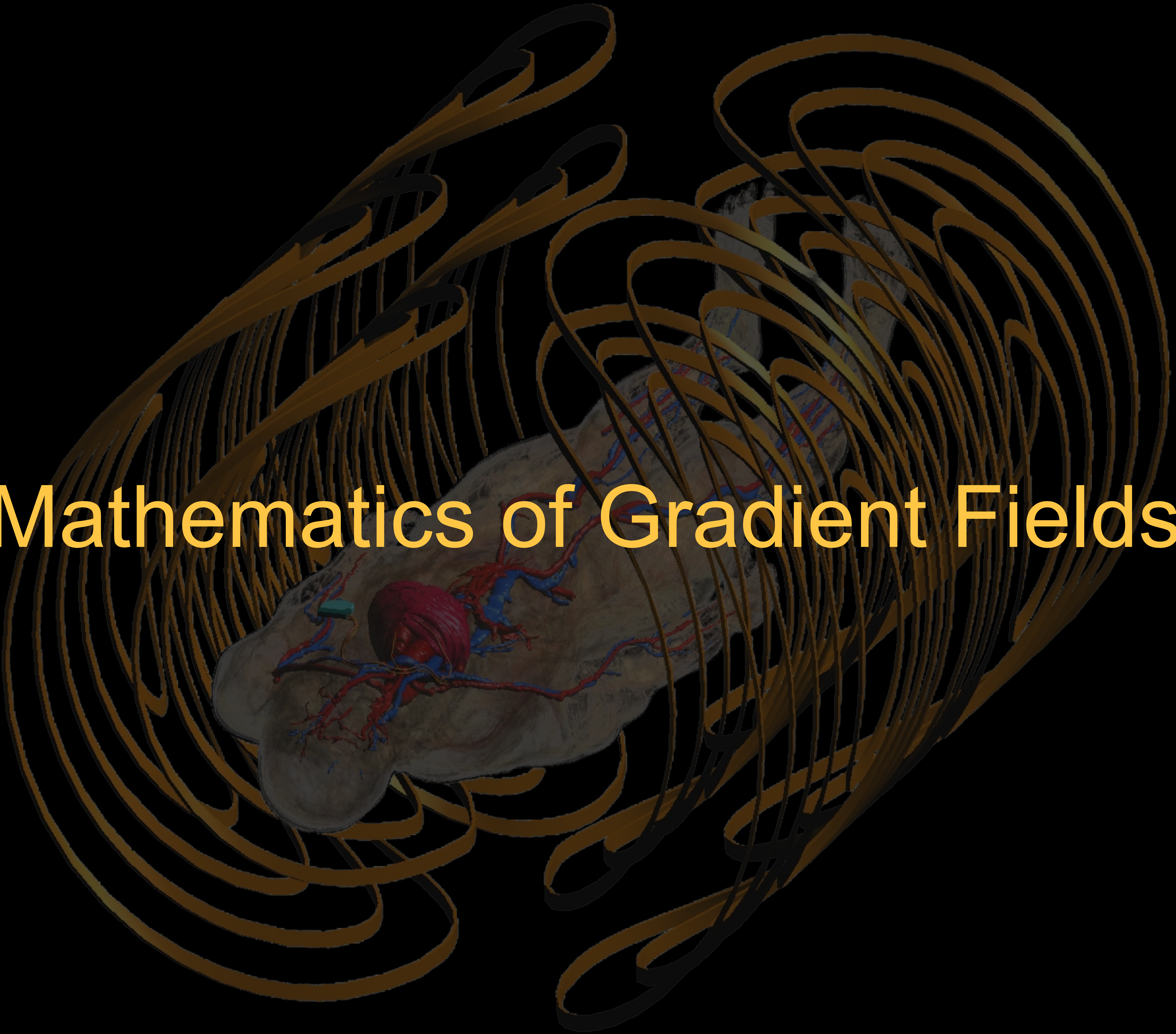
Gradients

- Gradients are a:
  - Small
    - <5G/cm (<0.0075T @ edge of 30cm FOV)
  - Spatially varying
    - Linear gradients
    - Adds to B_0 only in Z-direction
  - Time varying
    - Slewrate Max. ~150-200mT/m/ms
  - Magnetic field
    - Adds/Subtracts to the B_0 field
  - Parallel to B_0
- Gradients are NOT:
  - Fields perpendicular to B_0

Gradients



Gradients are “linear” over ~40-50cm on each axis.



Mathematics of Gradient Fields

Gradients

Gradients are a special kind of inhomogeneous field whose *z*-component varies linearly along a specific direction called the gradient direction.

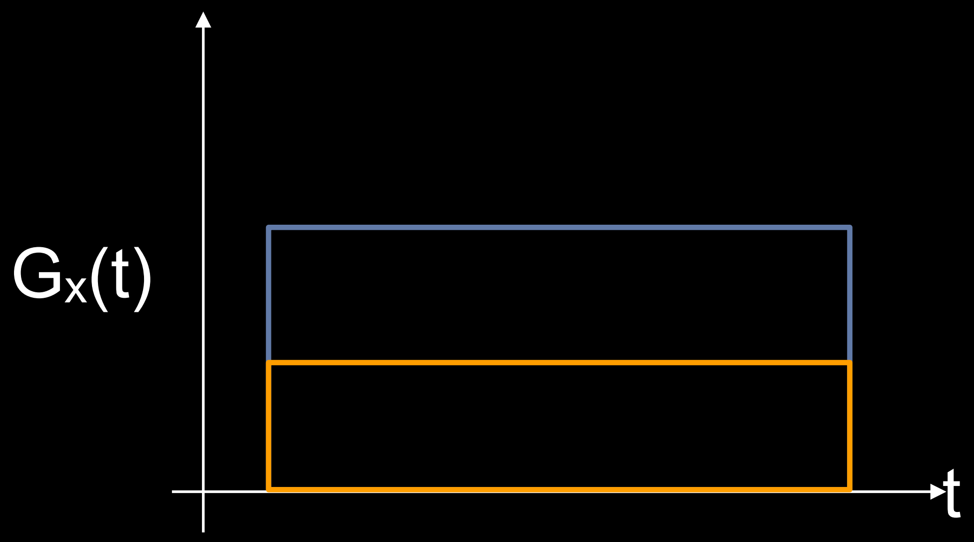
$$\underbrace{B_{G}}_{\text{B-field from a gradient}}{}_{,\underbrace{z}_{\text{Points along the z-direction}}}\underbrace{(x)}_{\text{Varies with the x-direction}} = \underbrace{G_x}_{\text{x-gradient amplitude}}\underbrace{x}_{\text{x-distance from isocenter}}$$

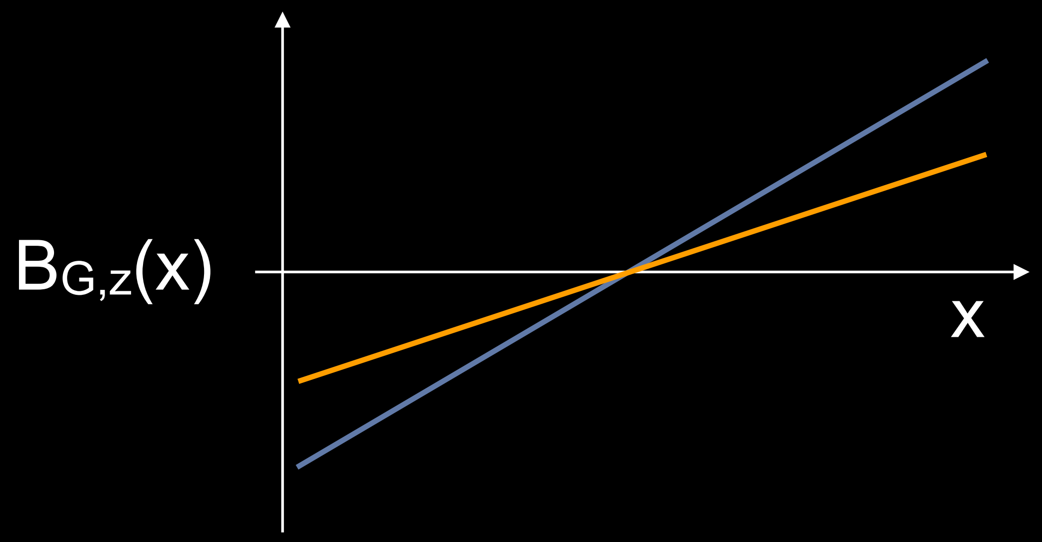
Gradient Induced B-Fields

$$B_{G,z}(x) = G_x x$$ x-gradient

$$B_{G,z}(y) = G_y y$$ y-gradient

$$B_{G,z}(z) = G_z z$$ z-gradient





Gradient Induced B-Fields

- Each gradient coil can be activated independently and simultaneously

$$
\begin{aligned}
B_{G,z}\vec{k} &= (G_x x + G_y y + G_z z)\vec{k} \\
&= (\vec{G}\cdot\vec{r})\vec{k}
\end{aligned}
$$

The magnetic field at a position depends on the magnitude of the applied gradient.

Combined B_0 and Gradient Fields

- Gradients contribute to the net B-field, but only along the z-direction

$$
\begin{aligned}
\vec{B}\left(\vec{r},t\right) &= \left(B_0 + B_{G,z}\right)\vec{k} \\
&= \left(B_0 + \vec{G}\left(t\right)\cdot\vec{r}\right)\vec{k}
\end{aligned}
$$

B-Field Assumptions in MRI

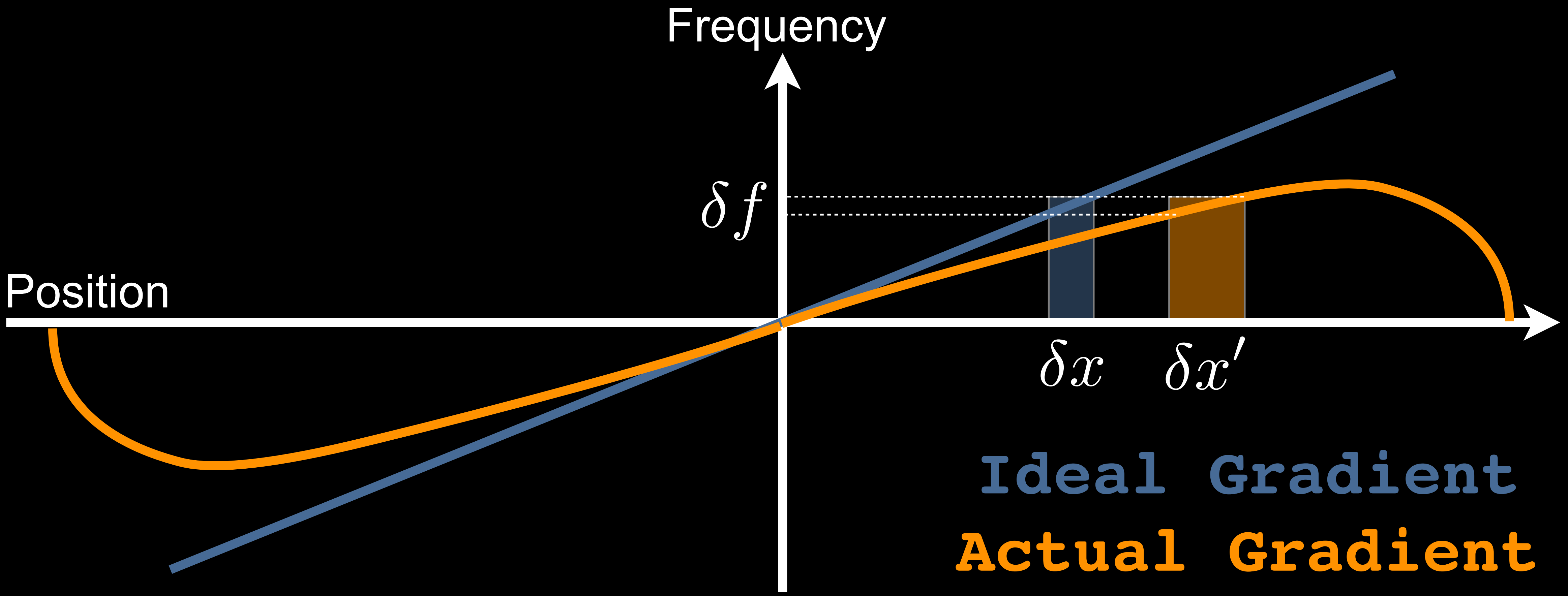
- **B_0-field is:**
  - Perfectly uniform over space.
    - “B_0 homogeneity”
  - Perfectly stable with time.
- **B_1-field is:**
  - Perfectly uniform over space.
    - “B_1 homogeneity”
  - Temporally modulated exactly as specified.
- **Gradient Fields are:**
  - Perfectly linear over space.
    - “Gradient linearity”
  - Temporally modulated exactly as specified

Imperfections of Gradient Fields

- Gradient coils aren’t perfect
  - –Non-linearity
  - –Eddy Currents
  - –Maxwell terms (Concomitant fields)
  - –But they are small
    - *<u>Much</u>* smaller than B_0

Gradient Non-linearity

Gradient Non-linearity

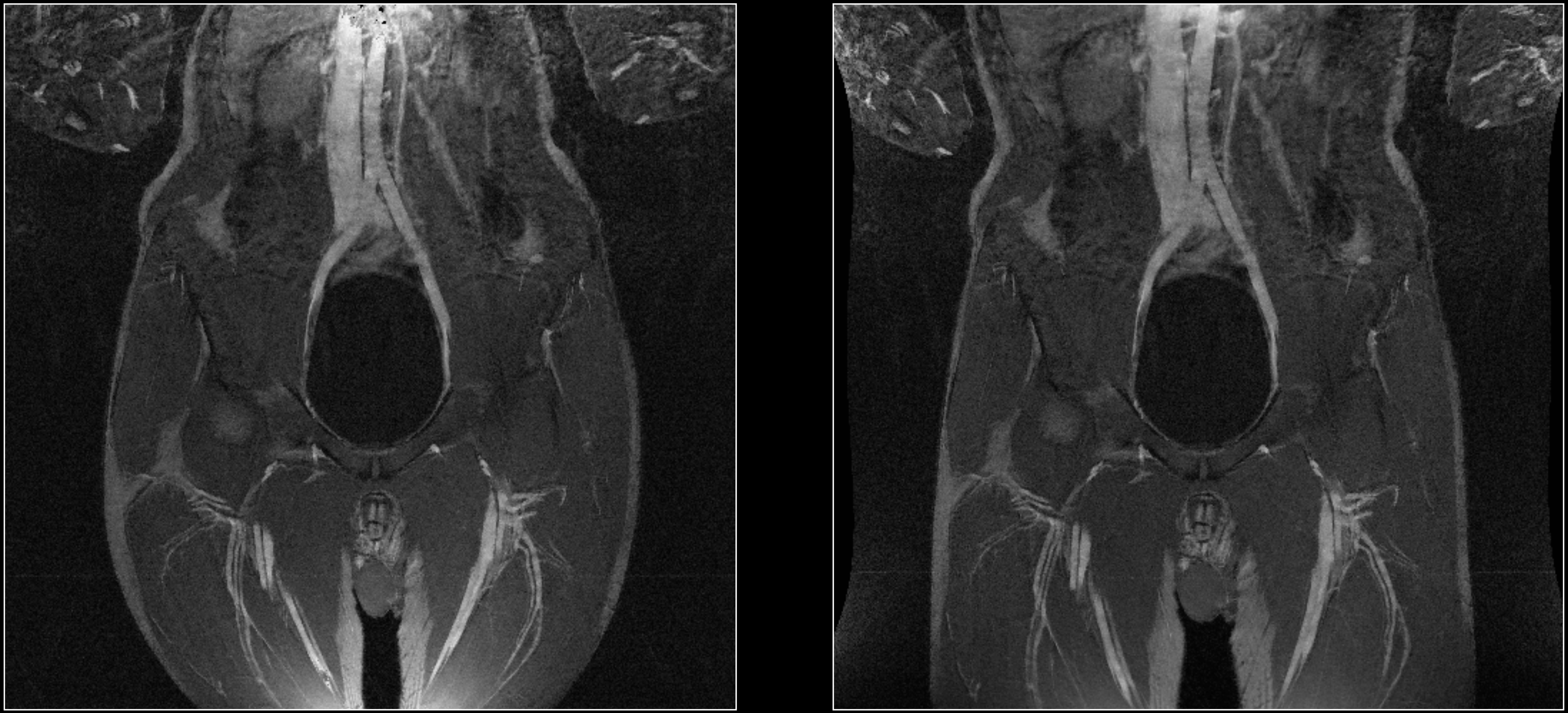


Ideally spatial position is linearly related to frequency.

Gradient Non-linearity

- **Basic *assumption* in MRI is that the z-component of the B-field created by the gradient coils varies linearly with x, y, or z over the FOV.**
- **Higher gradient amplitudes and slewrates can be achieved by compromising on spatial linearity.**
- **Gradient non-linearity causes geometric and intensity distortions.**

Gradient Non-linearity



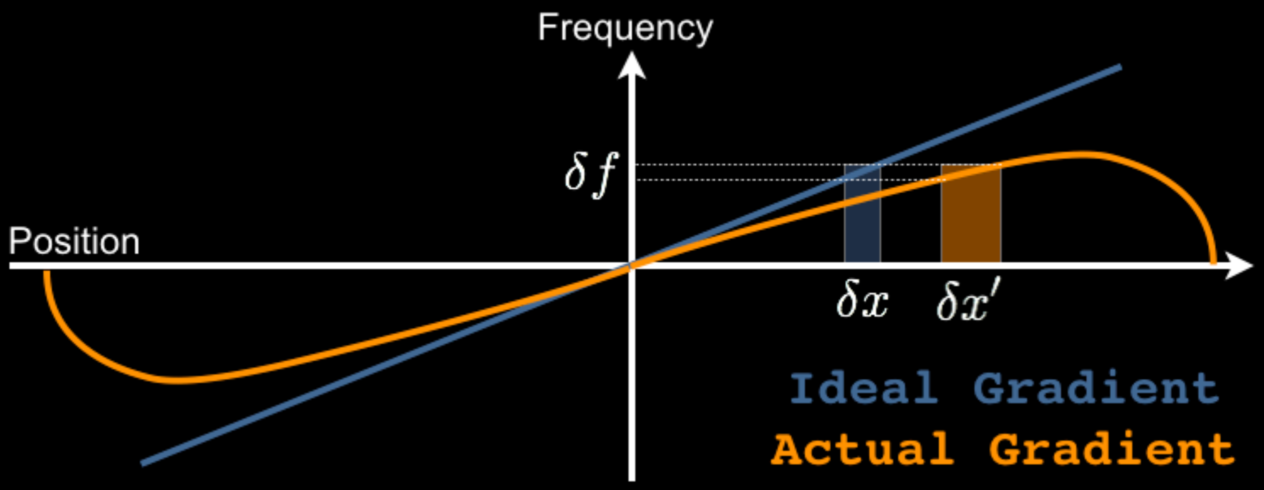


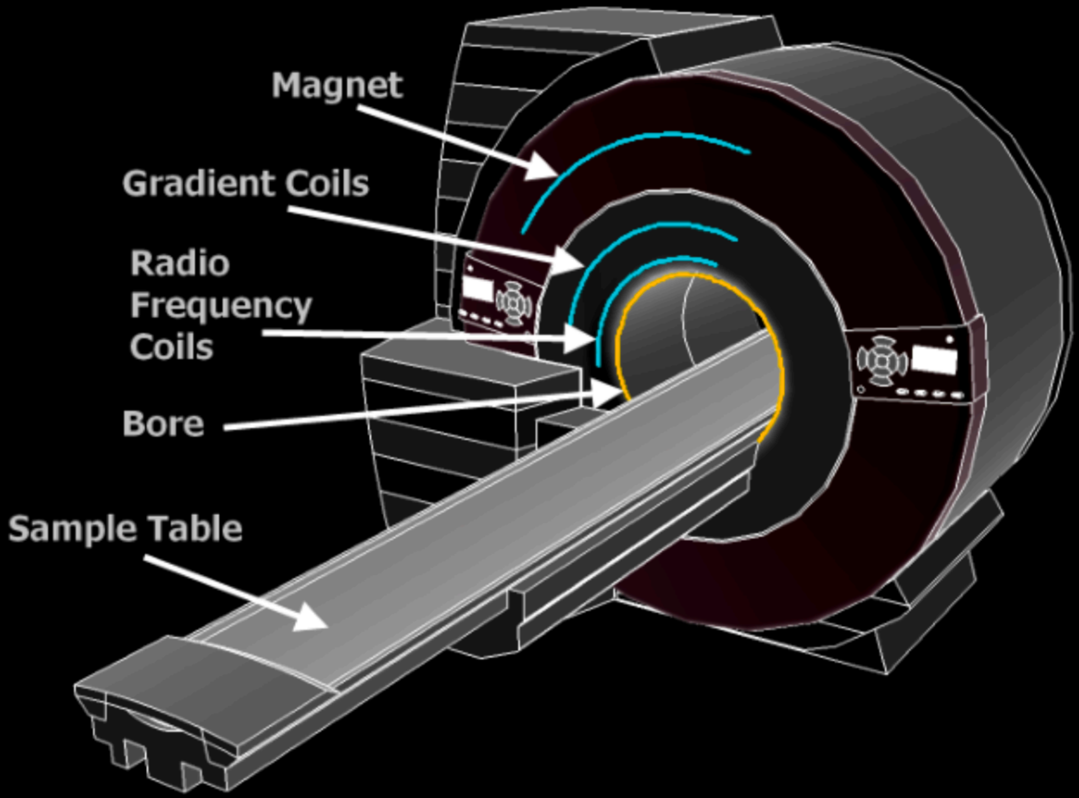
Image Courtesy of M.T. Alley & B.A. Hargeaves

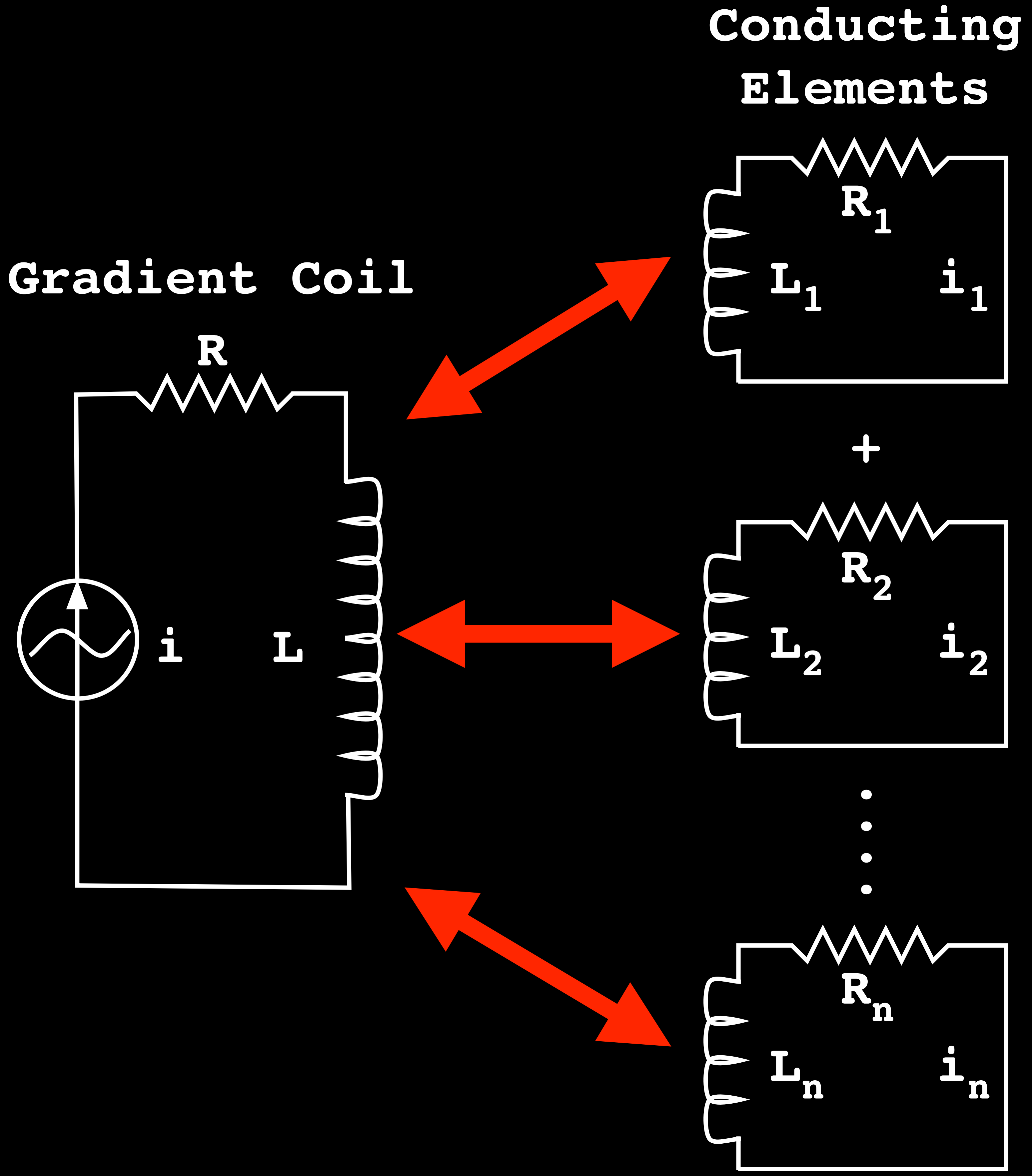
Solution

- Improve hardware and linearity!
- Pay attention to FOV!
- Image warping parameters that are system specific and applied to all images.
    - Works well qualitatively.
    - Can be problematic quantitatively.

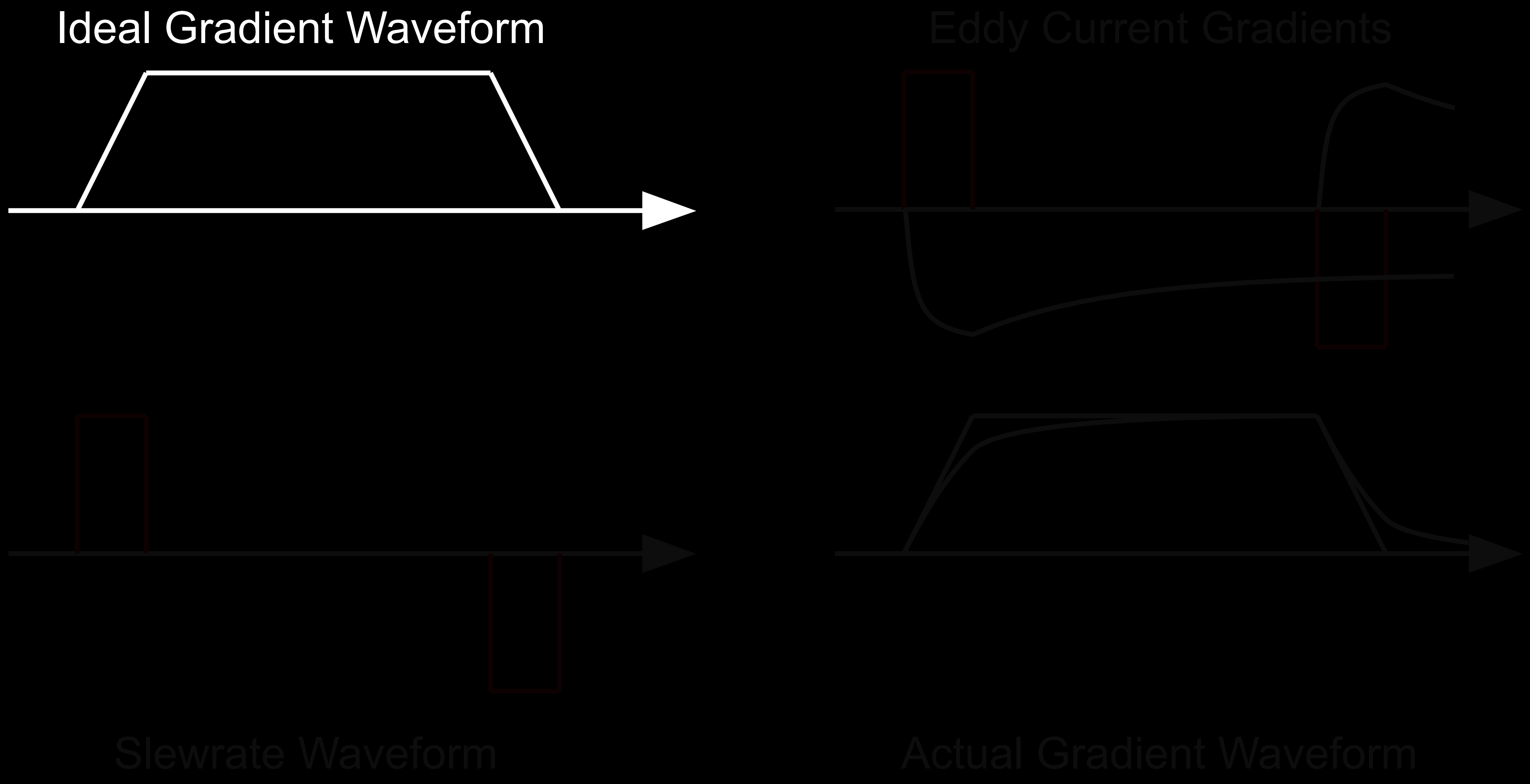
Eddy Currents

Eddy Current Origins: Diagram





Eddy Current Gradient Distortion



Ideal Gradient Waveform

Eddy Current Gradients

Slewrate Waveform

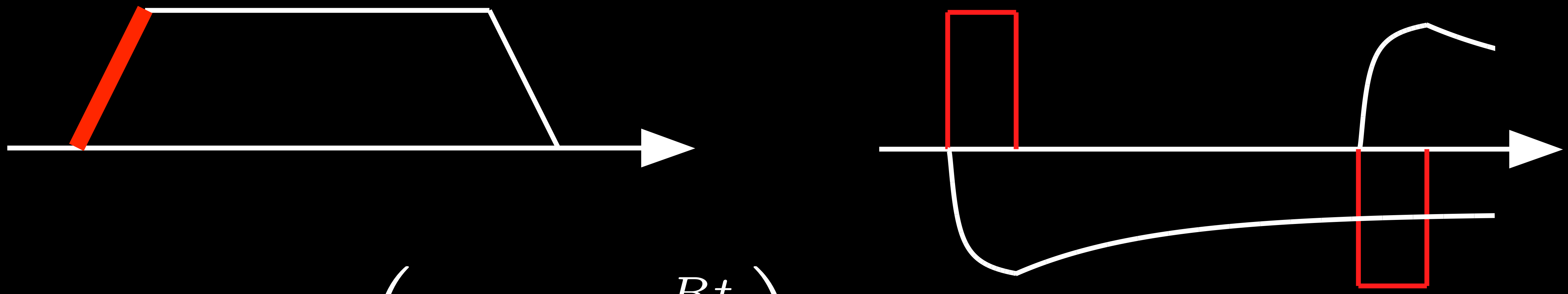
Actual Gradient Waveform

Eddy Current Origins: Mathematics

$$V_e = - \oint_{\vec{A}} \frac{\partial G}{\partial t} \cdot d\vec{A}$$

Faraday's Law

Lenz's Law



$$I_0\left(t\right) = I_f\left(1 - e^{-\frac{Rt}{L}}\right)$$

Ohm's Law

$$I_e\left(t\right) = I_0\left(t_r\right)e^{-Rt/L}$$

Source-Free
***RL* Circuit**

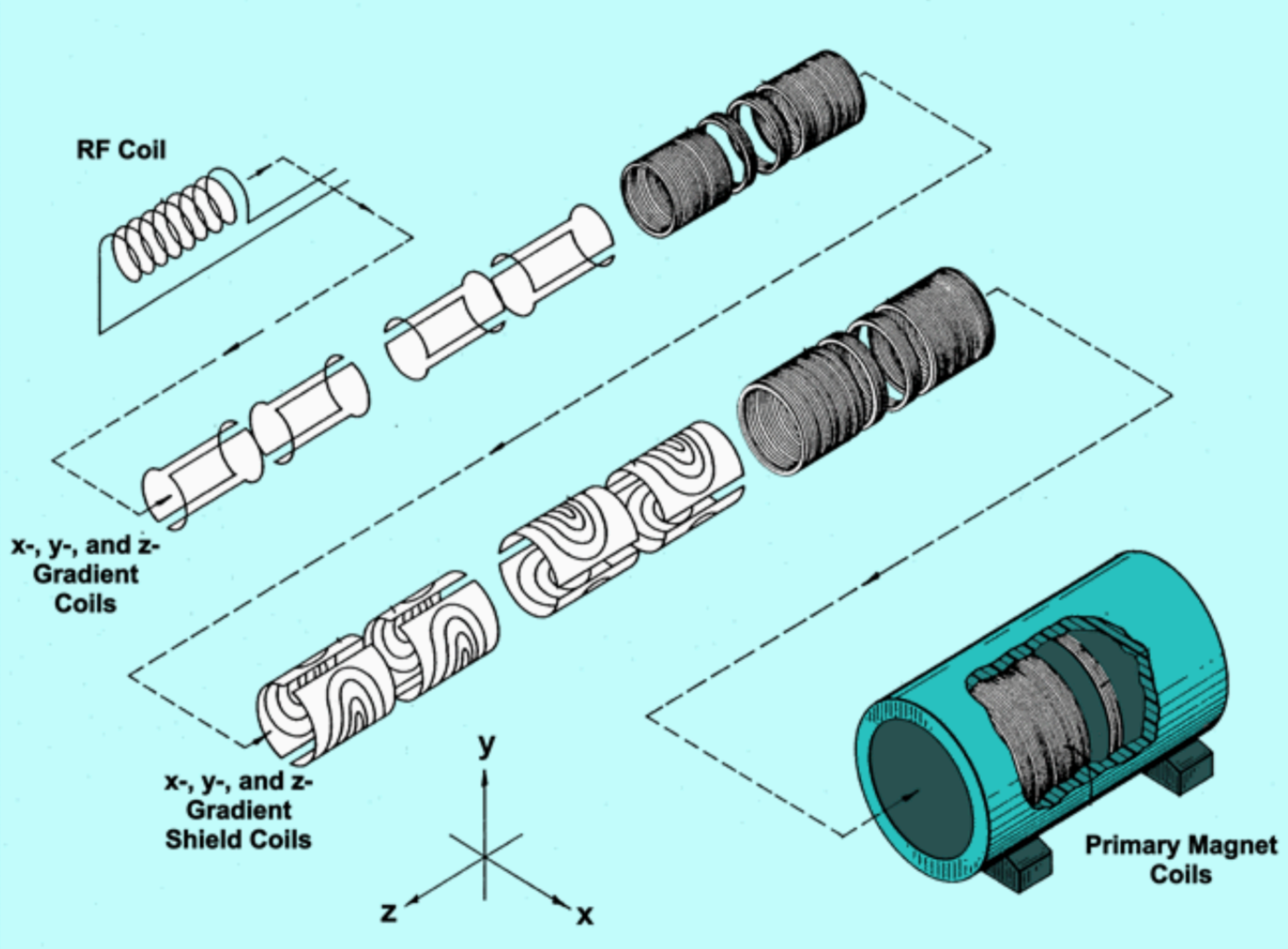
$$B_e\left(t\right) \propto I_e(t)$$

Eddy Current
Induced Field

Eddy Current Compensation

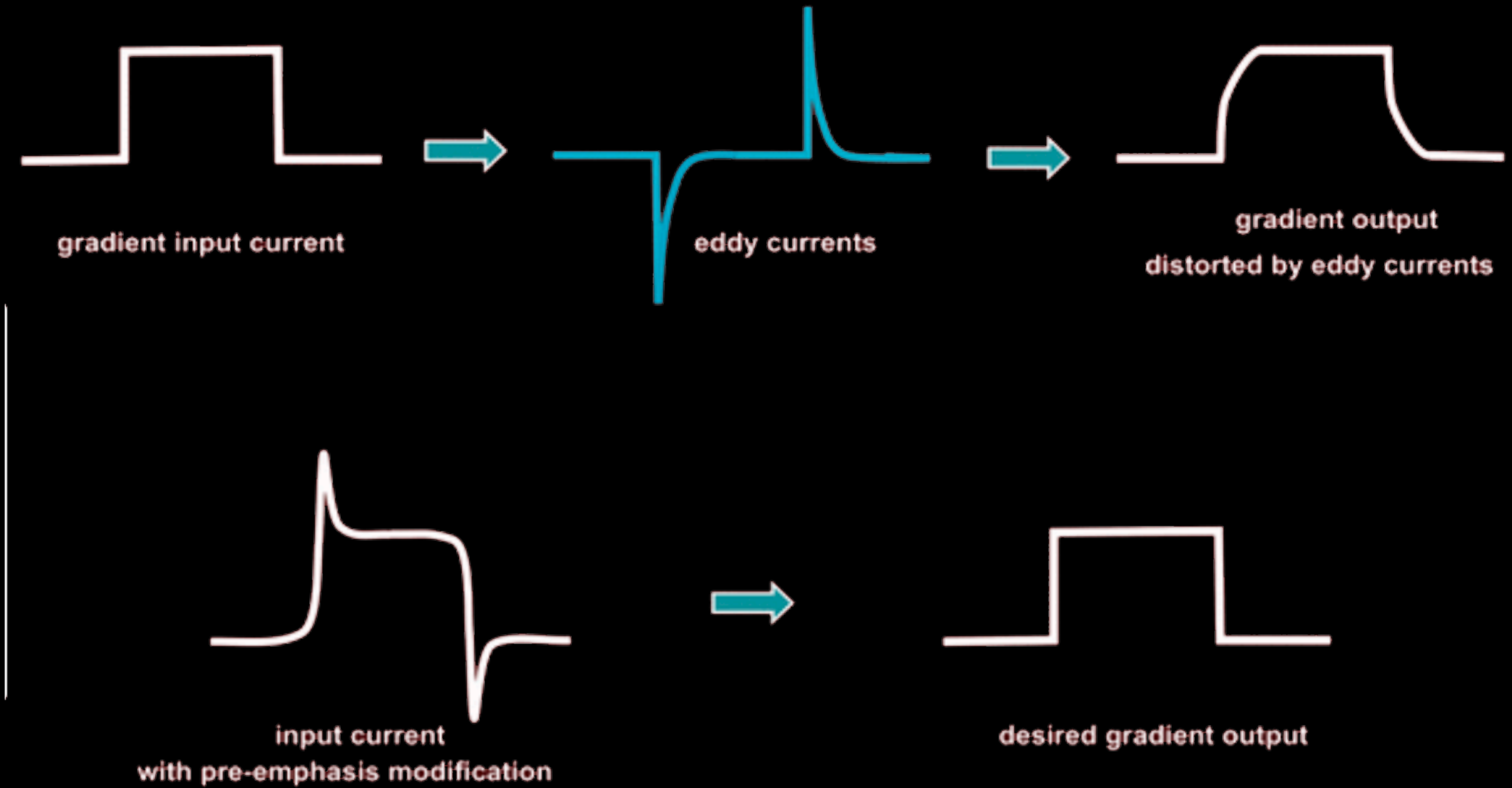
- **Hardware**
  - **Actively Shielded Gradient Coils**
  - **Waveform Pre-emphasis**
- **Pulse Sequence**
  - **Slewrate de-rating**
  - **Twice Re-focused Spin Echo**
- **Reconstruction**
  - **Measure & Subtract (PC)**
  - **Predict & Subtract**

Active Shielding



http://mriquestions.com/active-shielded-gradients.html

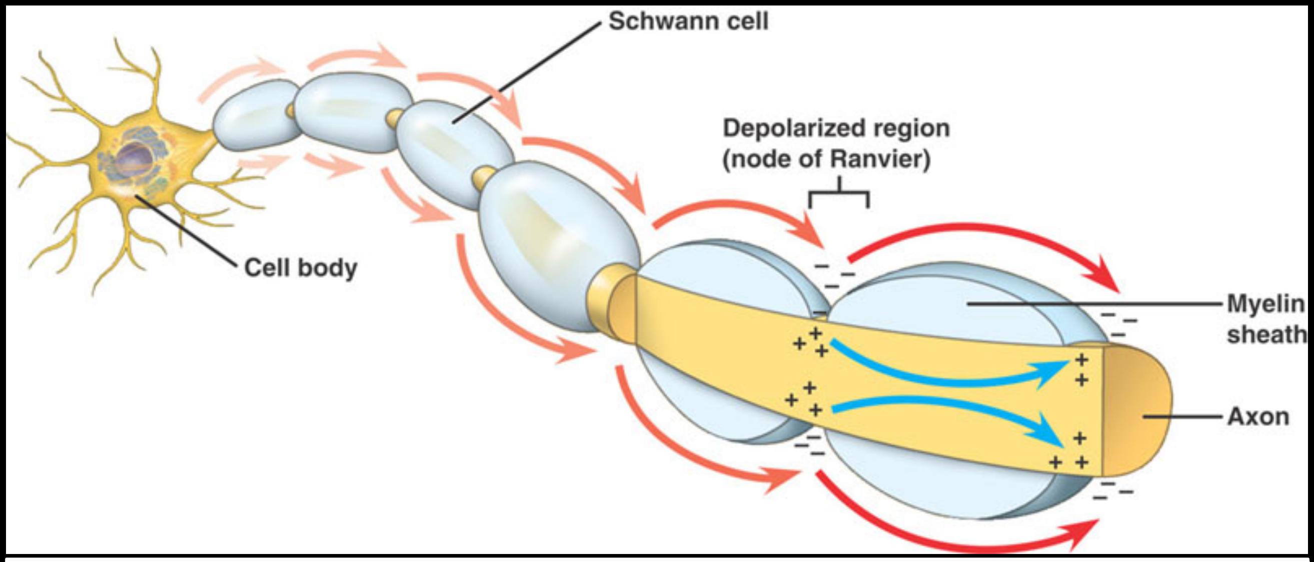
Eddy Current Pre-Emphasis

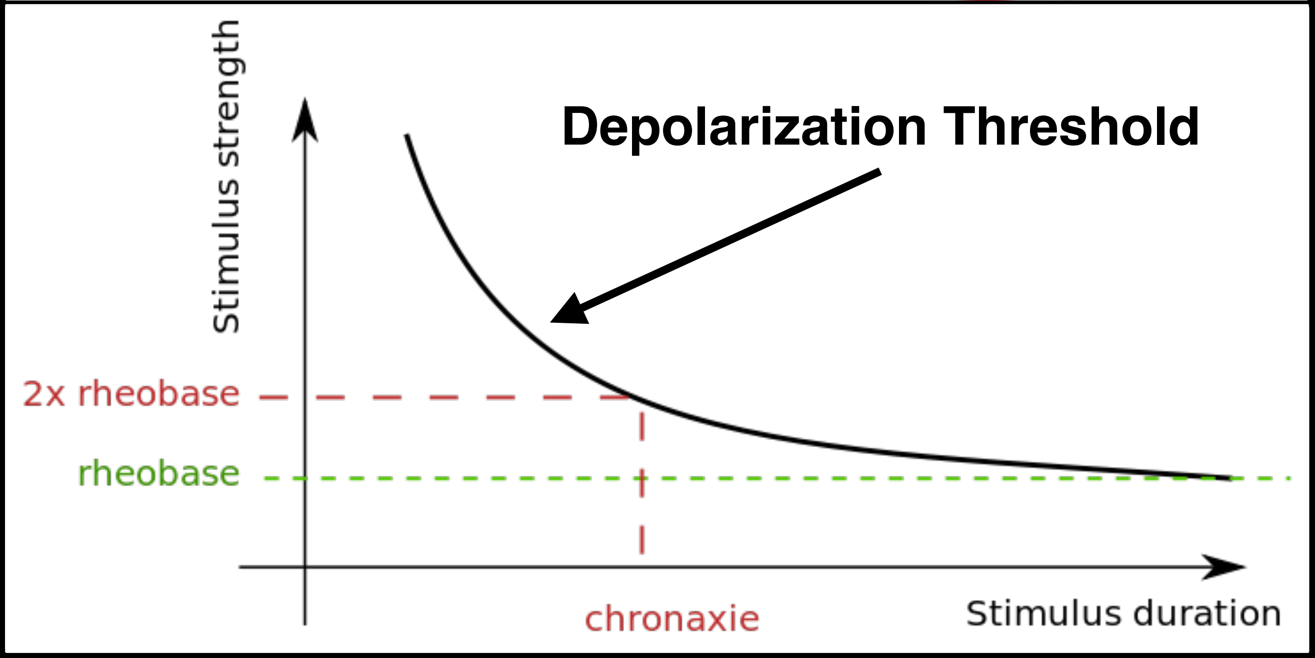


Gradient Safety

Gradient Safety

- **Noise**
- **Peripheral nerve stimulation (PNS)**





Solution: De-rate gradient slew rates, but this increases scan time.

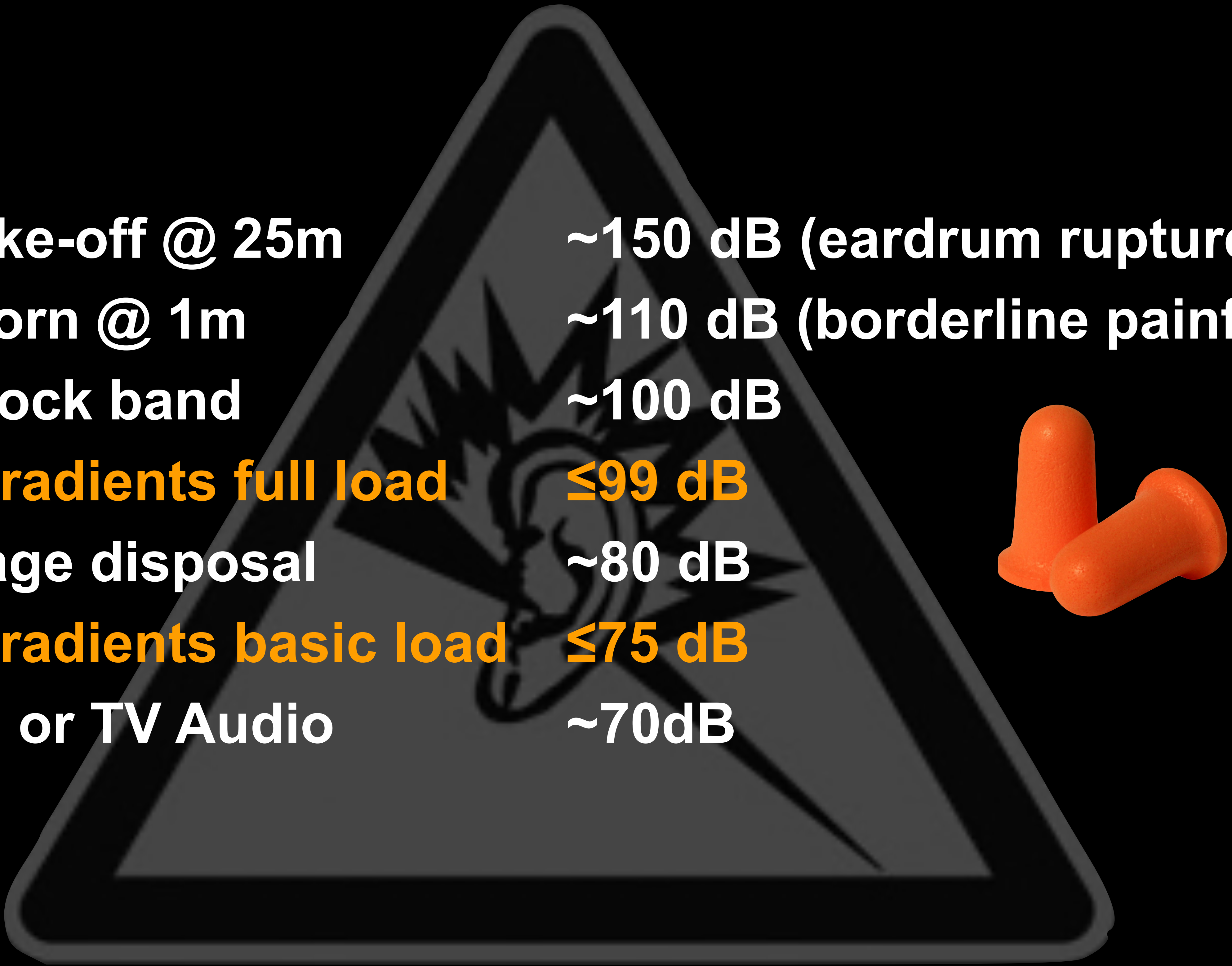
Solution: Ear plugs Head phones

Time-varying gradients induce mechanical vibrations and PNS.

MRI Gradient Noise

Switching the gradients on ms time scales (kHz) generates acoustic noise.

Gradient Noise

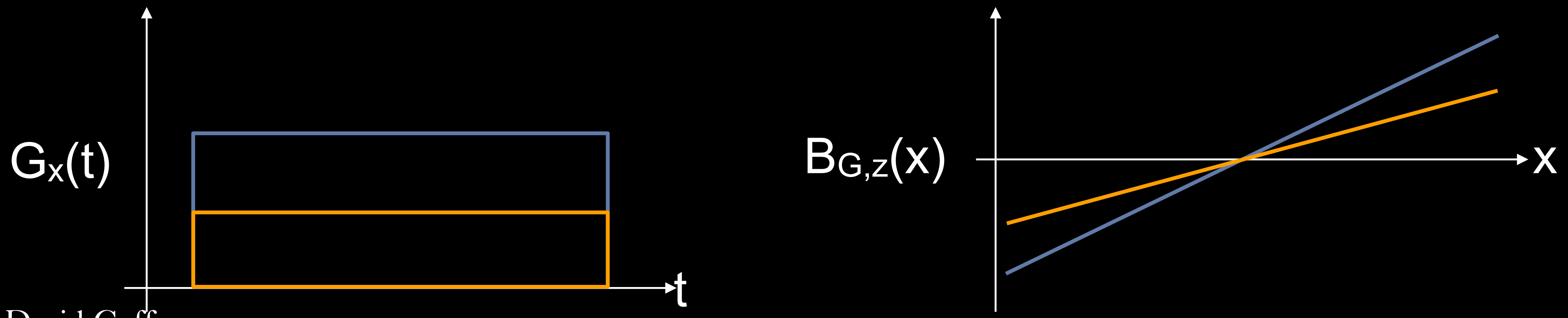


Jet take-off @ 25m	~150 dB (eardrum rupture)
Car horn @ 1m	~110 dB (borderline painful)
Live rock band	~100 dB
MRI gradients full load	**≤99 dB**
Garbage disposal	~80 dB
MRI gradients basic load	**≤75 dB**
Radio or TV Audio	~70dB

Siemens: mri-magnetom-essenza-environmental_product_declaration-00079271.pdf

Gradient Safety – G_{Max}

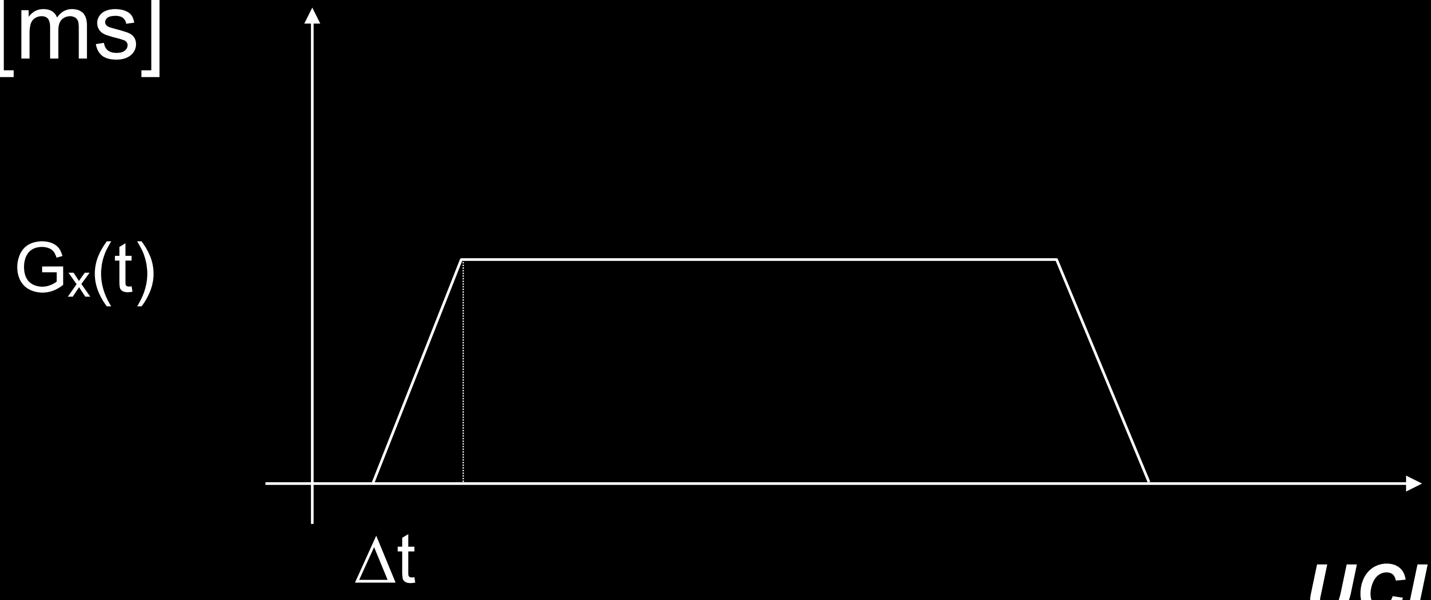
- **G_{max} limitations:**
  - Concern: None known.
    - B_0 is already pretty big.
  - Conventional Gradients
    - G_{Max} = 4 to 5G/cm (=50mT/m)
  - Cutting Edge Gradients
    - G_{Max} = 8G/cm (=80mT/m)
  - Connectome Gradients
    - G_{Max} = 30G/cm (=300mT/m)
  - Consider the ΔB contributed by a gradient...



http://www.aapm.org/meetings/02AM/pdf/8356-48054.pdf

Gradient Slewrate

- **Gradient slew rate**
  - T/m/s (or G/cm/s)
  - dG/dt – Rate of change of gradient amplitude
- **Slew rate limited by dB/dt:**
  - Concern: Peripheral Nerve Stimulation
  - Regulated by FDA
  - Normal Mode: dB/dt=16 T/s•(1+0.36/ß)
  - First Level Mode: dB/dt=20 T/s•(1+0.36/ß)
  - ß=stimulus duration [ms]



Questions?

- Related reading materials
  - Liang/Lauterbur - Chap 4.4.1
  - Nishimura - Chap 5.1.2, 7.3.2

Kyung Sung, Ph.D.
KSung@mednet.ucla.edu
http://mrrl.ucla.edu/sunglab