
Fast Imaging, Advanced Image Reconstruction

M219 Principles and Applications of MRI

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Class Business

- Syllabus and materials
 - https://mrri.ucla.edu/pages/m219_2022

Outline

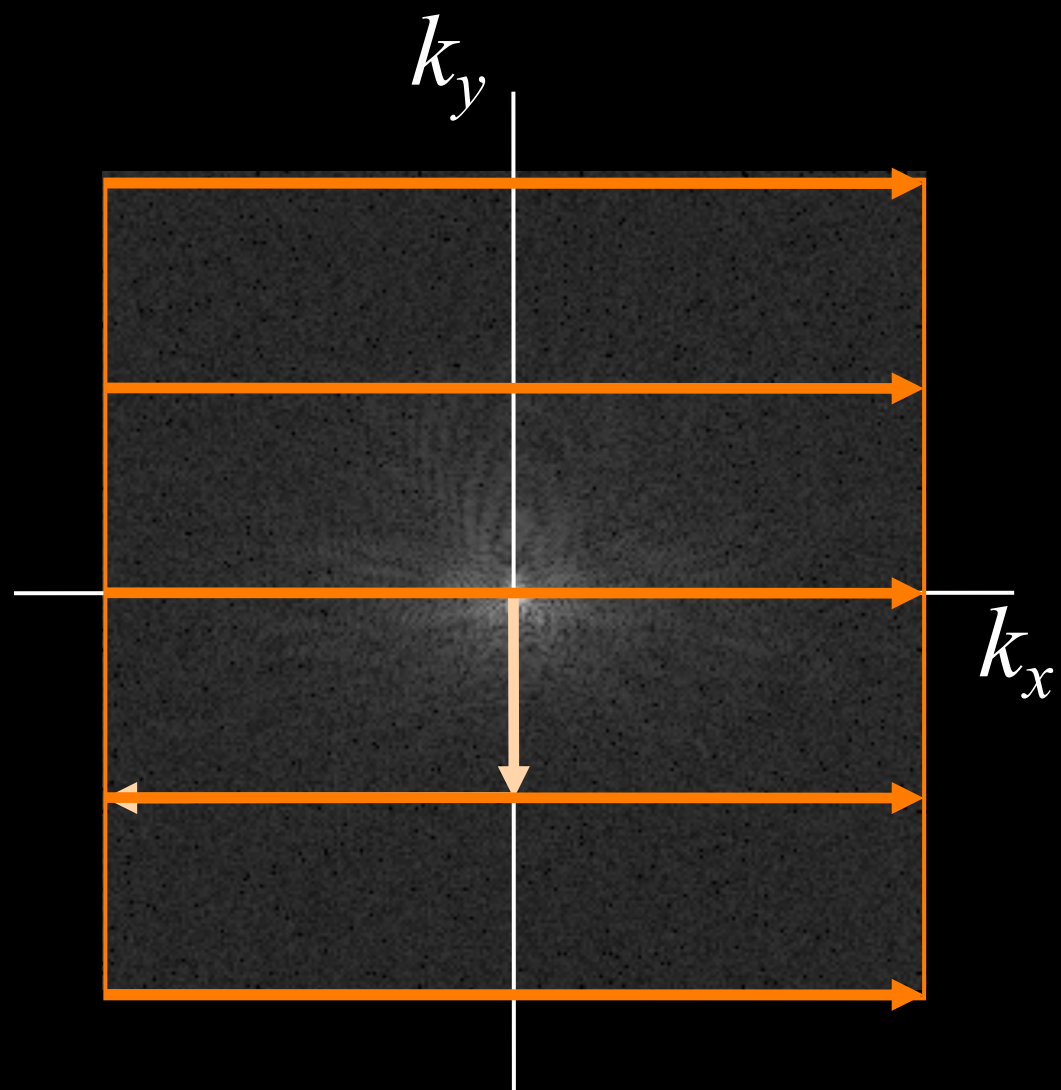
- Fast Imaging
 - Non-Cartesian MRI
 - Echo-planar imaging (EPI)
- Advanced MR Image Reconstruction
 - Parallel imaging
 - Compressed sensing

Overview

- Motivation
 - MRI is relatively slow; need to accelerate
- Strategies
 - Efficient pulse sequences
 - Fast k-space sampling trajectories
 - Data undersampling + advanced recon
- Many challenges and trade-offs
- Key drivers for MRI research

Fast Imaging

k-Space Sampling



set of $s(t)$ covers $m(k_x, k_y)$

Pulse Sequence Diagram

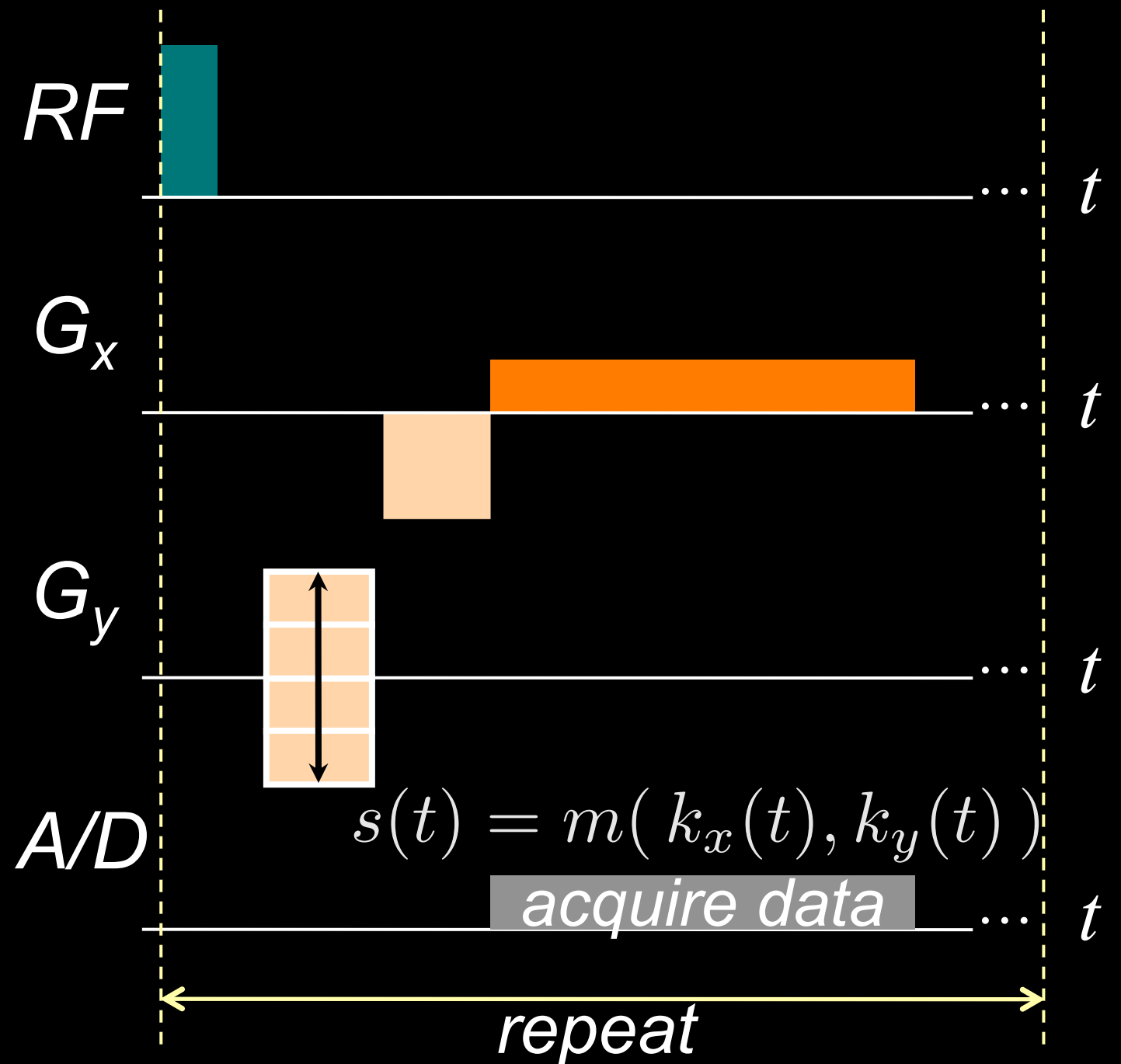
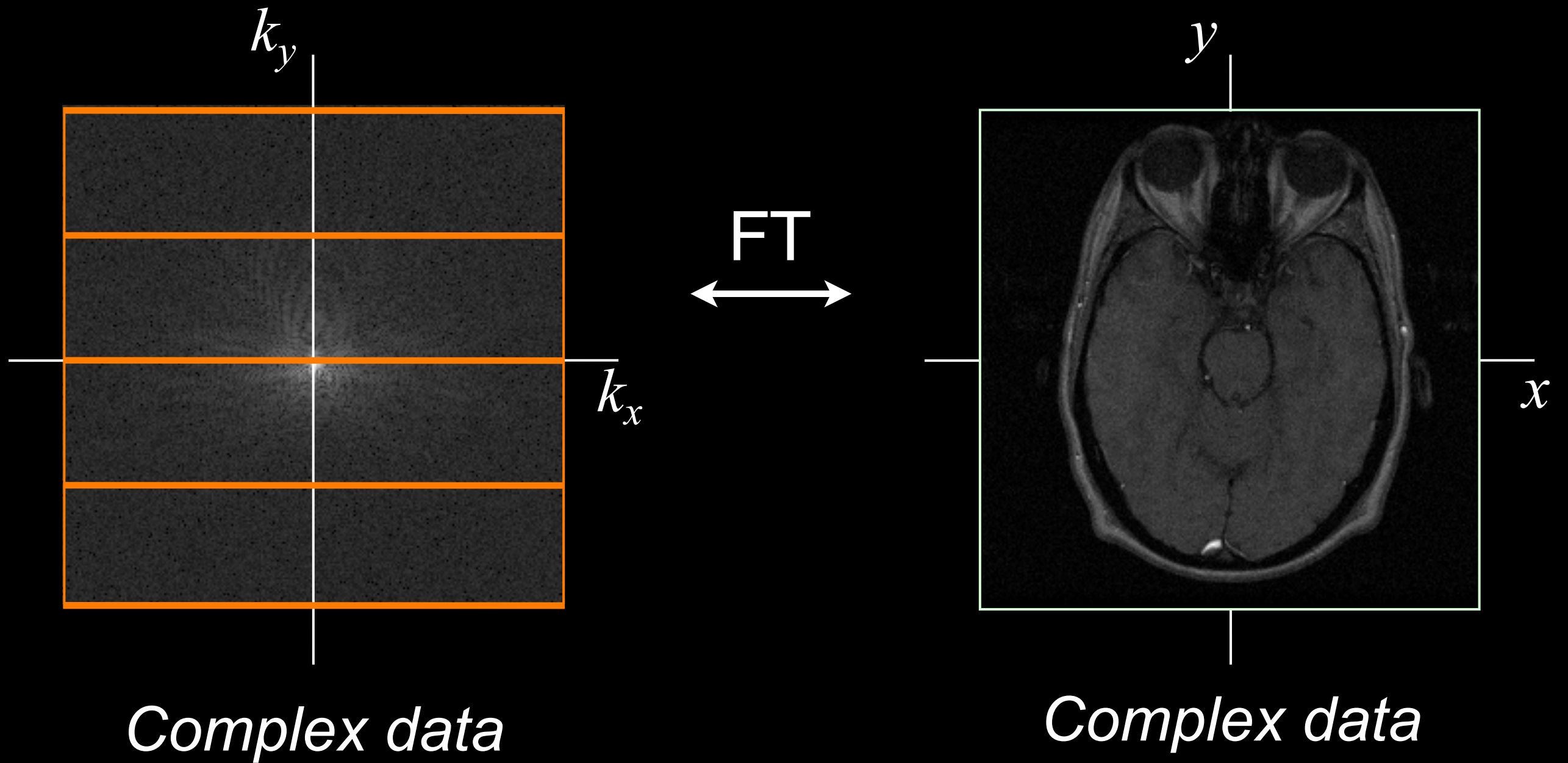
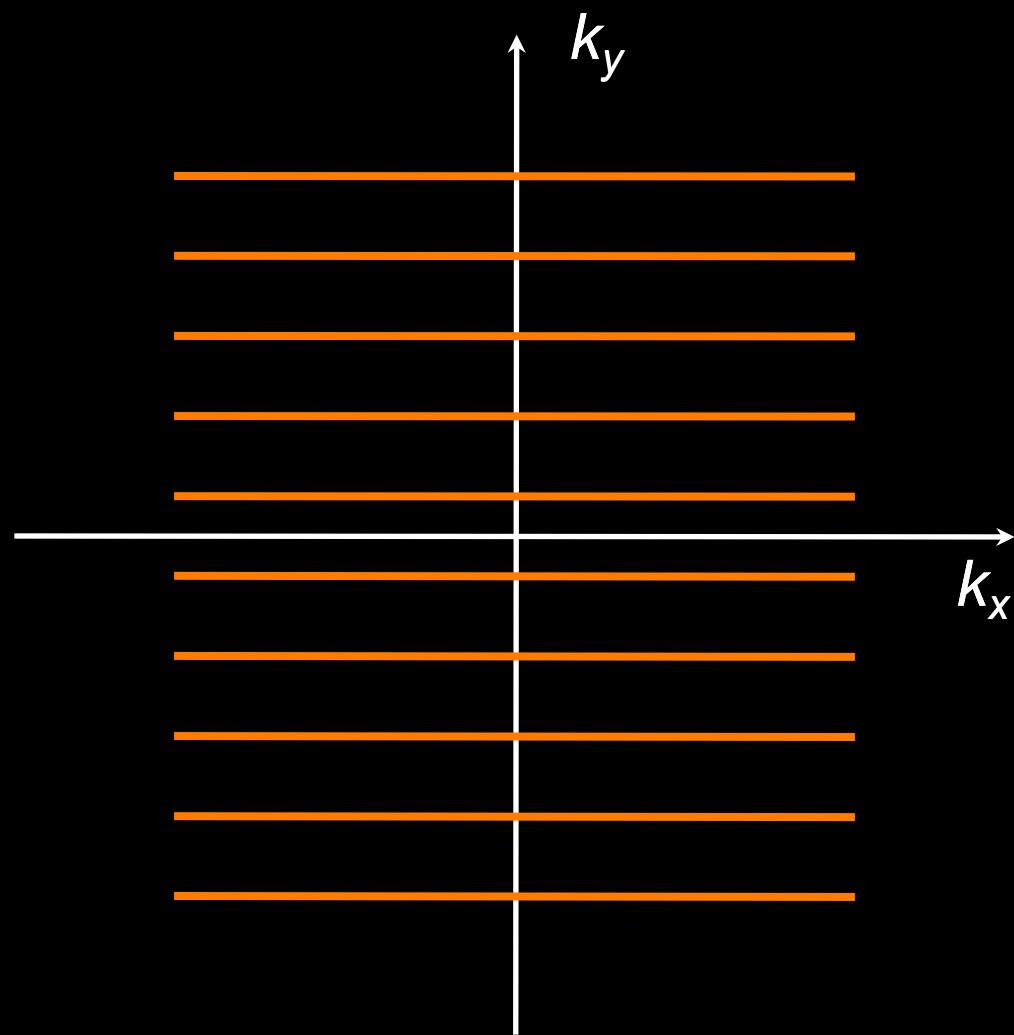


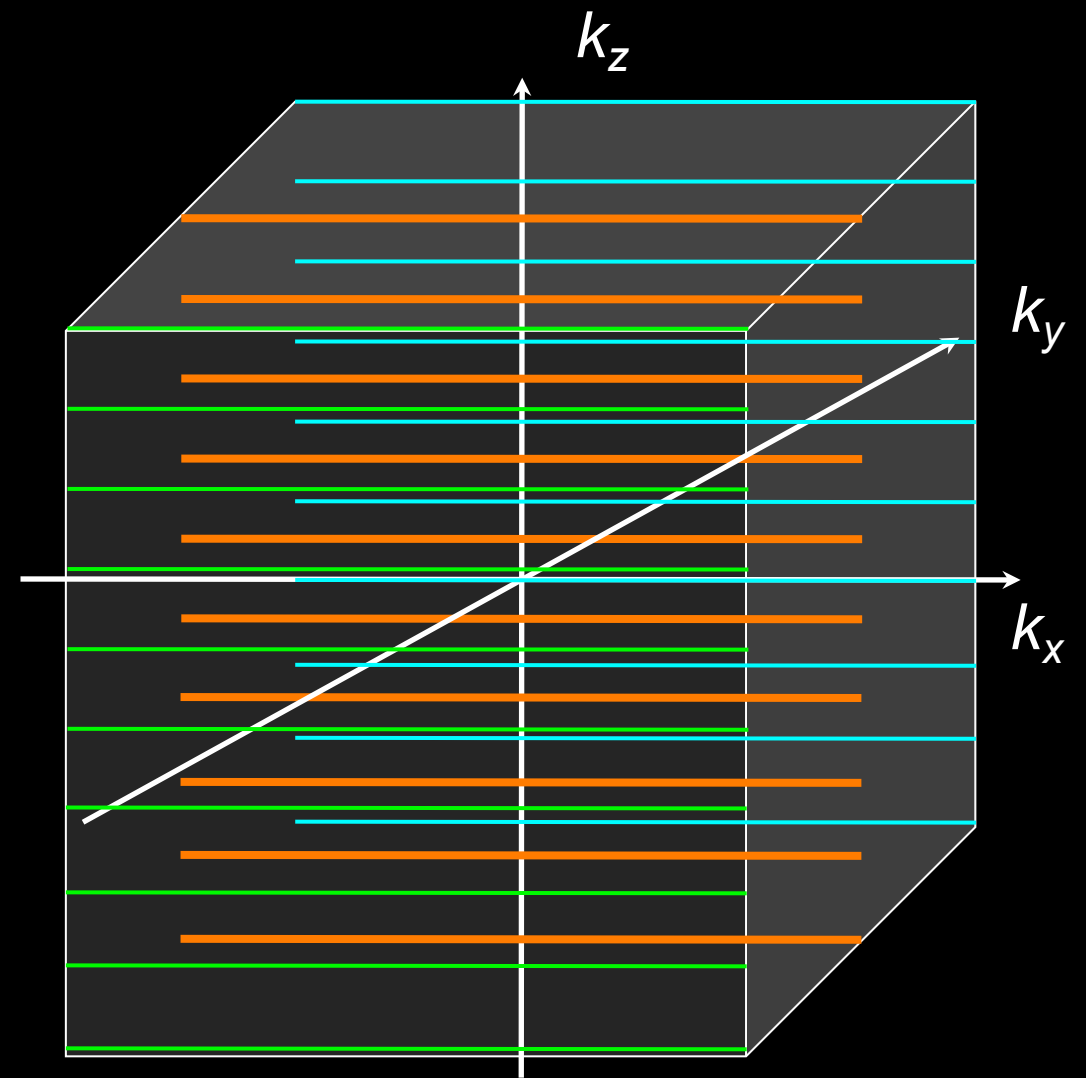
Image Reconstruction



Cartesian Sampling



Cartesian 2DFT



Cartesian 3DFT

MR Signal Equation

$$s(t) = \iint_{X,Y} M(x, y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) dx dy$$

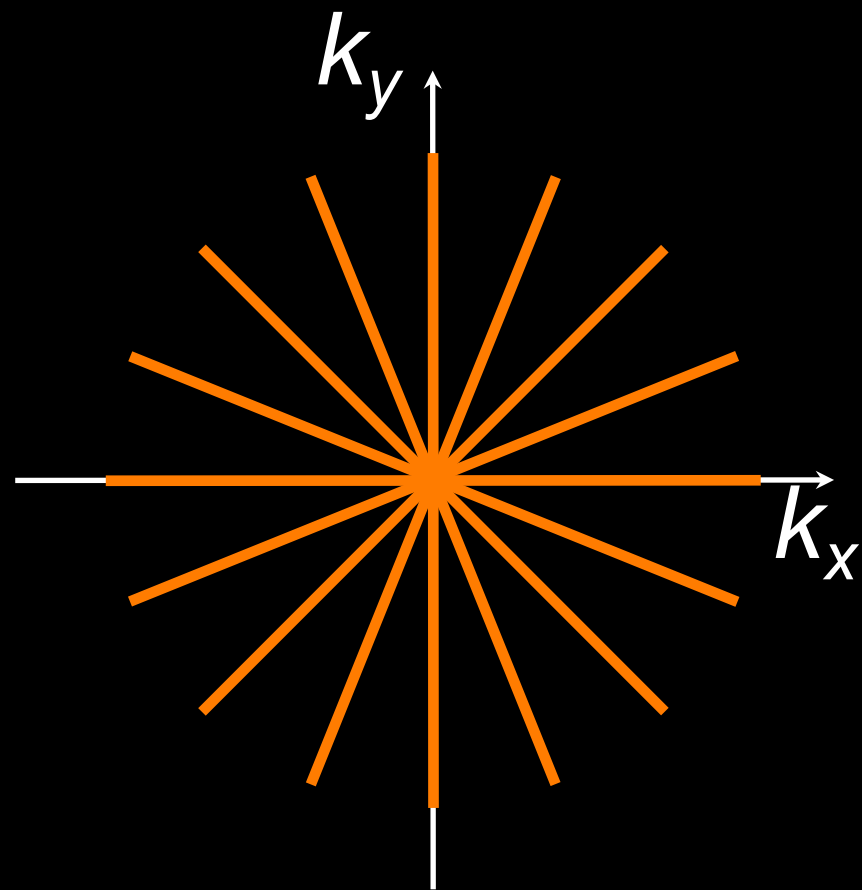
$$= m(k_x(t), k_y(t))$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t, \quad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

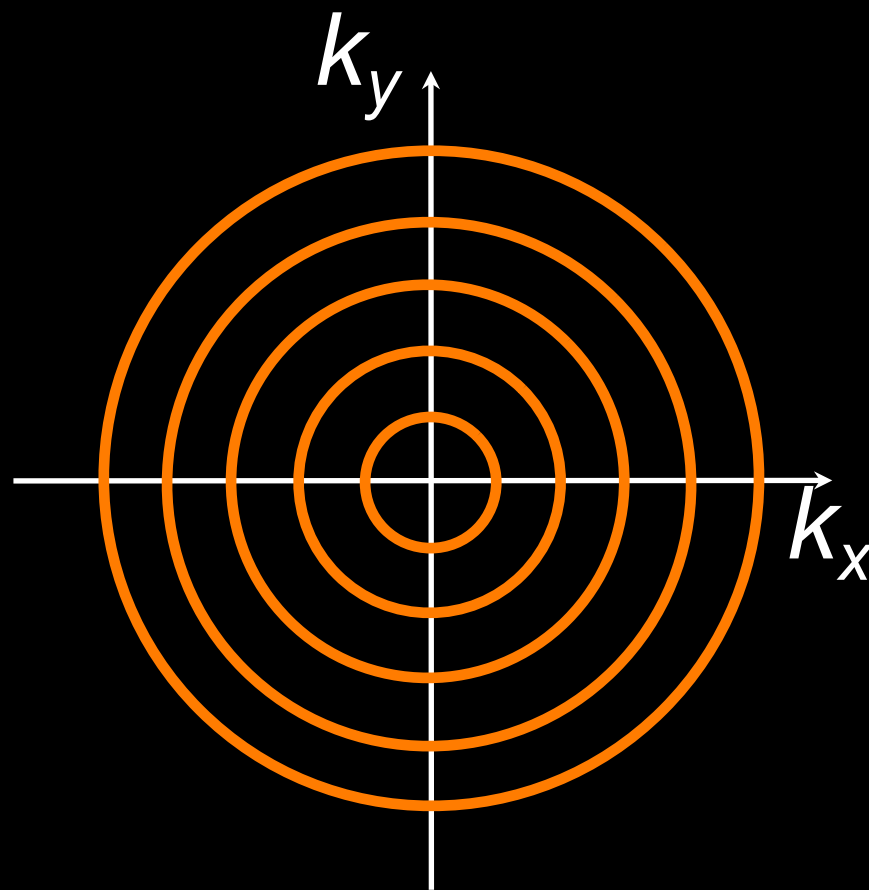
$$m = \mathcal{FT}(M(x, y))$$

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau, \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$$

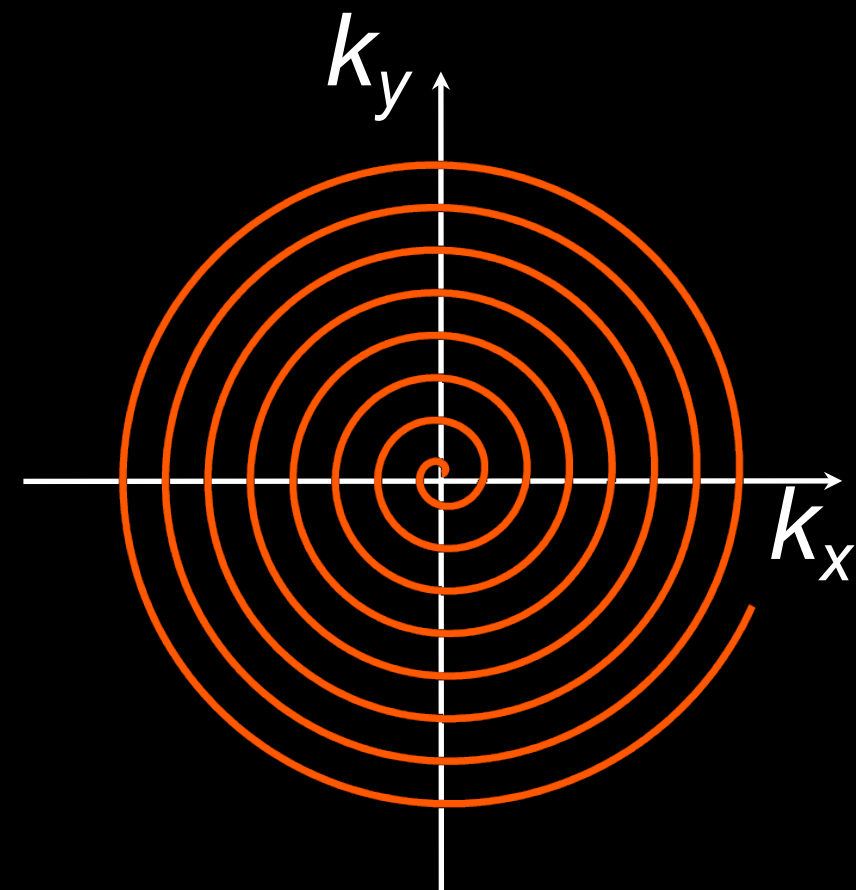
Non-Cartesian Sampling



2D Radial



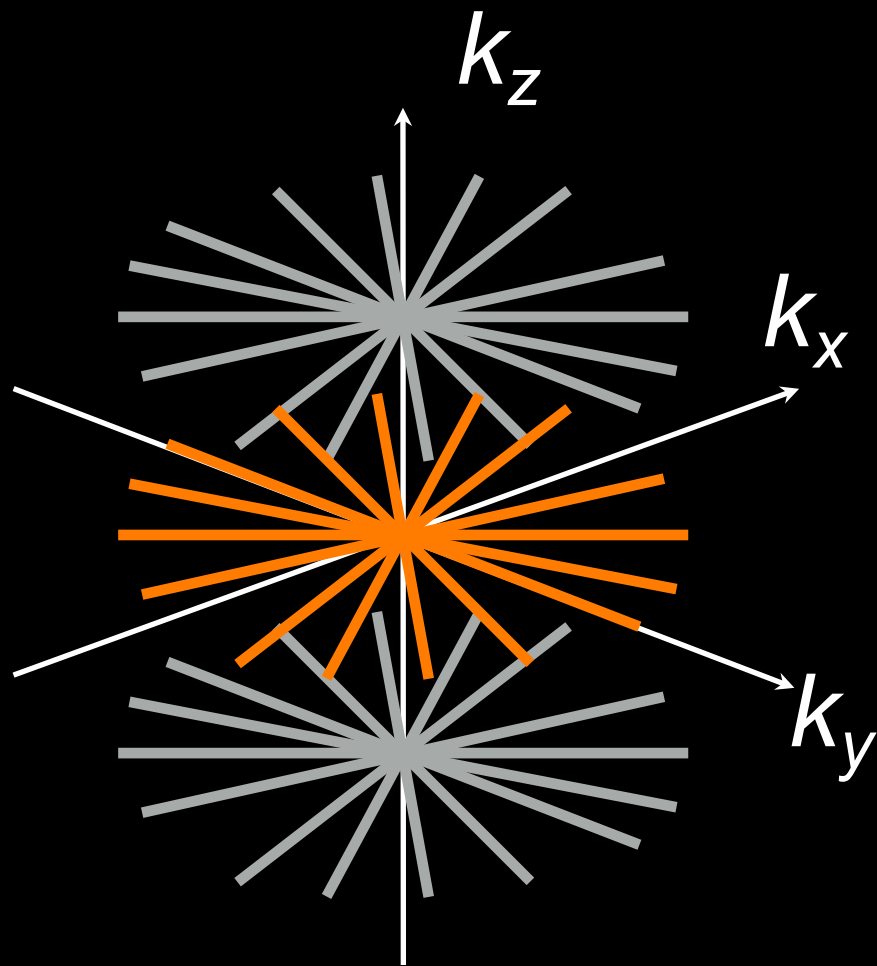
2D Concentric Rings



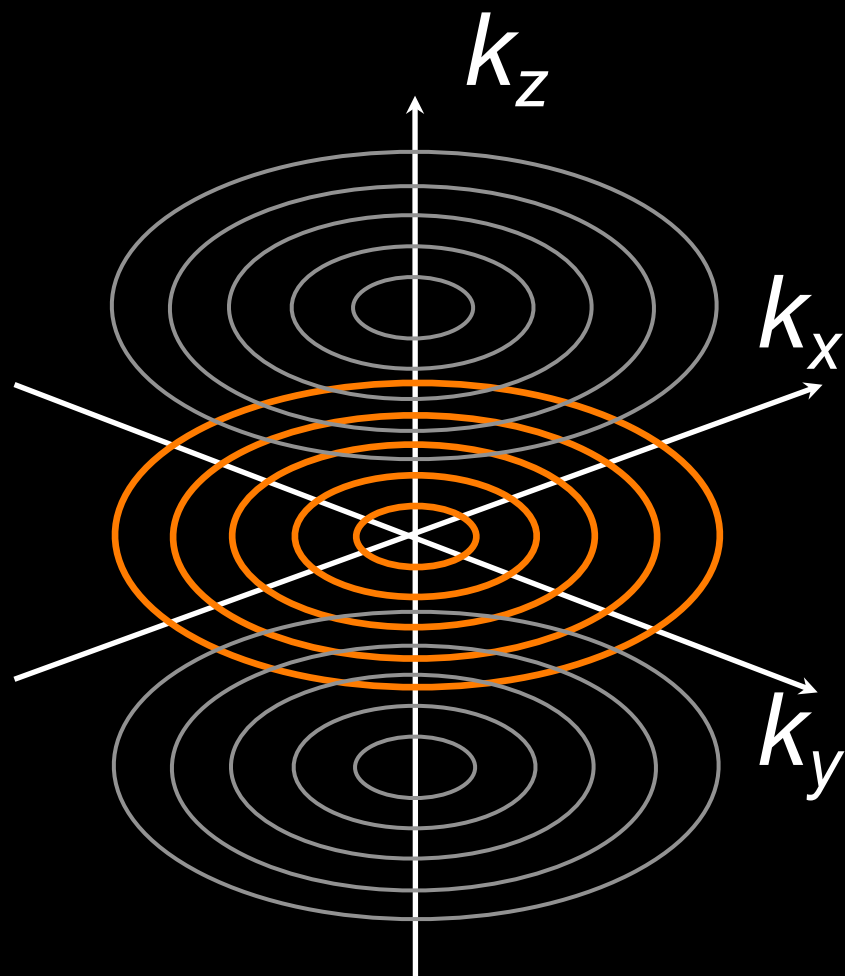
2D Spiral

and much more ...

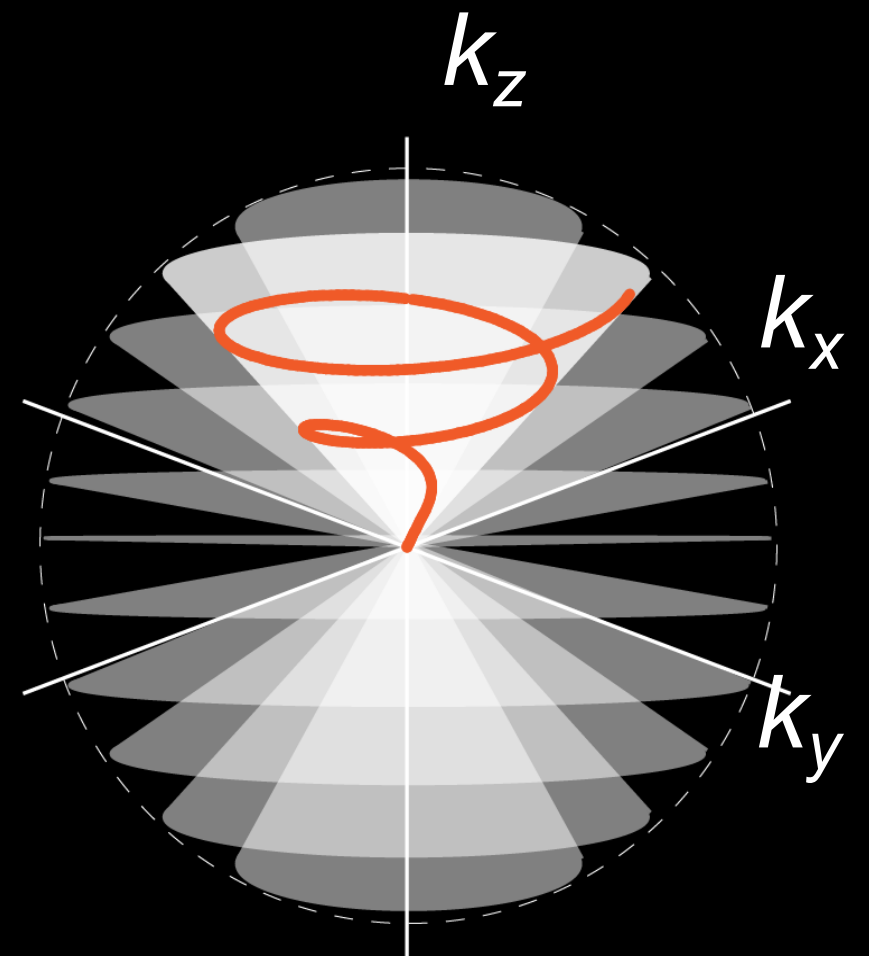
Non-Cartesian Sampling



3D Stack of Stars



3D Stack of Rings



3D Cones

and much more ...

Radial: Real-time MRI

Radial FLASH

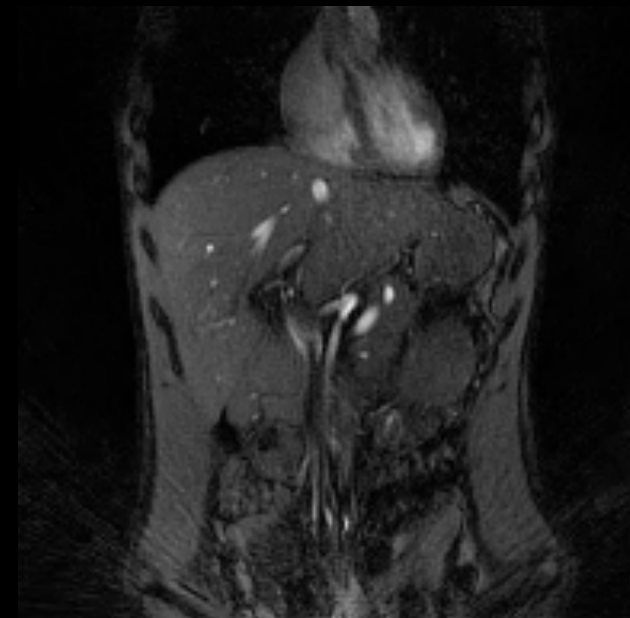
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms
(1 spoke per TR)
- 3.0 T

Reconstruction

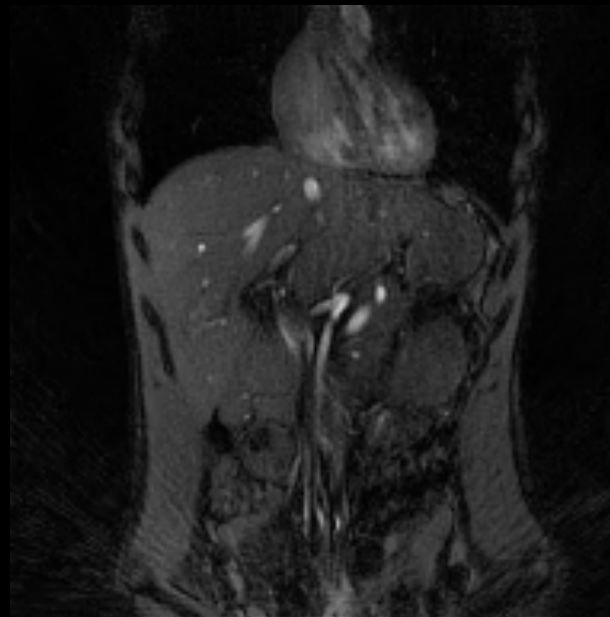
- sliding window of 20 TRs
(display at 16 frames/sec)
- **parallel imaging (SPIRiT)**
(300 spokes for Nyquist)



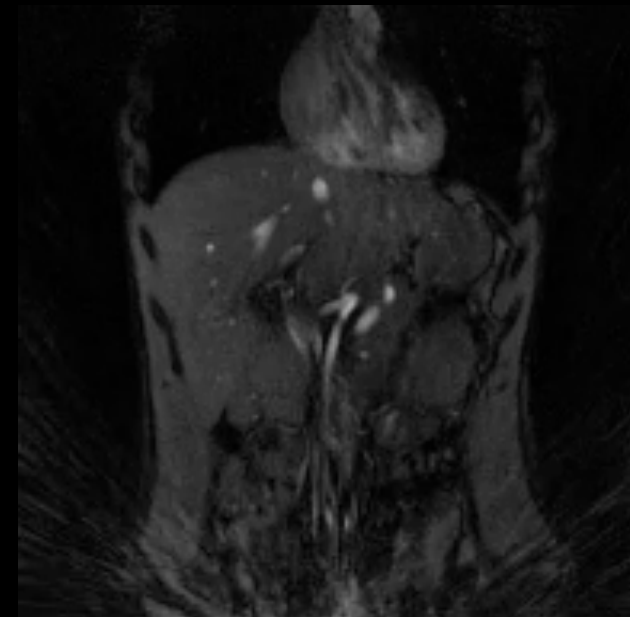
255 spokes/frame
(791 ms/frame)



144 spokes/frame
(446 ms/frame)



89 spokes/frame
(276 ms/frame)



55 spokes/frame
(171 ms/frame)

courtesy of Samantha Mikael

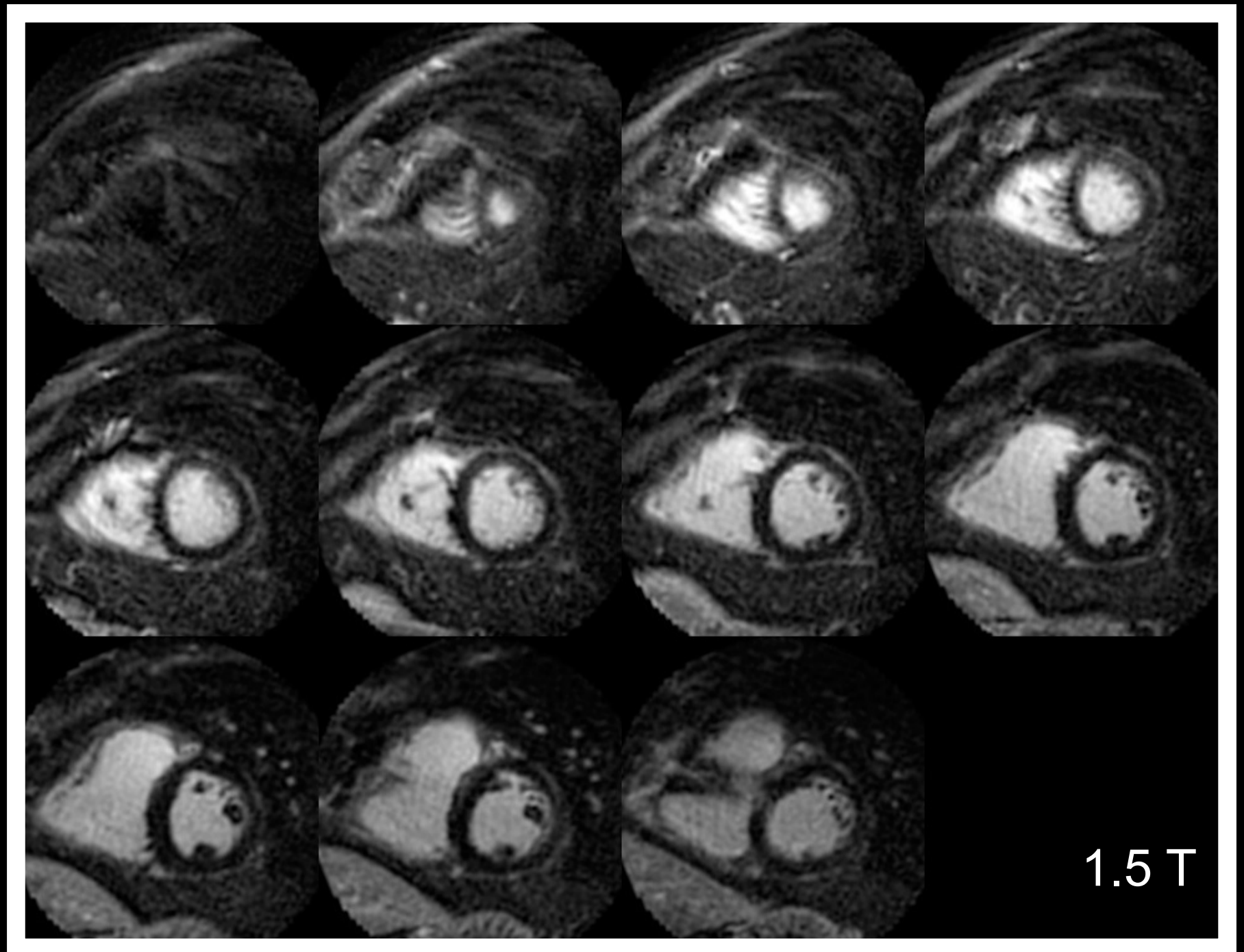
Spirals: 3D LGE MRI

3D Spiral IR-GRE

- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

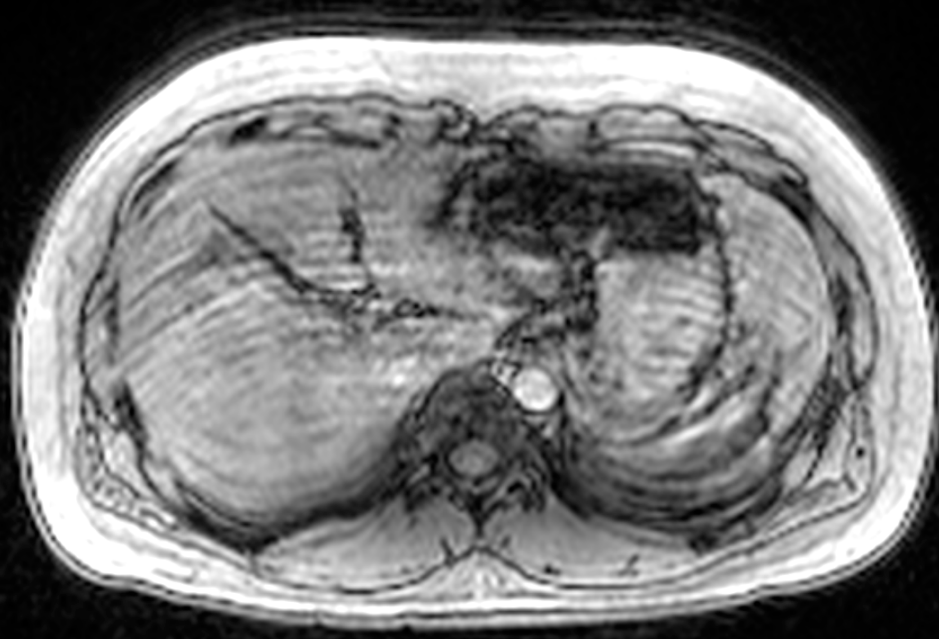
Reconstruction

- SPIRiT ($R = 2$)
- ~5-sec recon



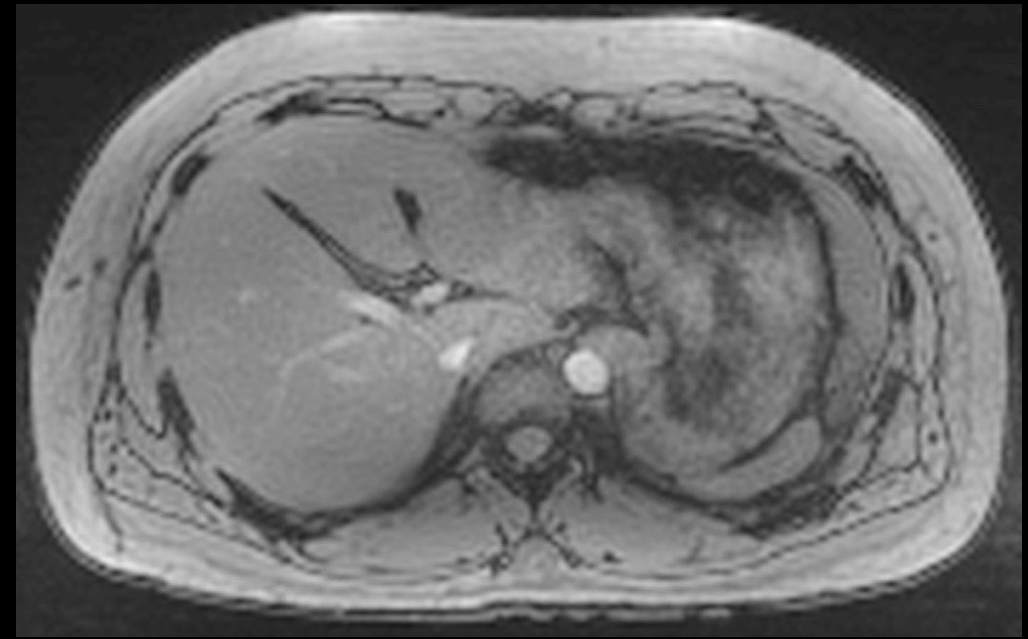
3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

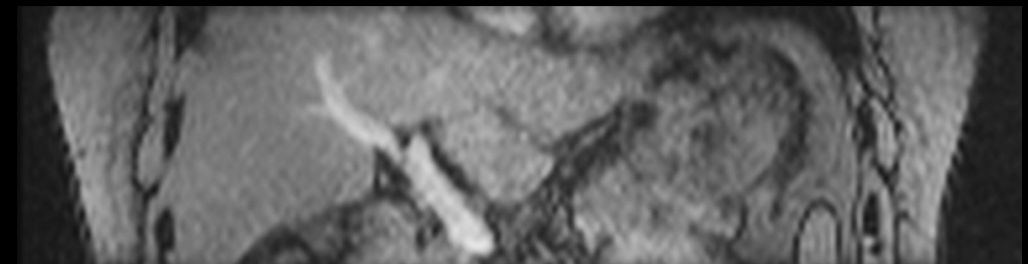


Insufficient breath-holding

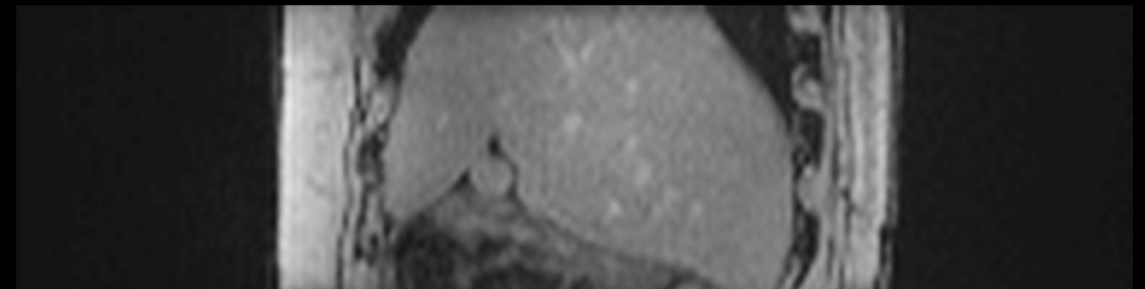
Free-breathing 3D Stack-of-Radial MRI



Axial



Coronal



Sagittal

3D Radial: Coronary MRA

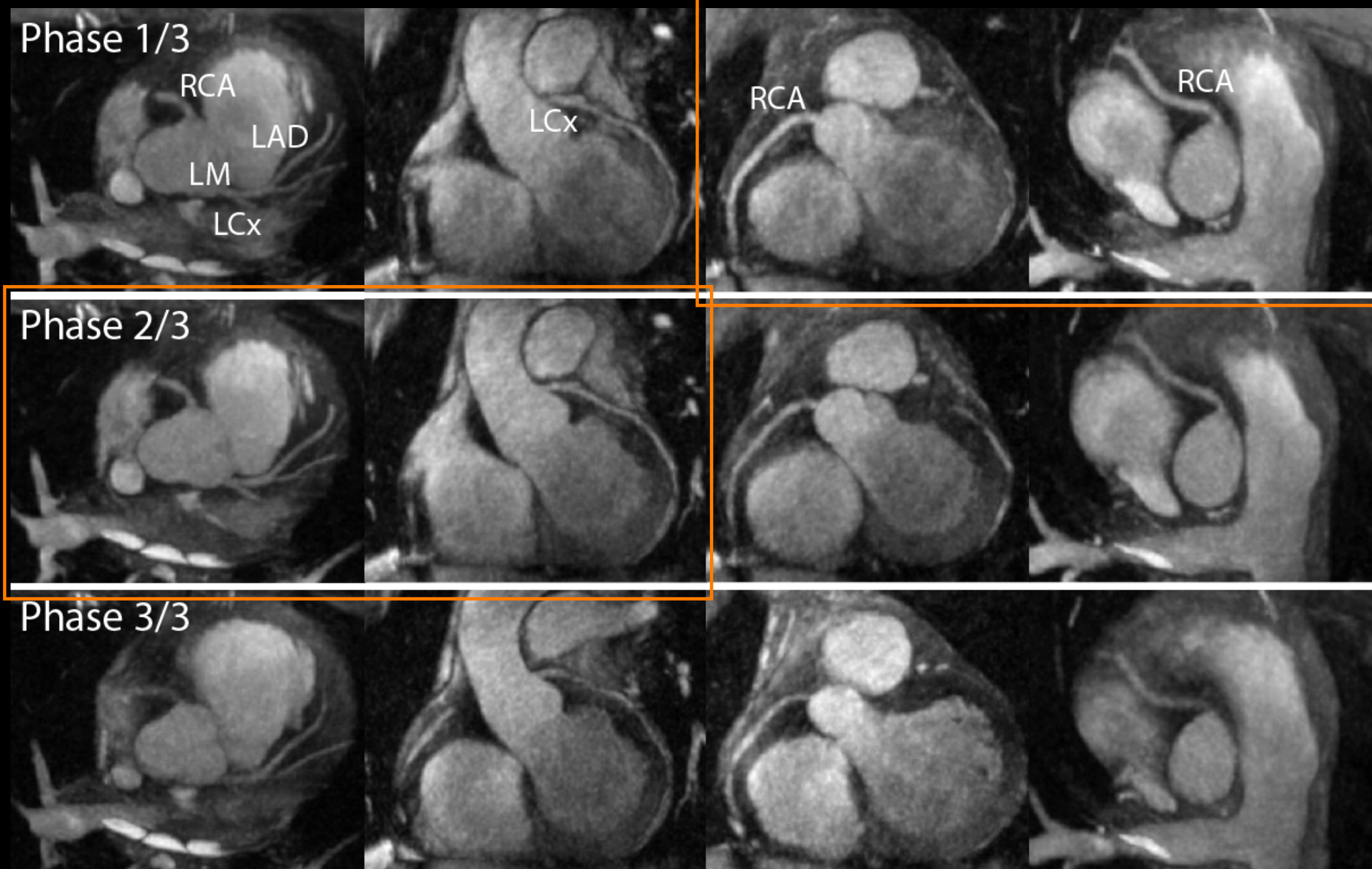
Contrast-Enhanced MRA at 3.0T



ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence
(1.0 mm)³ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

3D Cones: Coronary MRA

Multi-Phase Thin-Slab MIP Reformats

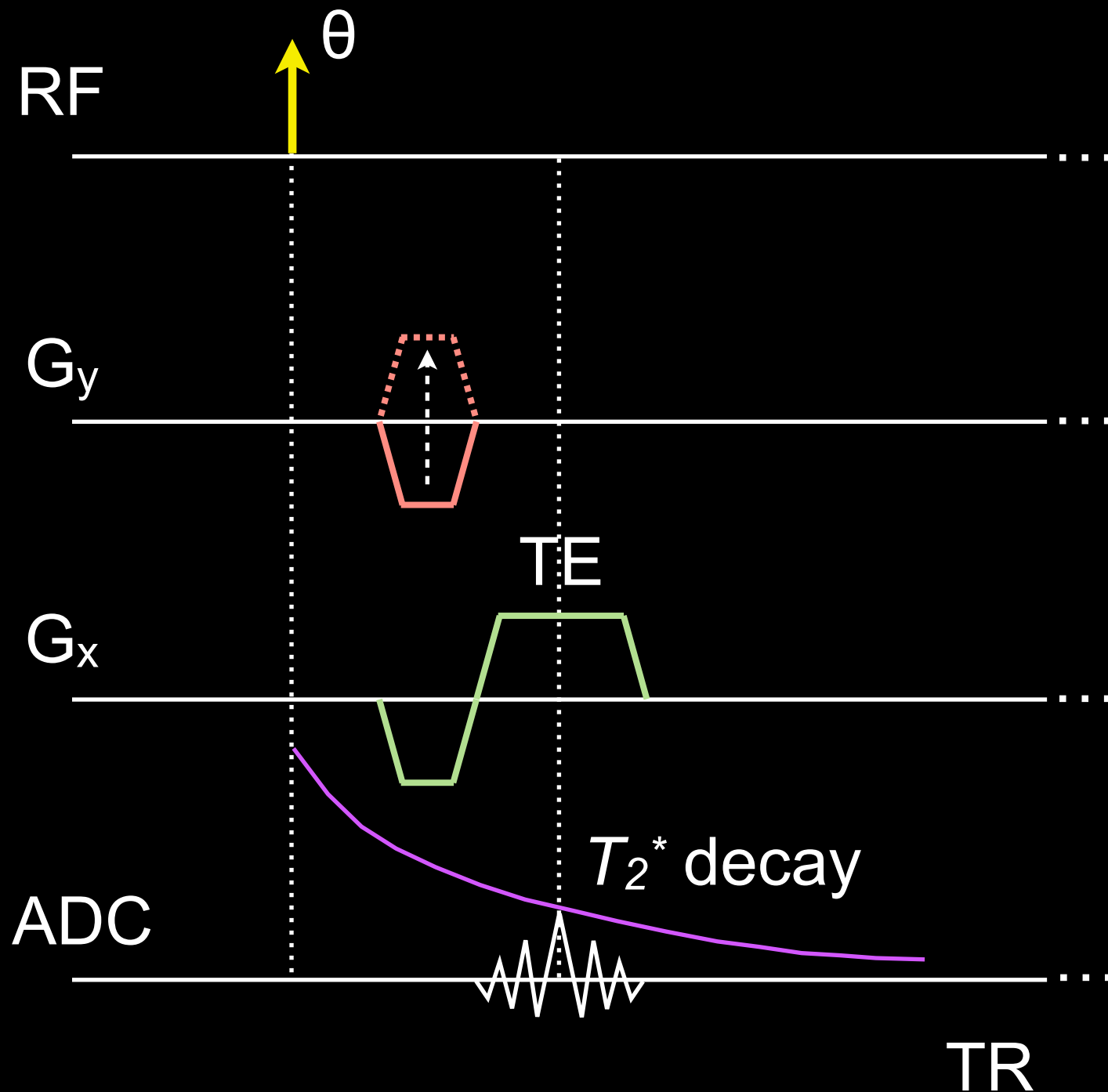


Echo-Planar Imaging

- Echo-Planar Imaging (EPI)¹
- Ultra-fast imaging (<100 ms/frame)
- Imperfections and artifacts
- Ongoing topic of rapid MRI research

¹*Mansfield P, J Phys C: Solid State Phys 1977*

Gradient Echo

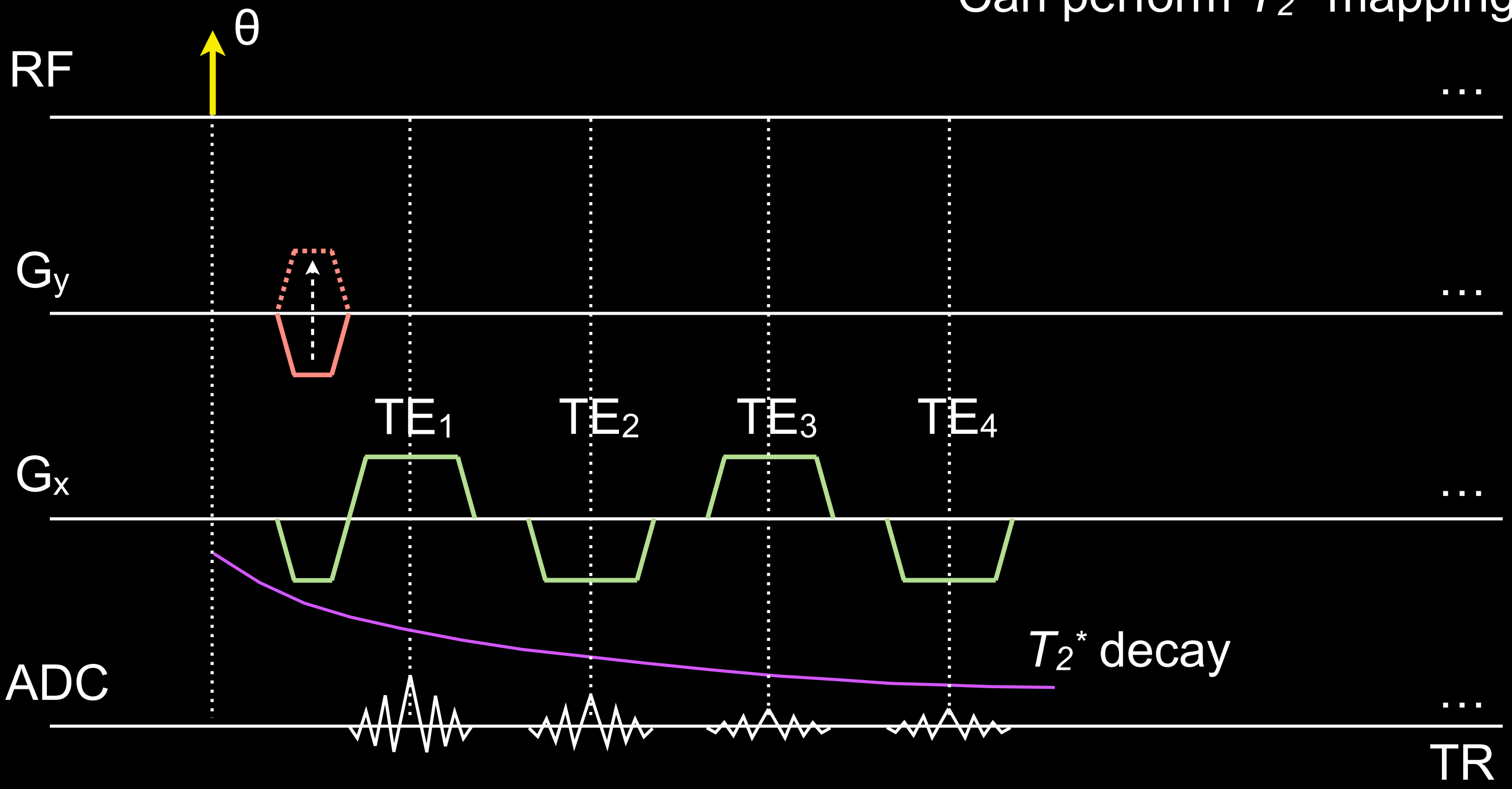


- Utilization of transverse magnetization
 - With $T_s = 8 \mu s$ and $N_x = 128$, $T_{acq} = 1.024 ms$
 - $<2\%$ of T_2^* in brain at 3 T!¹
- Scan time
 - $T_{GRE} = N_{pe} \times TR$
 - $TR = 10 ms$, $N_{pe} = 256$:
 $T_{GRE} = 2.56 sec$

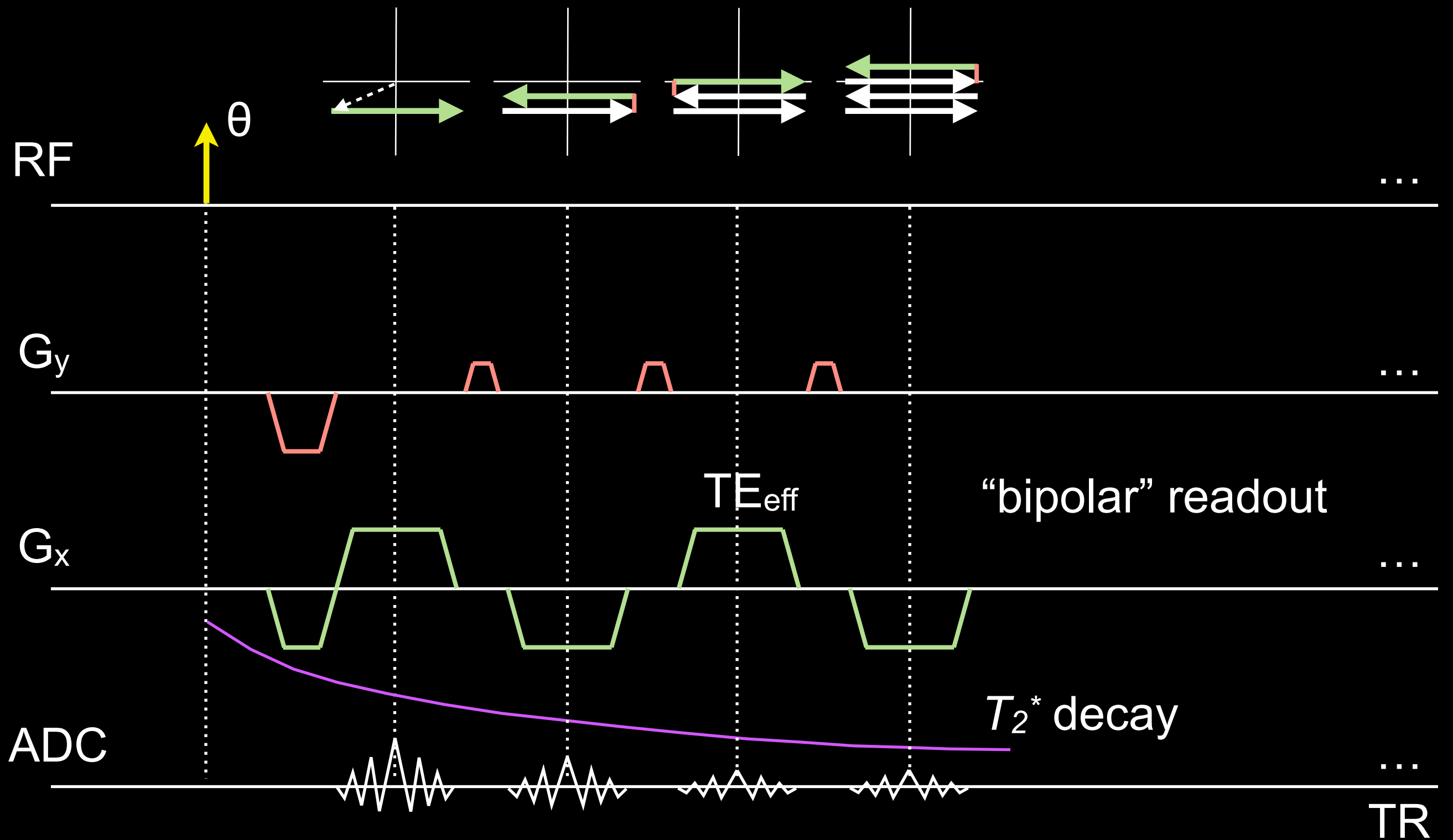
¹Peters, et al., Proc ISMRM 2006

Multi-echo Gradient Echo

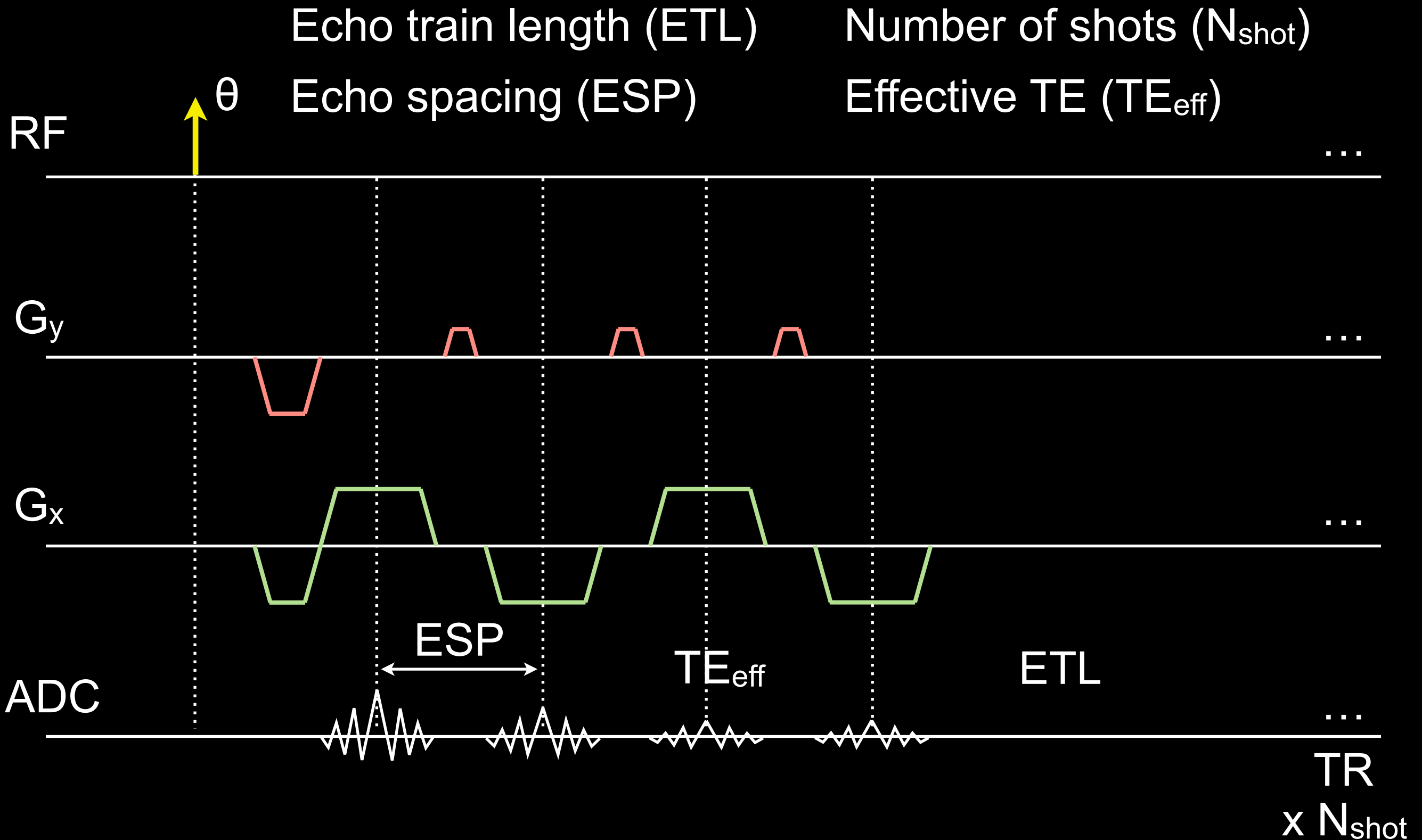
ΔTE can be non-uniform
Can perform T_2^* mapping



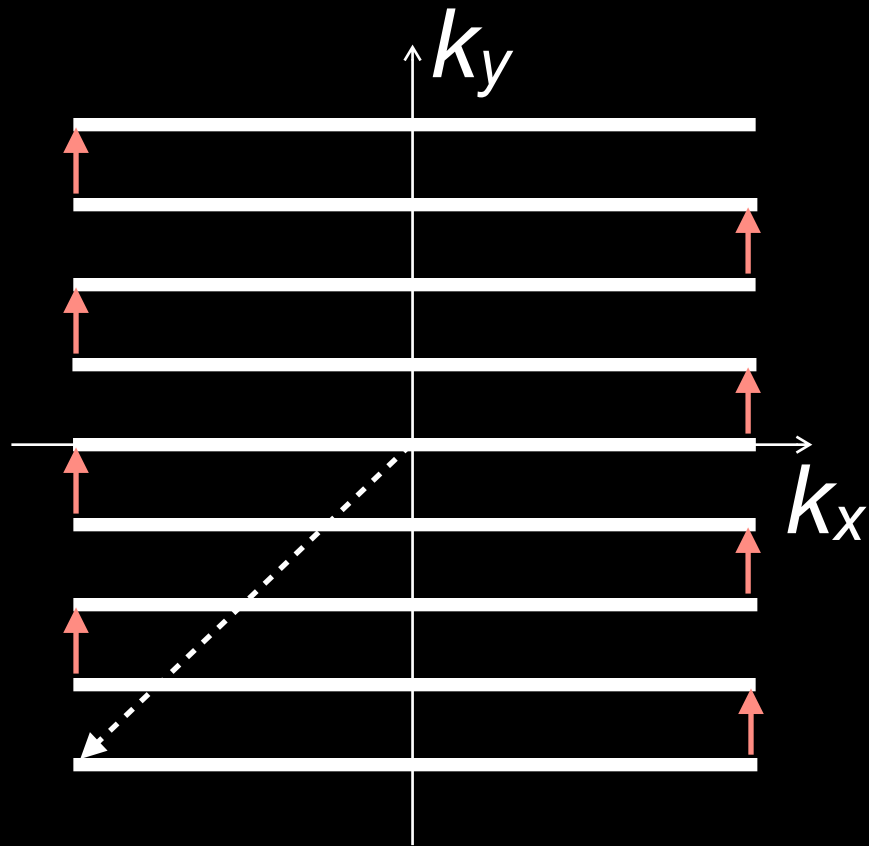
Gradient-Echo EPI



EPI Sequence Parameters



EPI k-Space Sampling



- ETL can be 4-64 or higher
 - Limited by T_2^* decay, off-resonance effects
 - aka “EPI factor”
- ESP typically ~ 1 ms
 - Must accommodate RF, gradients, ADC
 - Short ESP facilitates high ETL

Fast Sampling Trajectories

- **Benefits**

- Reduced scan time
- Robustness to motion and flow
- Short echo time

- **Applications**

- Dynamic MRI
- Real-time MRI
- Cardiovascular MRI
- Short-TE MRI

- **Challenges**

- Hardware performance
- Gradient fidelity
- Off-resonance effects
- Design and implementation

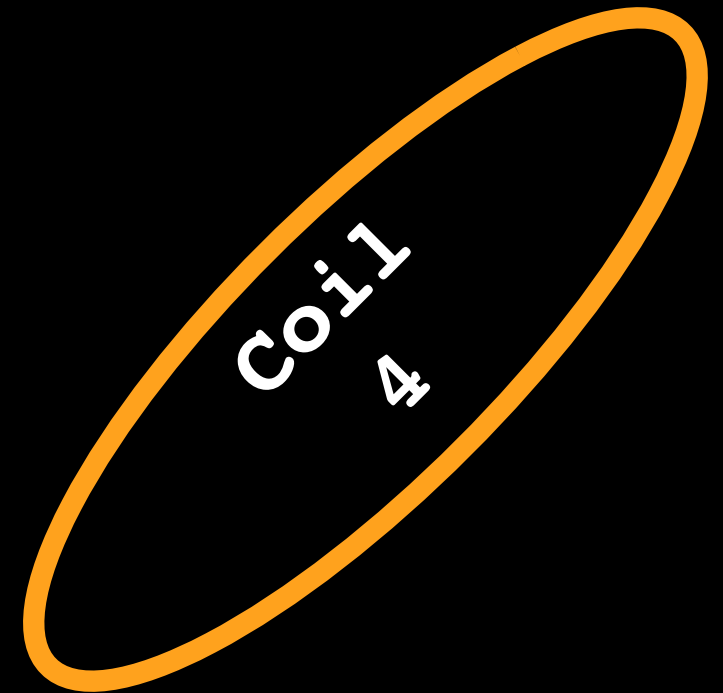
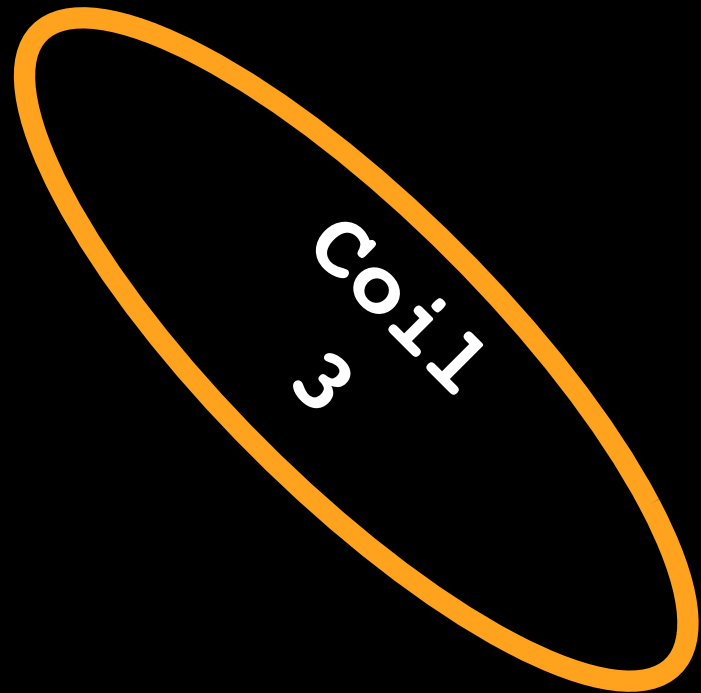
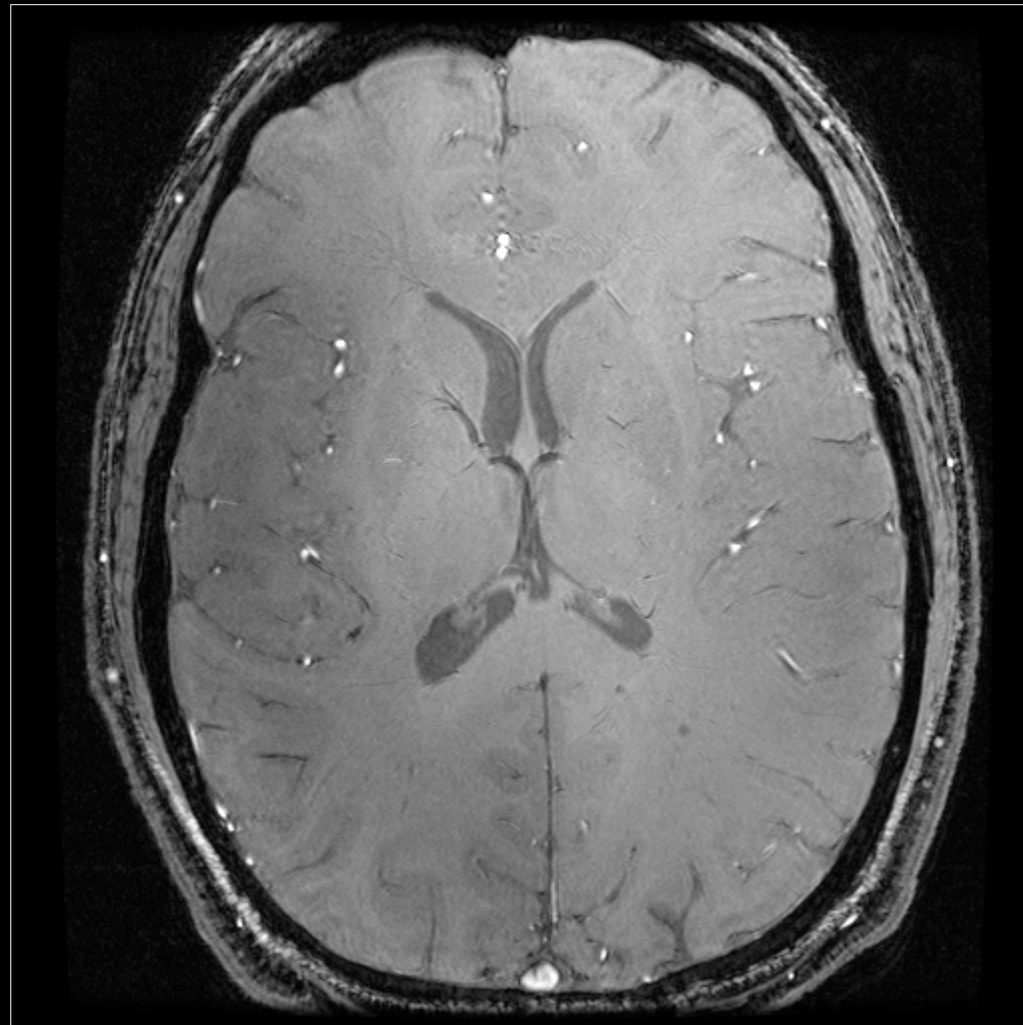
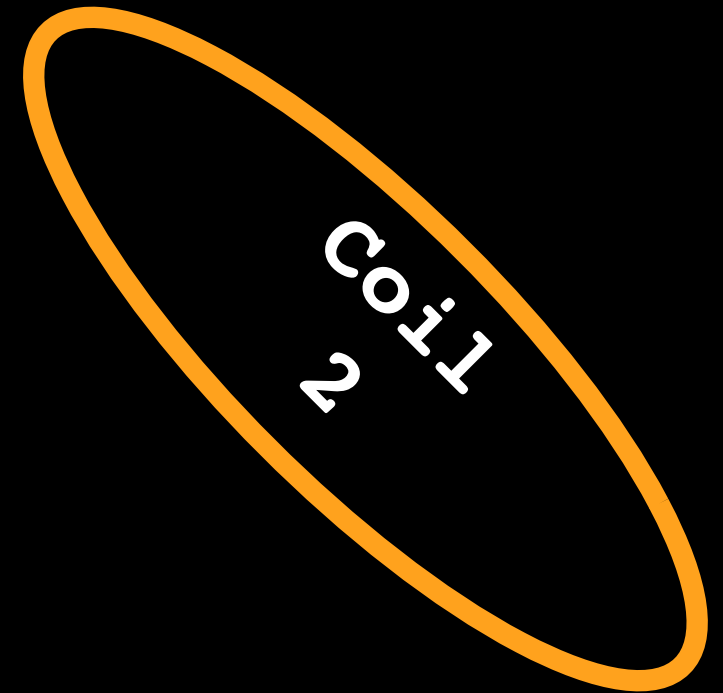
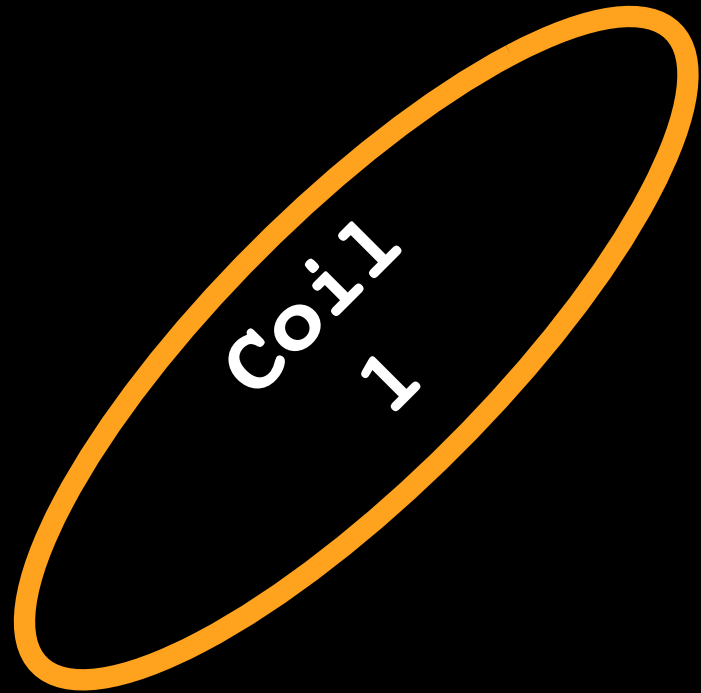
- **Challenges addressed**

- **On-going research**

- **Use judiciously!**

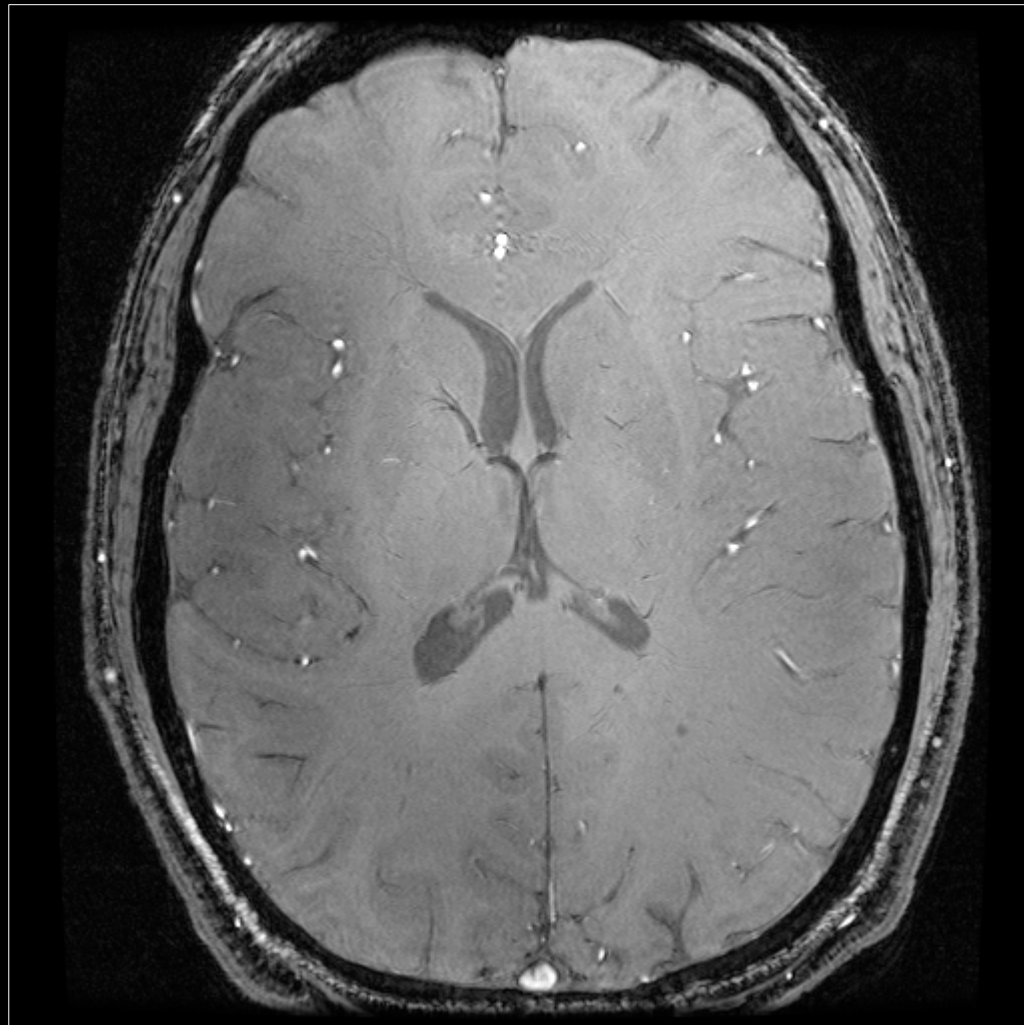
Parallel Imaging

Multi-coil Arrays



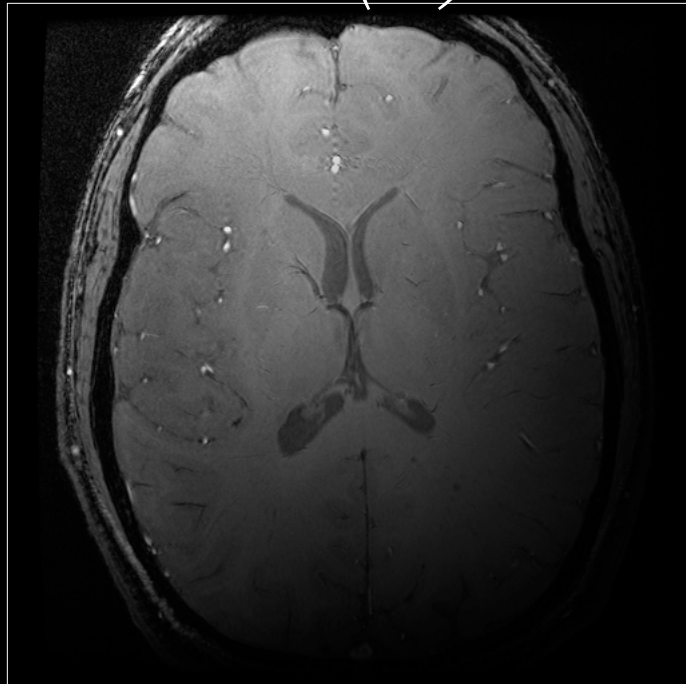
Multi-coil Sensitivity

$$\| \vec{B}(\vec{r}) \|$$

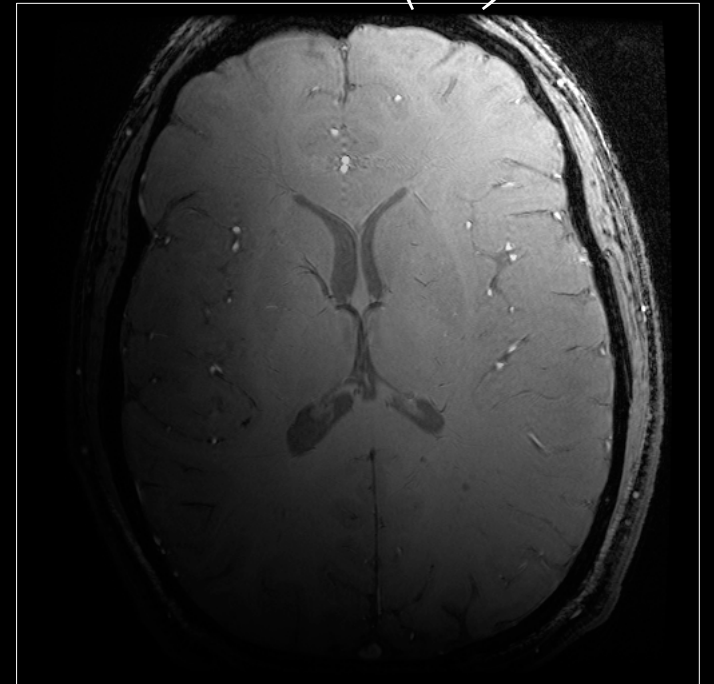


Multi-coil Images

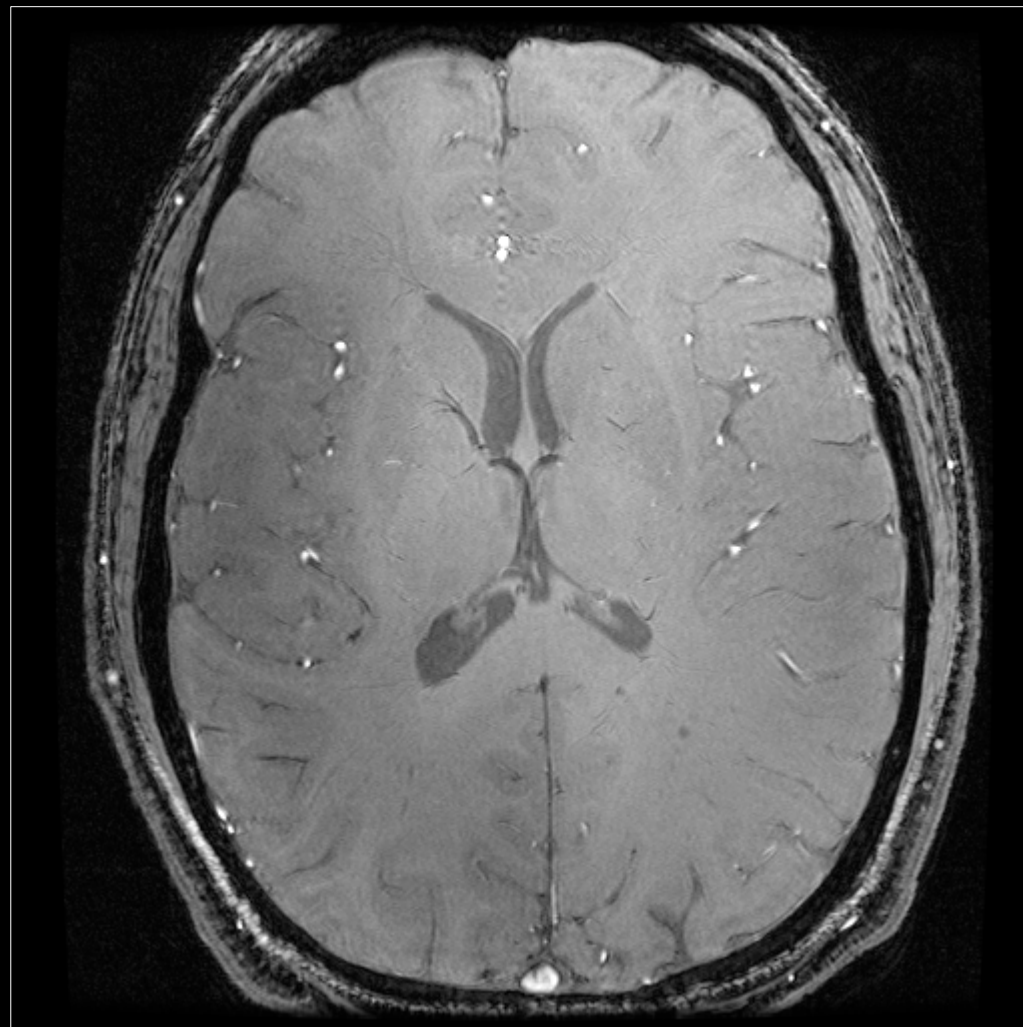
$m_1(x)$



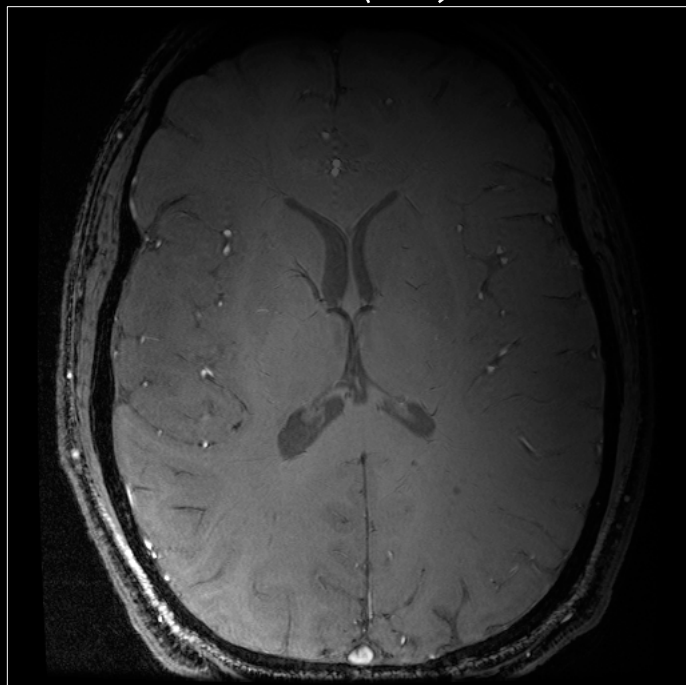
$m_2(x)$



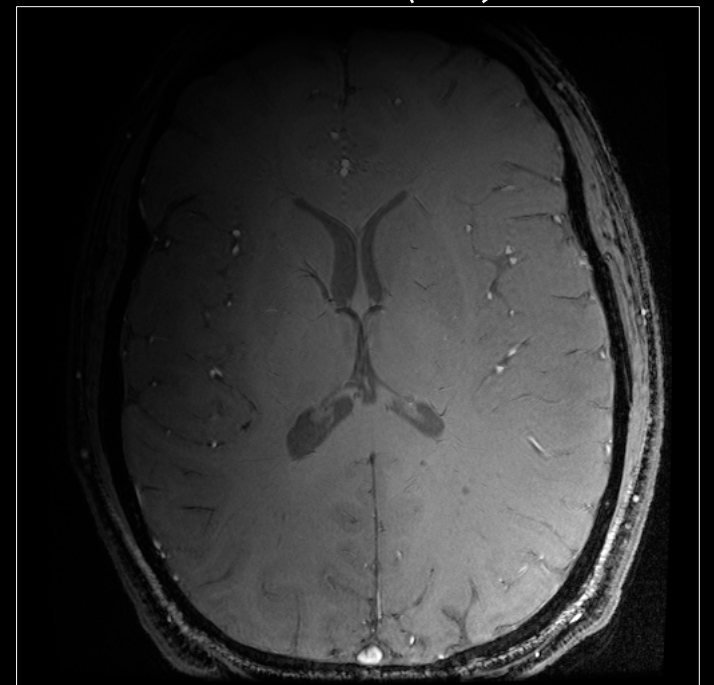
$m_s(x)$



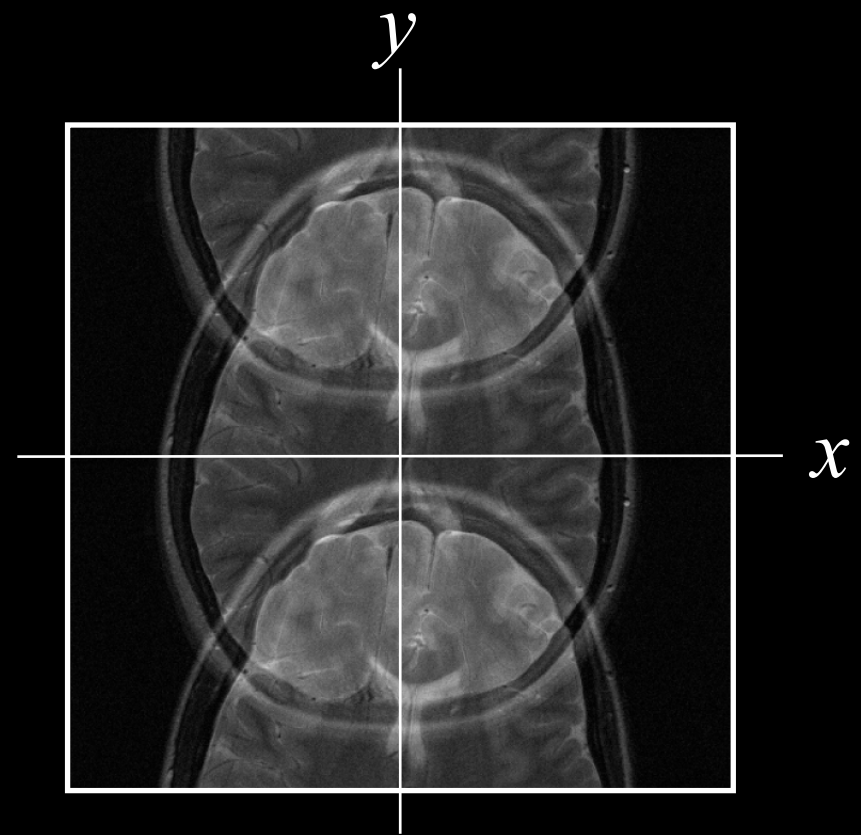
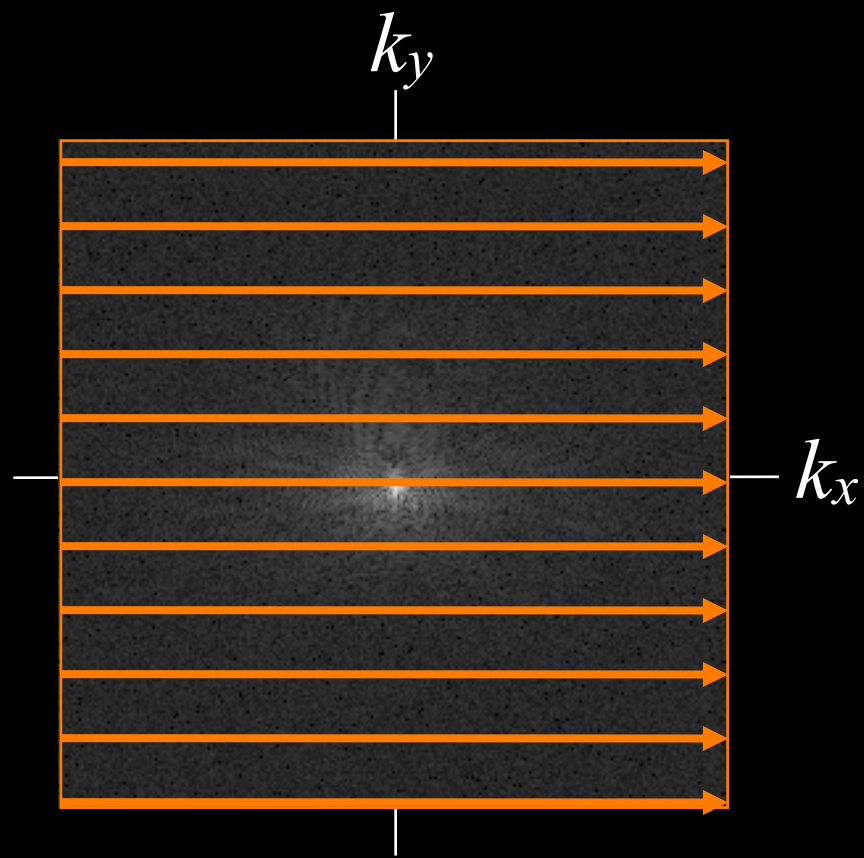
$m_3(x)$



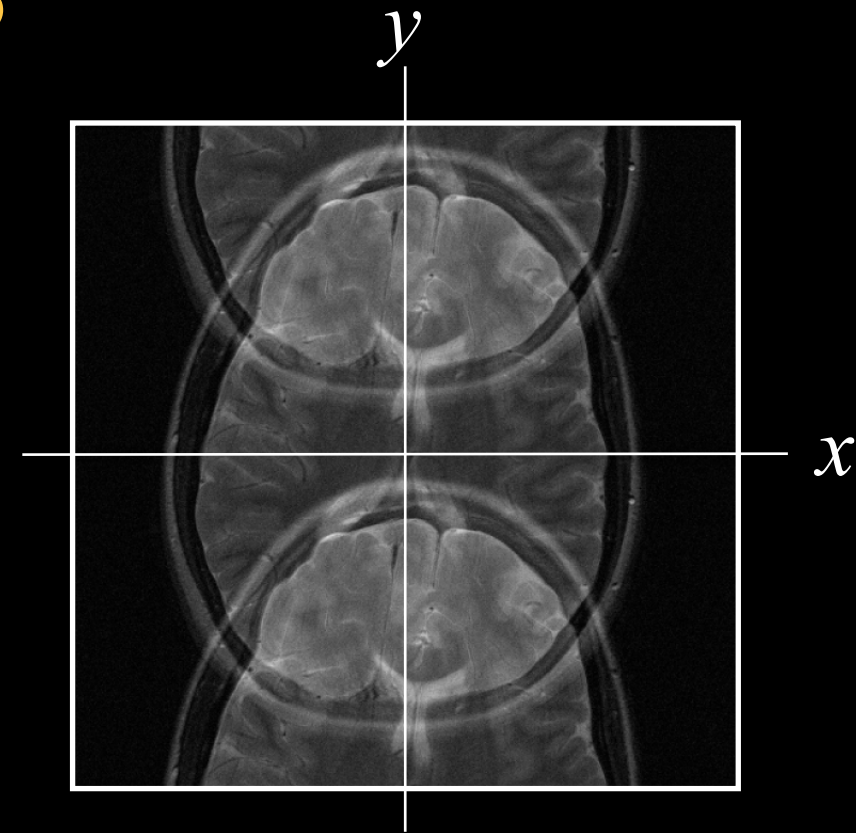
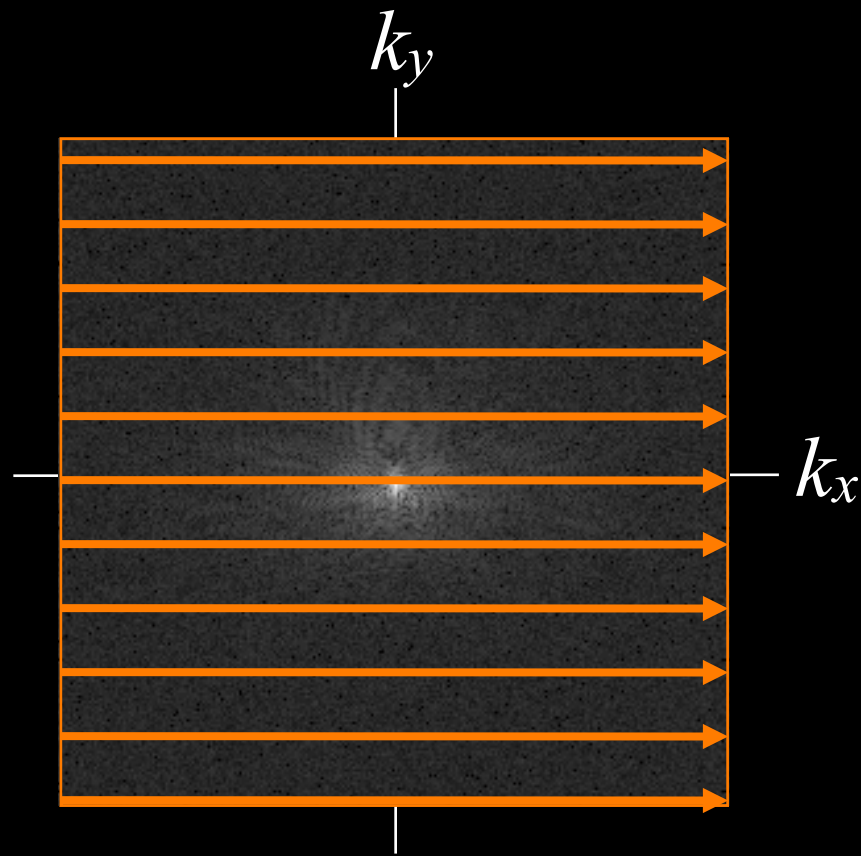
$m_4(x)$



Accelerate Imaging with Array Coils



Accelerate Imaging with Array Coils



- Parallel Imaging
 - Coil elements provide some localization
 - Undersample in k-space, producing aliasing
 - Sort out in reconstruction

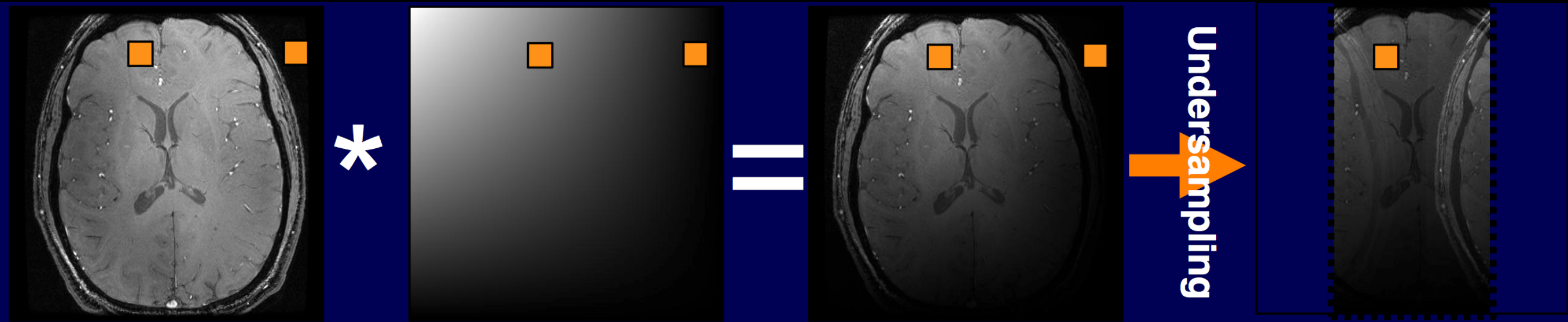
Parallel Imaging

- Many approaches:
 - Image domain - SENSE
 - k-space domain - SMASH, GRAPPA
 - Hybrid - ARC

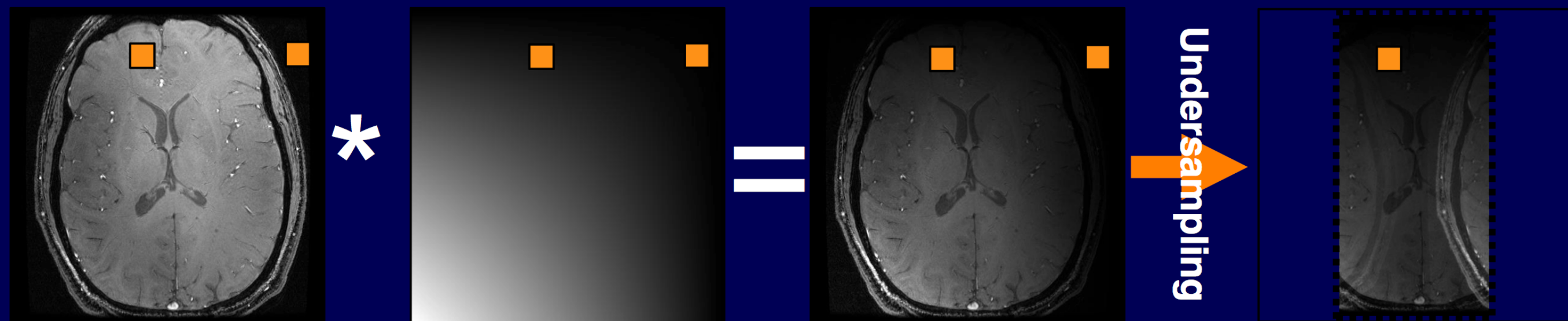
- We will introduce one:
 - SENSE: optimal if you know coil sensitivities

Cartesian SENSE

$$m_1(\vec{x}_1) = C_1(\vec{x}_1)m(\vec{x}_1) + C_1(\vec{x}_2)m(\vec{x}_2)$$



$$m_2(\vec{x}_1) = C_2(\vec{x}_1)m(\vec{x}_1) + C_2(\vec{x}_2)m(\vec{x}_2)$$



$$\begin{pmatrix} m_1(\vec{x}_1) \\ m_2(\vec{x}_1) \\ \cdot \\ \cdot \\ \cdot \\ m_L(\vec{x}_1) \end{pmatrix} = \begin{pmatrix} C_1(\vec{x}_1) & C_1(\vec{x}_2) \\ C_2(\vec{x}_1) & C_2(\vec{x}_2) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ C_L(\vec{x}_1) & C_L(\vec{x}_2) \end{pmatrix} \begin{pmatrix} m(\vec{x}_1) \\ m(\vec{x}_2) \end{pmatrix} + \begin{pmatrix} n_1(\vec{x}_1) \\ n_2(\vec{x}_1) \\ \cdot \\ \cdot \\ \cdot \\ n_L(\vec{x}_1) \end{pmatrix}$$

Aliased
Images

Sensitivity at
Source Voxels

Source
Voxels

OR

$$\begin{matrix} & & 2 \times 1 \\ m_s = & C & m + n \\ L \times 1 & L \times 2 & L \times 1 \end{matrix}$$

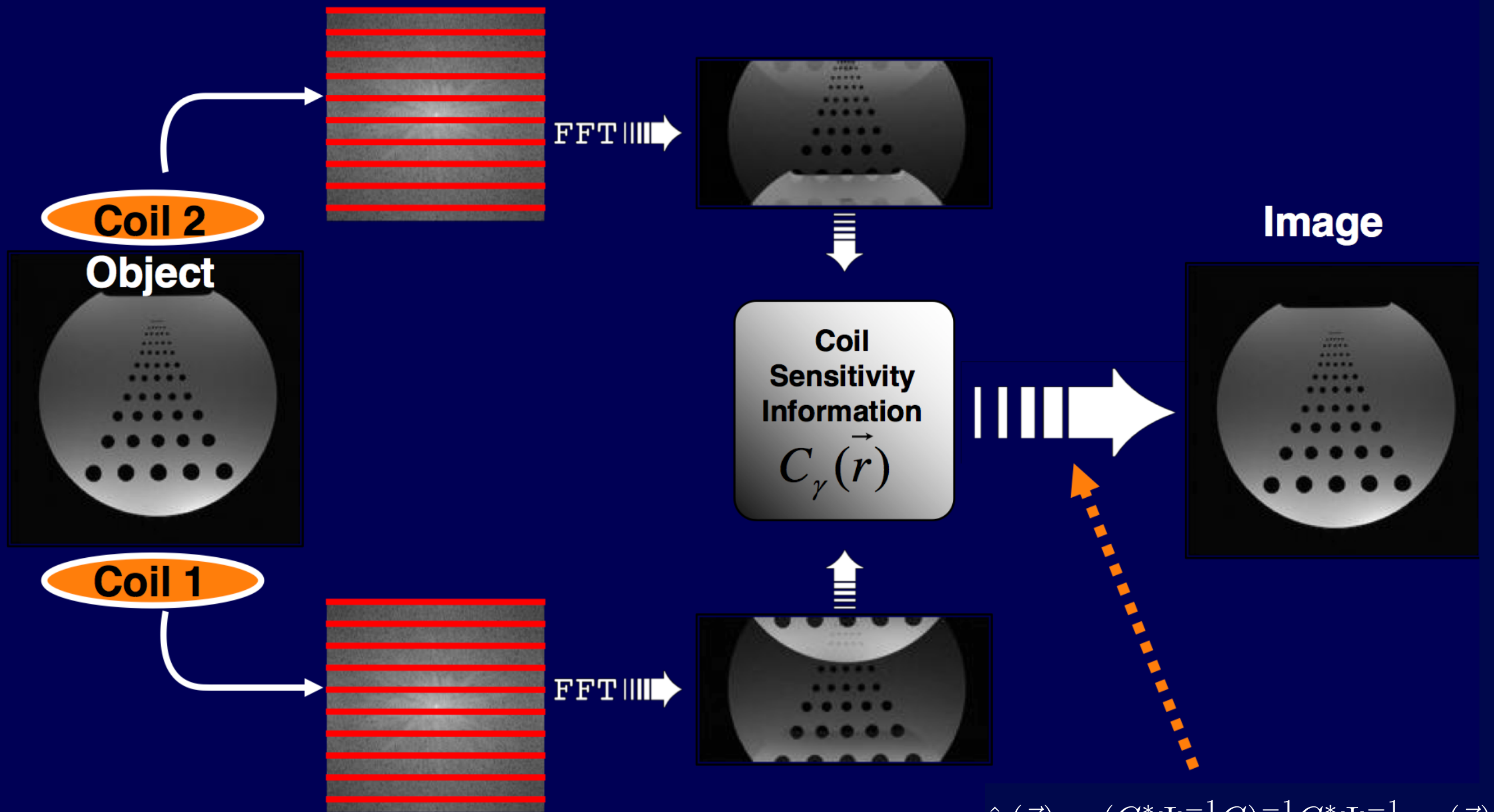
$$\hat{m}(\vec{x}) = \underbrace{(C^* \Psi^{-1} C)^{-1}}_{2 \times 2} \underbrace{C^* \Psi^{-1}}_{2 \times L} \underbrace{m_s(\vec{x})}_{L \times 1}$$

L aliased reconstruction resolves 2 image pixels

For an N x N image, we solve (N/2 x N)
2 x 2 inverse systems

For an acceleration factor R, we solve (N/R x N)
R x R inverse systems

SENSE Reconstruction



Unwrap fold over in image space

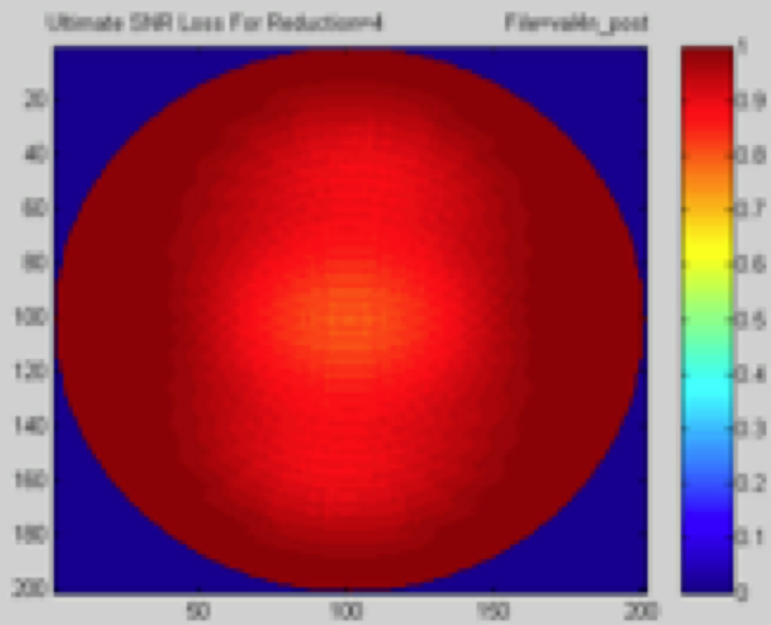
SNR Cost

- How large can R be?
- Two SNR loss mechanisms
 - Reduced scan time
 - Condition of the SENSE decomposition
- SNR Loss

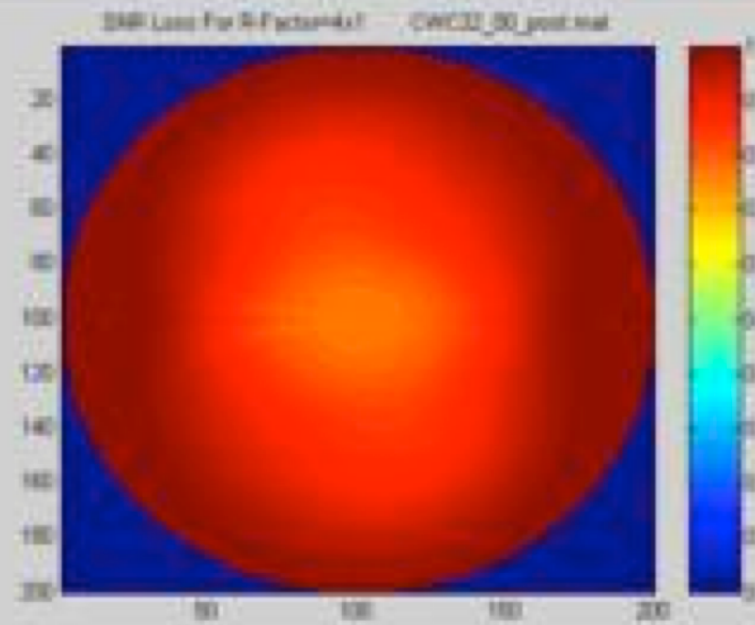
$$SNR_{SENSE} = \frac{SNR}{g\sqrt{R}}$$

Geometry Reduced
Factor Scan Time

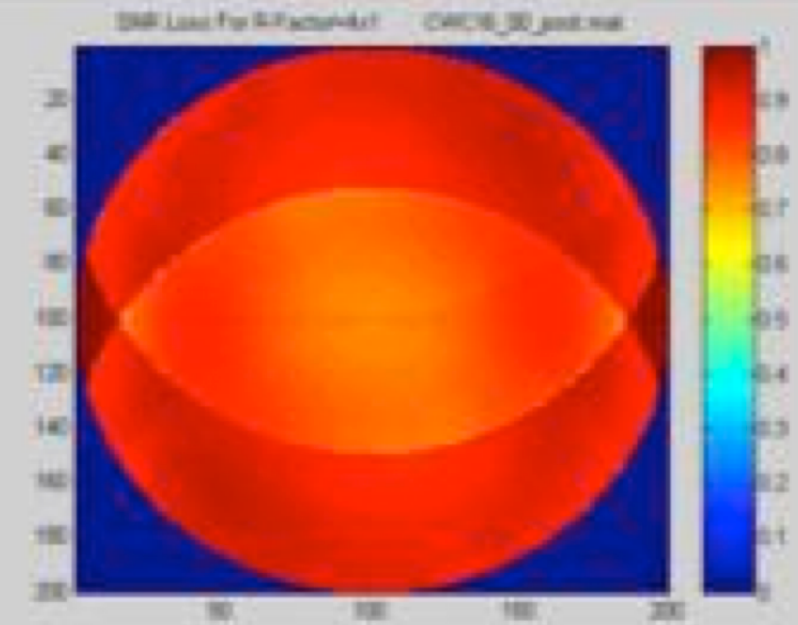
1/g-factor Map for R=4



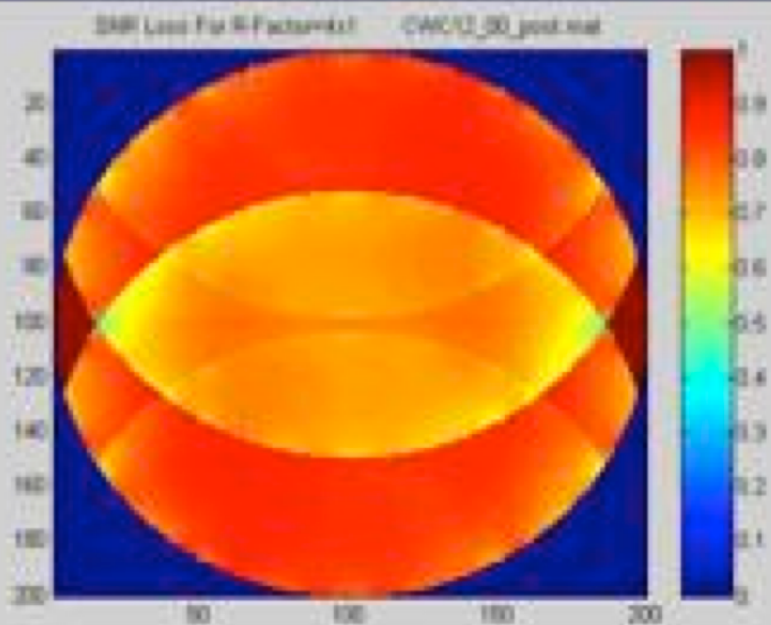
∞ elements



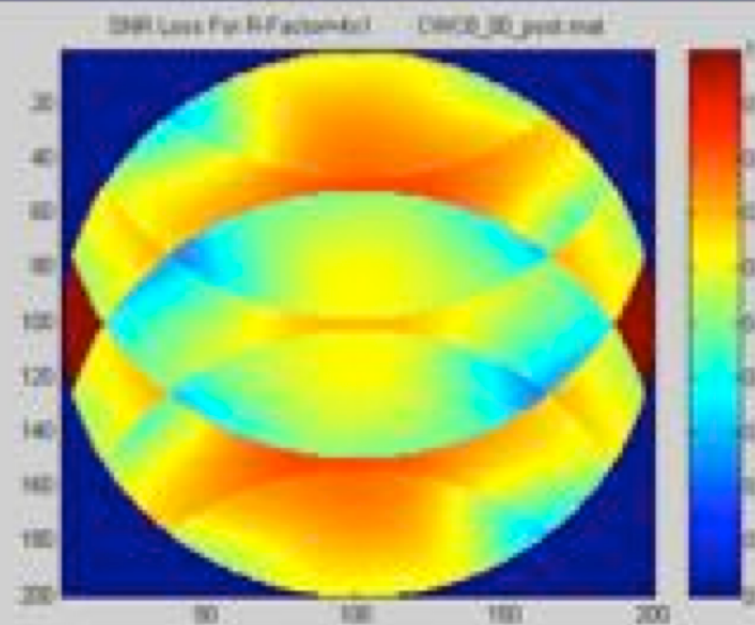
32 elements



16 elements



12 elements



8 elements

Relative
SNR
Scale

g-factor and its impact on images

Rate 1

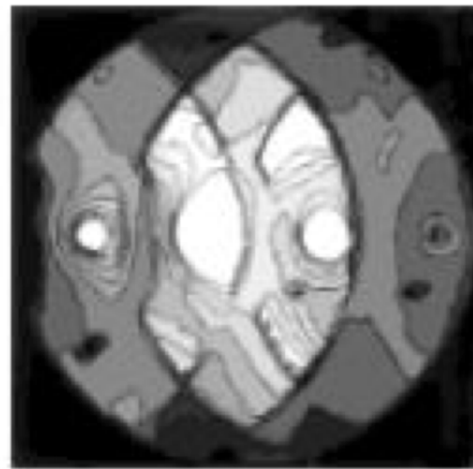
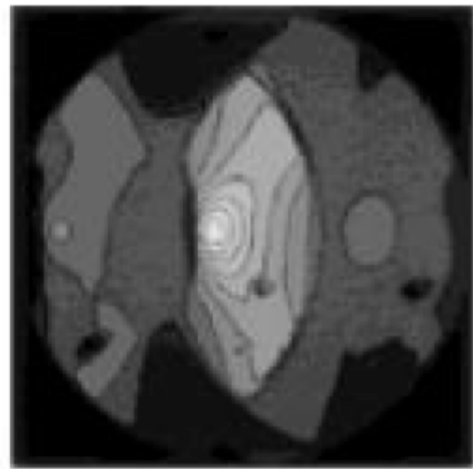
2

2.4

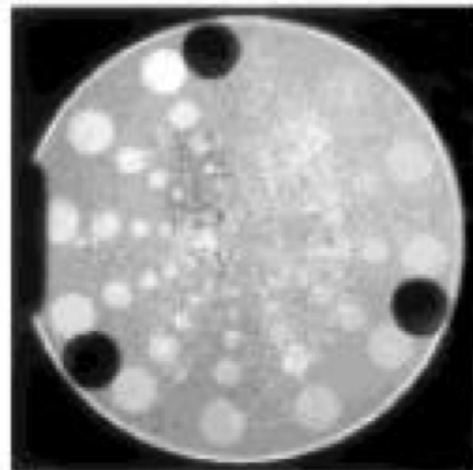
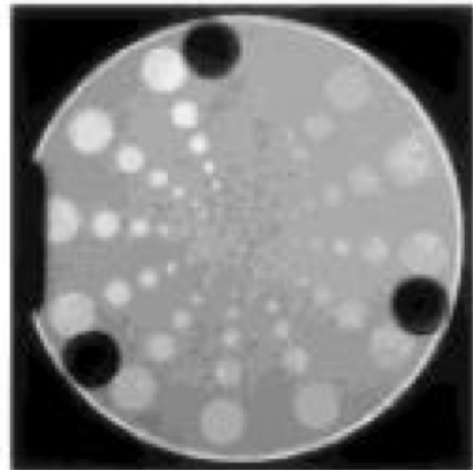
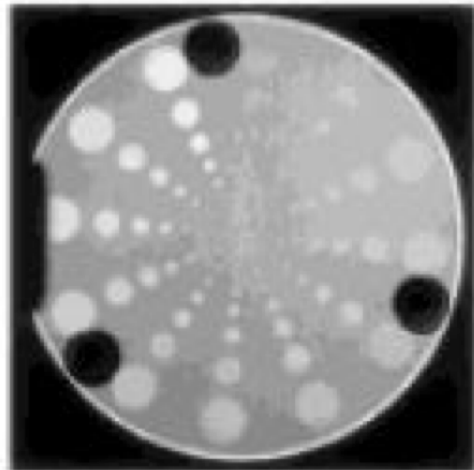
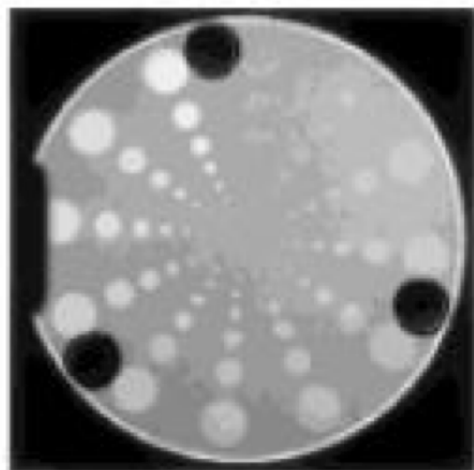
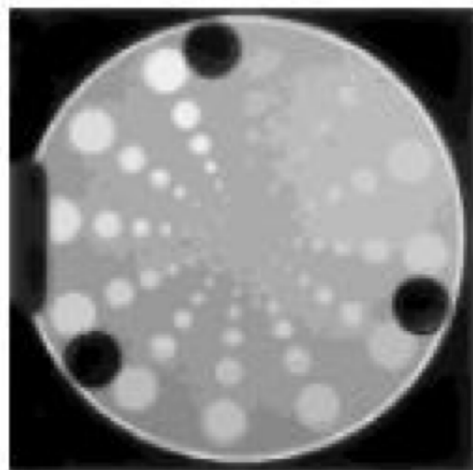
3

4

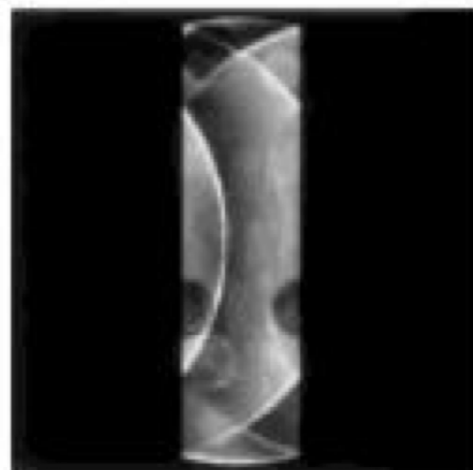
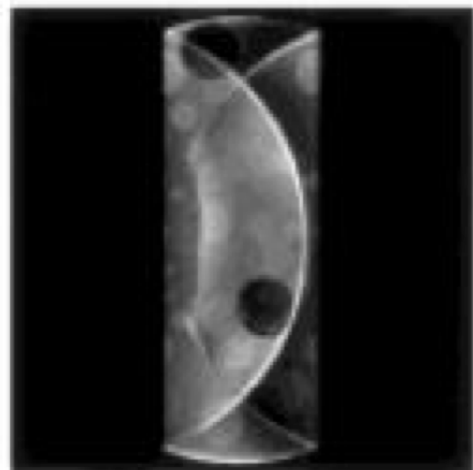
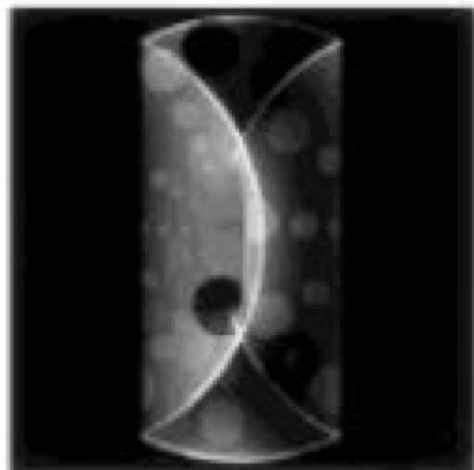
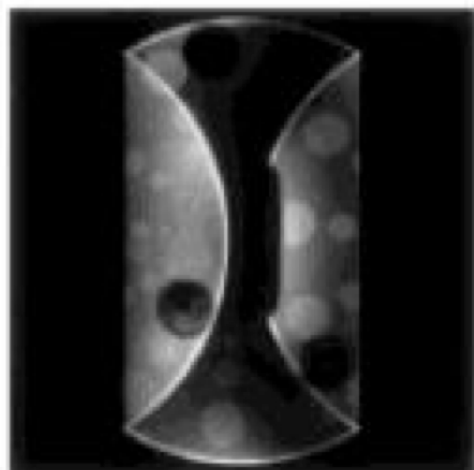
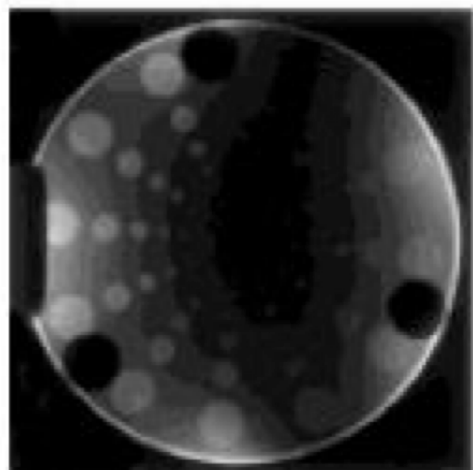
g-map



SENSE



aliased



Parallel Imaging

- Utilizes coil sensitivities to increase the speed of MRI (typical $R=2-4$)
- Cases for parallel imaging
 - Higher patient throughput
 - Real-time imaging/Interventional imaging
 - Motion suppression
- Cases against parallel imaging
 - Low SNR applications

Compressed Sensing (CS)

What is CS?

- CS is about acquiring a **sparse** signal in a most efficient way (subsampling) with the help of an **incoherent** projecting basis

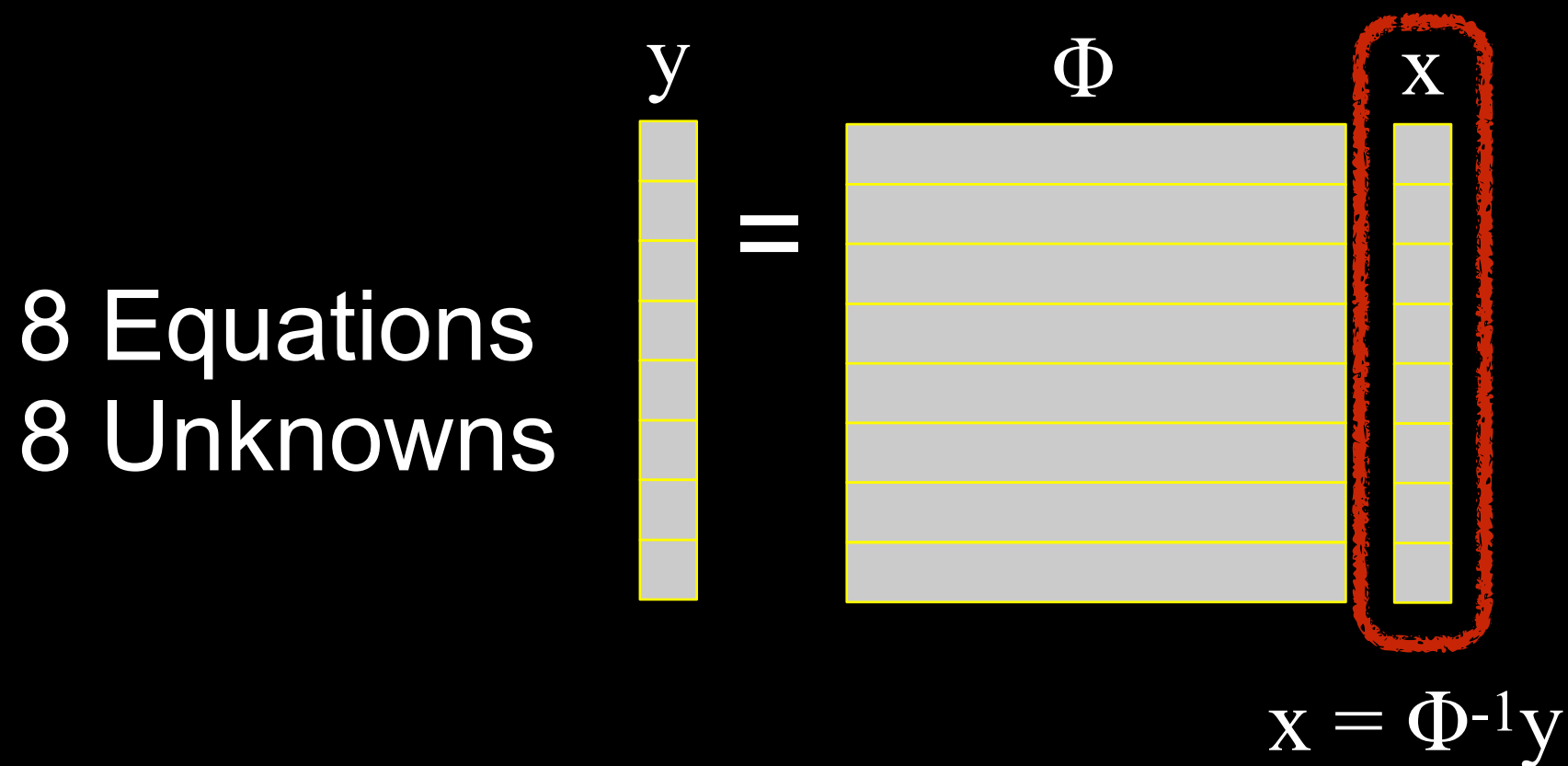


Donoho, IEEE TIT, 2006

Candes et al., Inverse Problems, 2007

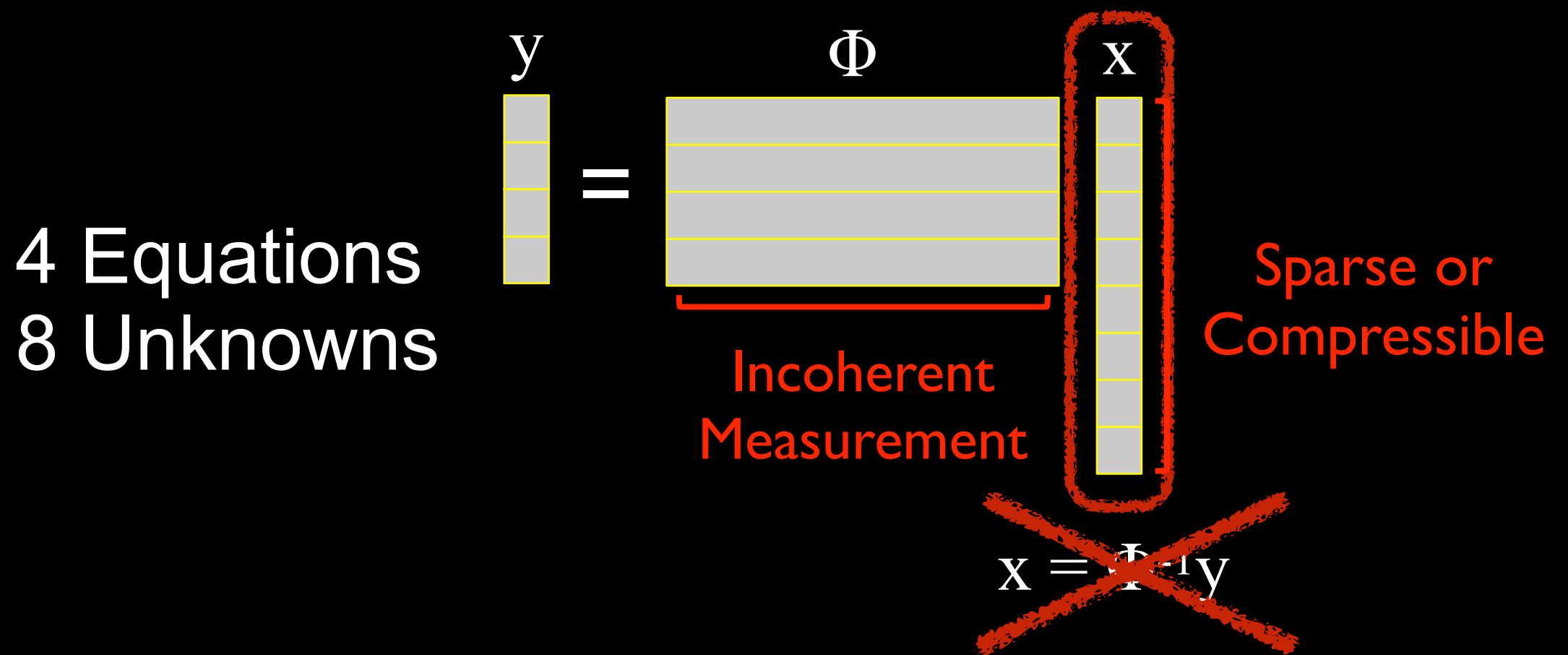
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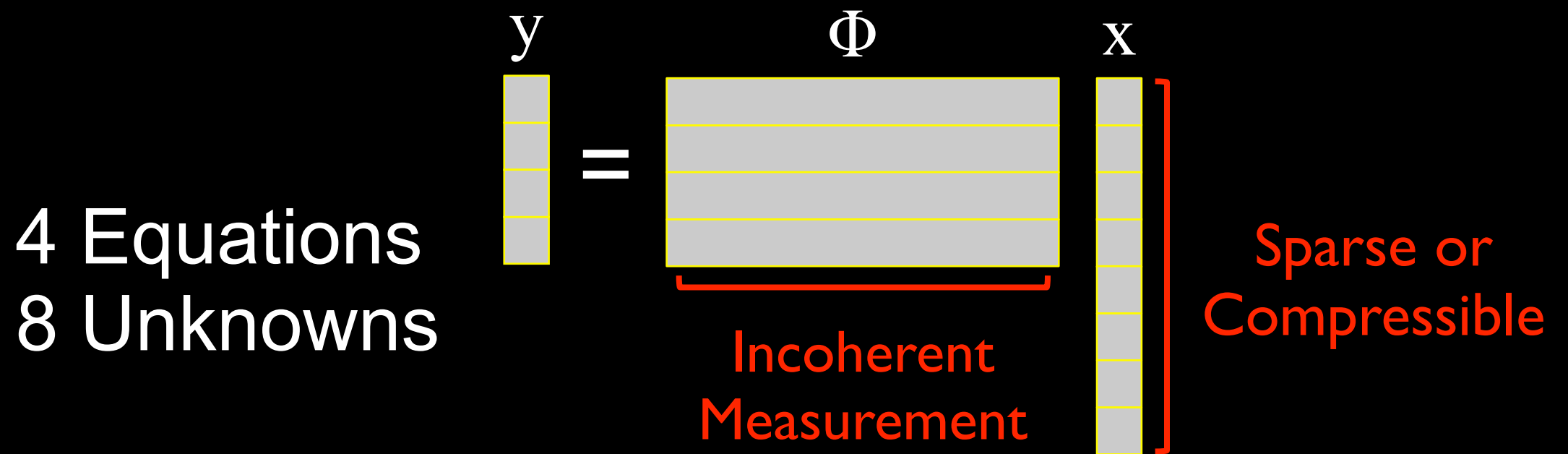
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What is CS?

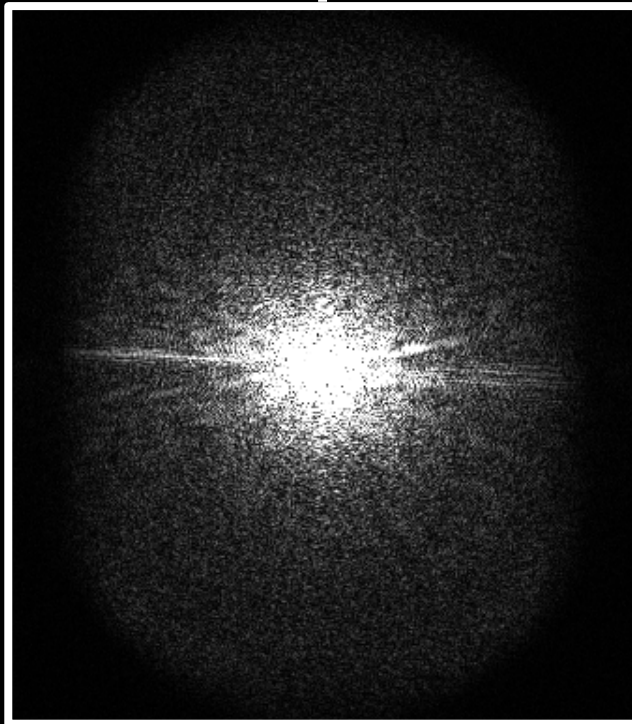
- CS is about acquiring a **sparse** signal in a most efficient way (subsampling) with the help of an **incoherent** projecting basis



We still can find 8 unknowns!

Compressed Sensing MRI

k-space

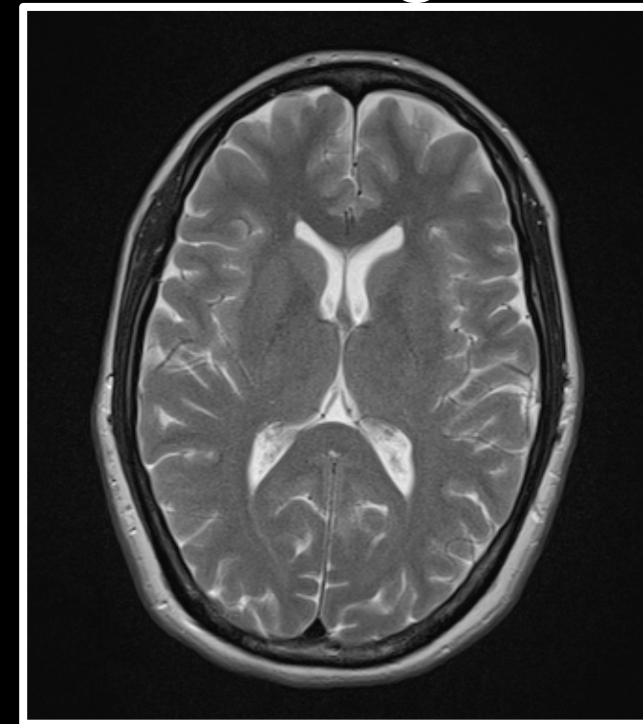


Inverse Fourier
Transform Φ^{-1}



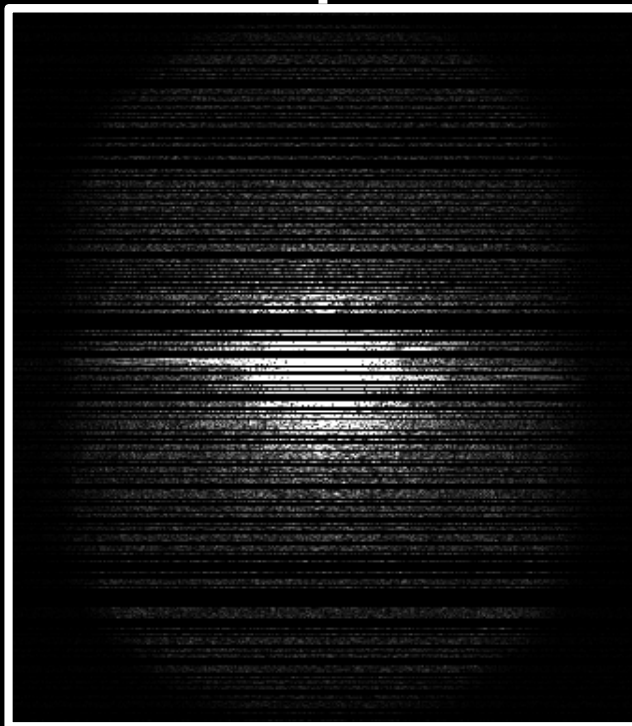
$$x = \Phi^{-1}y$$

Image



Compressed Sensing MRI

k-space

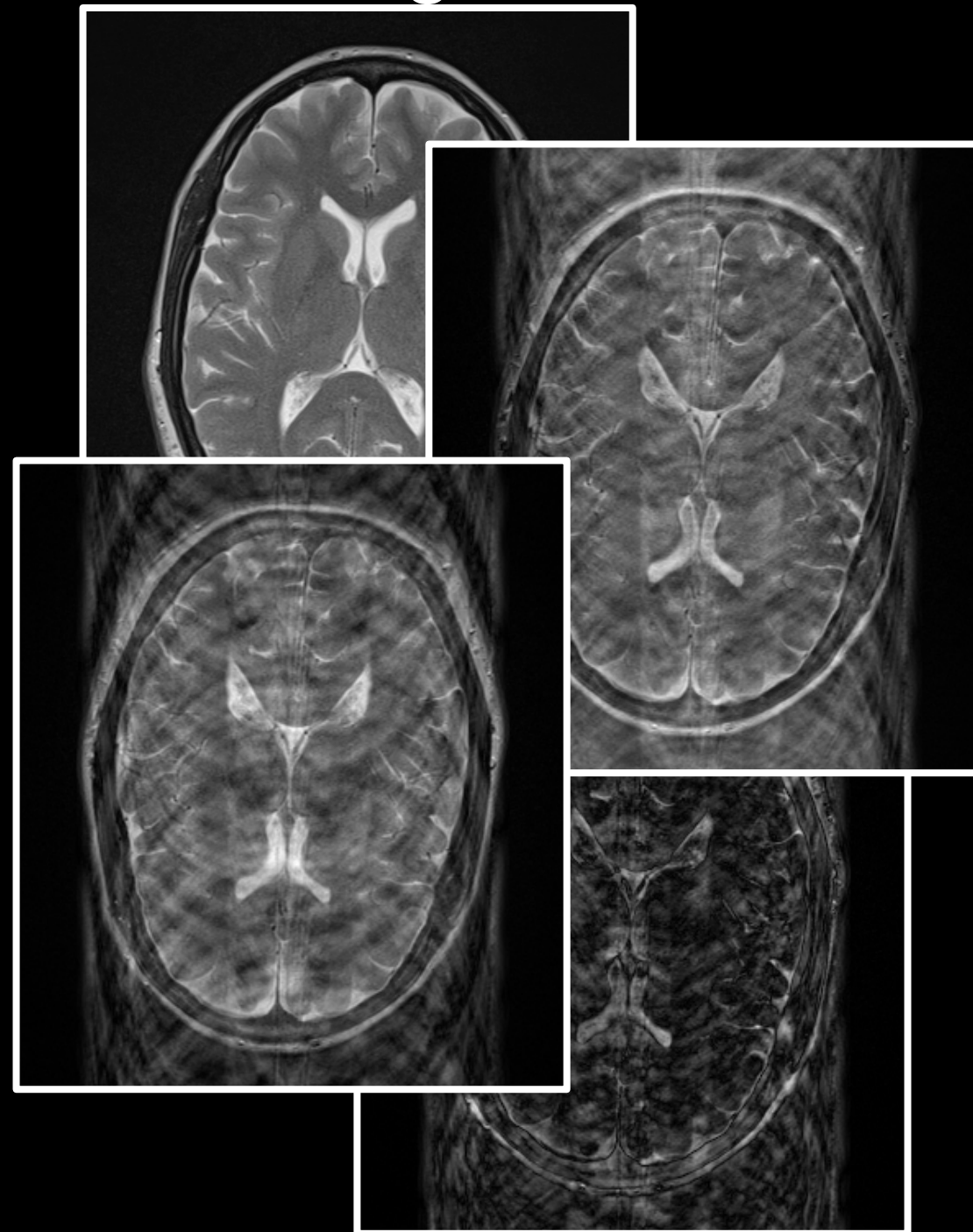


~~Inverse Fourier Transform Φ^{-1}~~



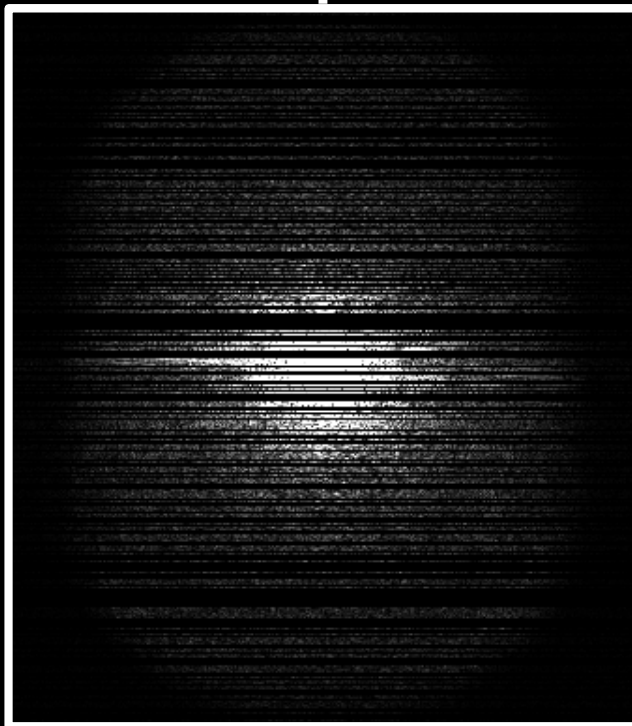
~~$x = \Phi^{-1}y$~~

Image



Compressed Sensing MRI

k-space

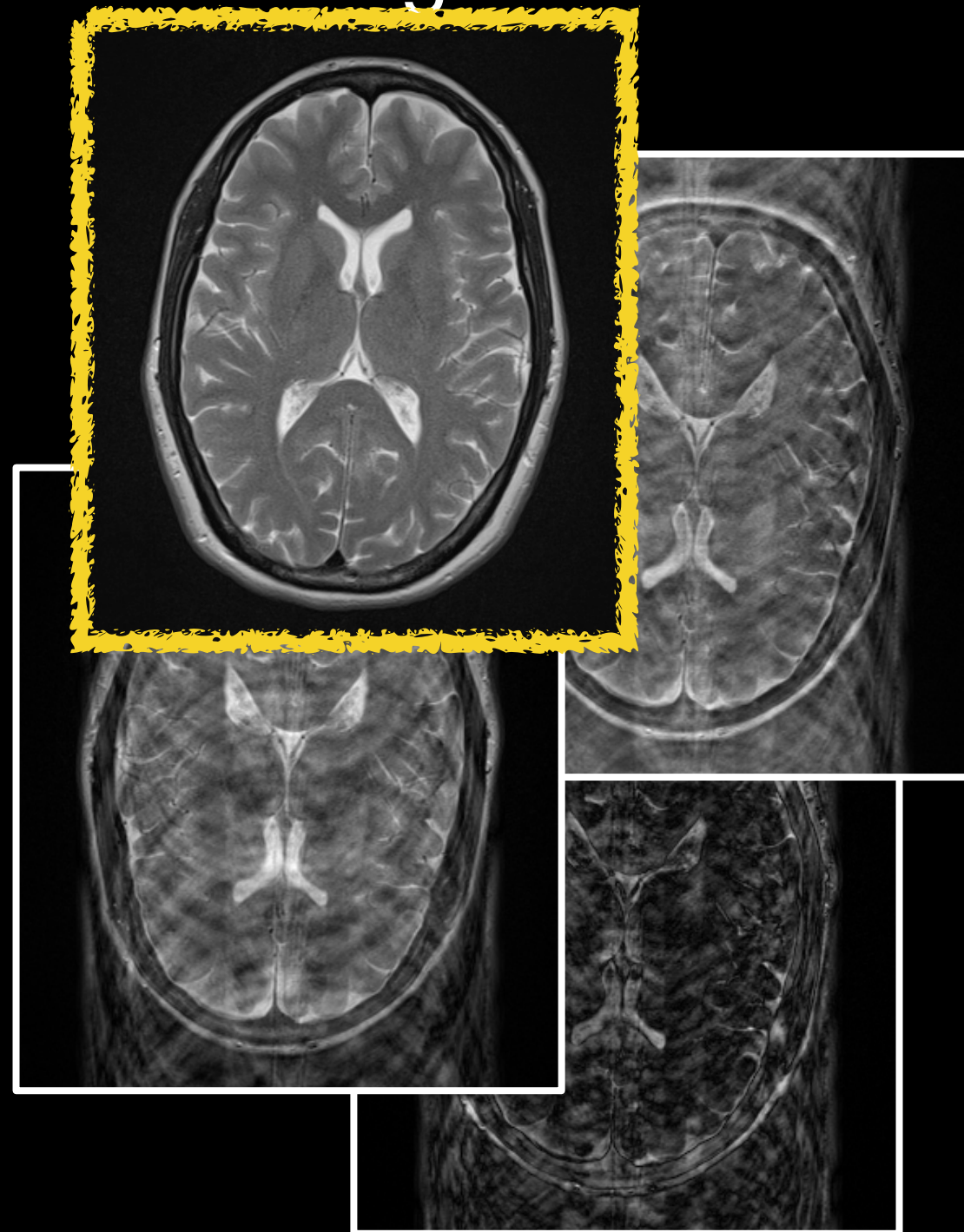


~~Inverse Fourier Transform Φ^{-1}~~



~~$x = \Phi^{-1}y$~~

Image



Choose the most compressible image matching data
(systematic optimization)

CS-MRI Reconstruction

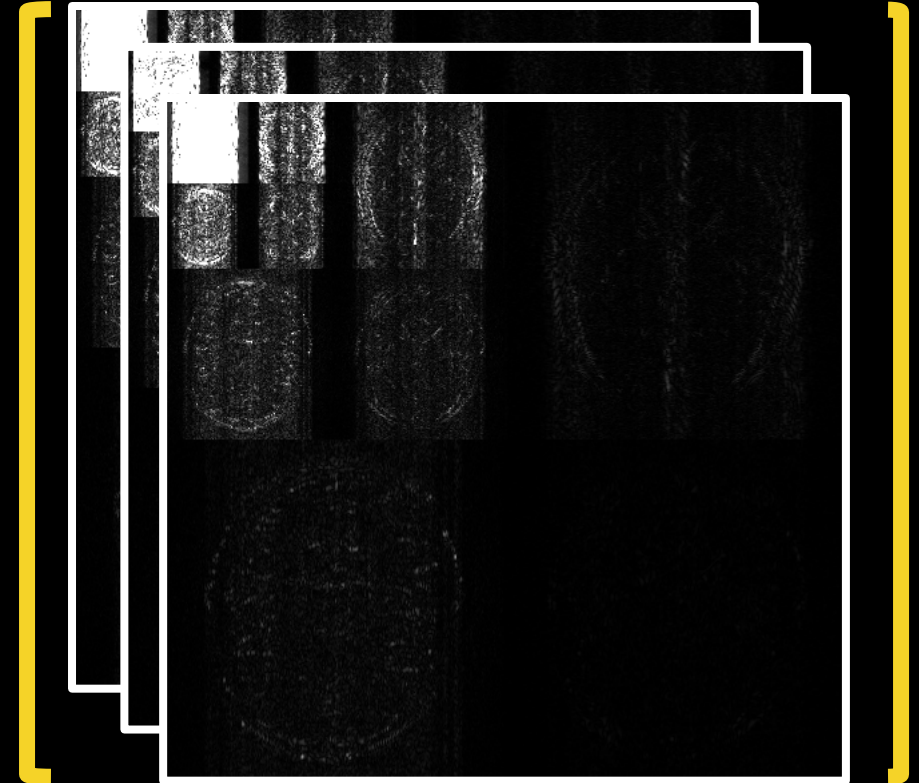
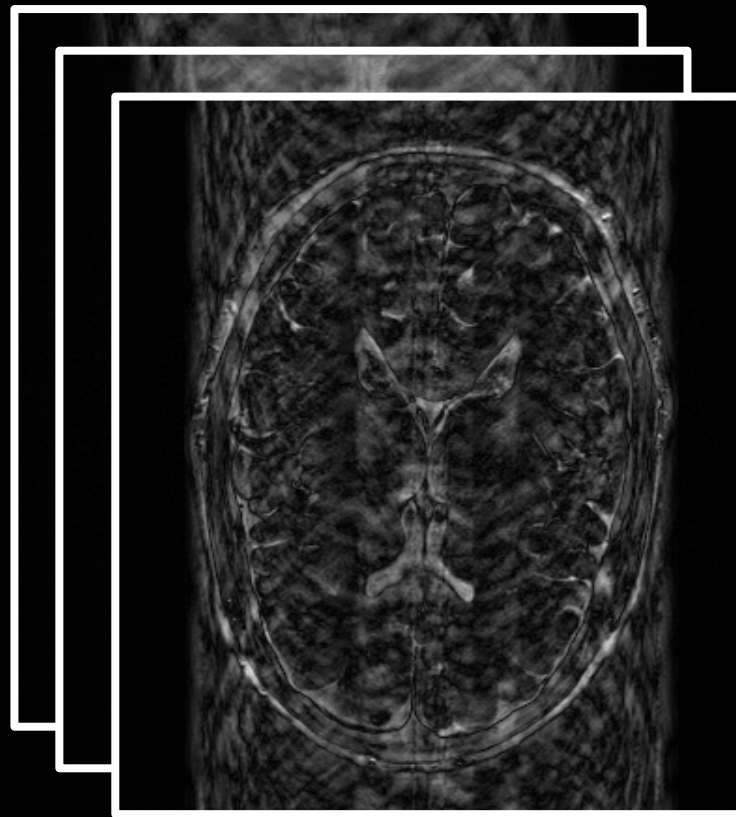
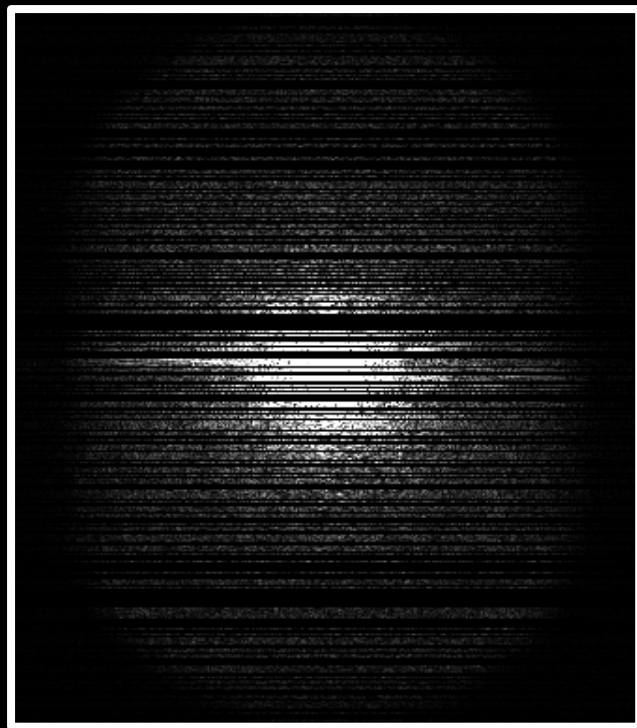
$$|y - \Phi x|^2 < \epsilon$$

$$w = \Psi x$$

y: k-space

x: Image

w: Wavelet



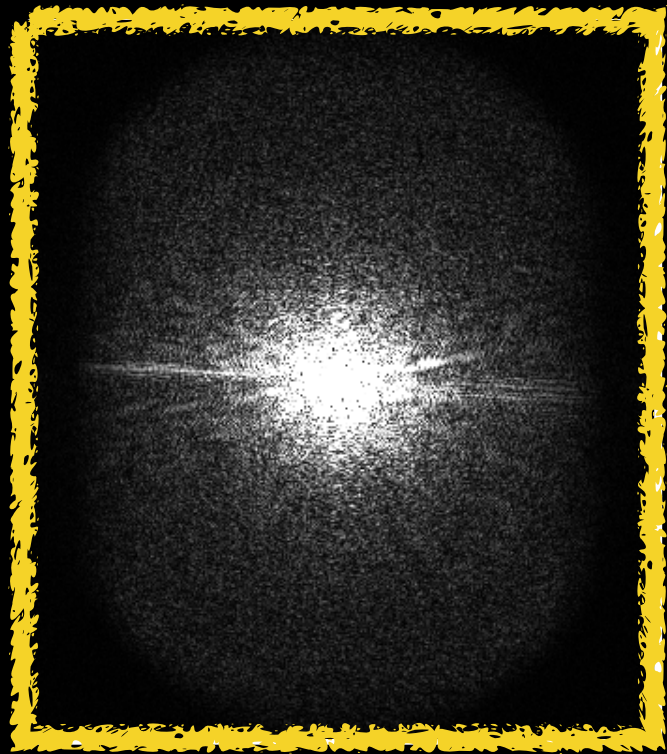
L1-norm

minimize $|\Psi x|_1$

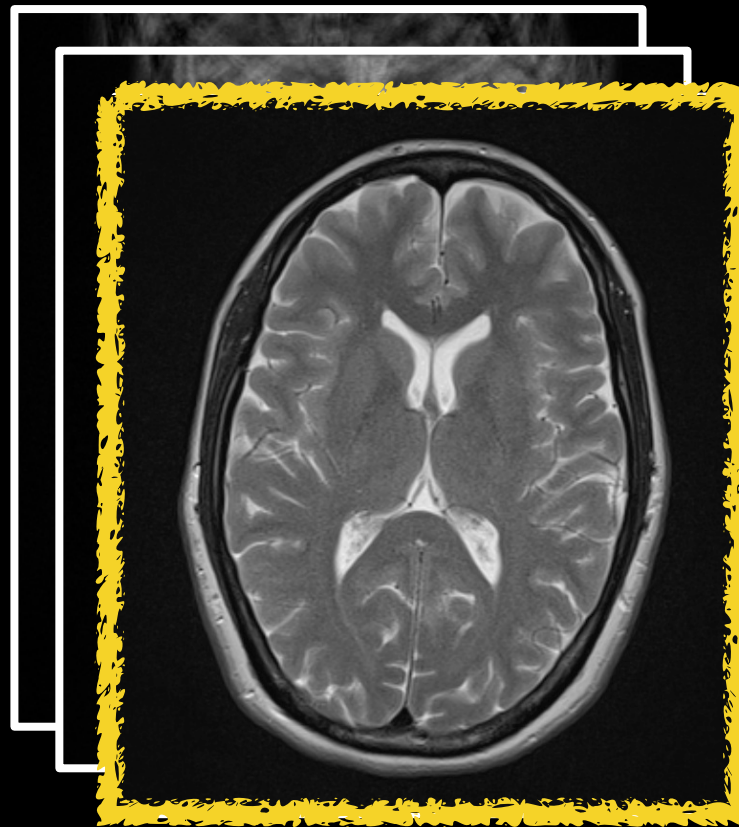
CS-MRI Reconstruction

$$\text{minimize } F(\mathbf{x}): |\mathbf{y} - \Phi\mathbf{x}|^2 + R(\mathbf{x})$$

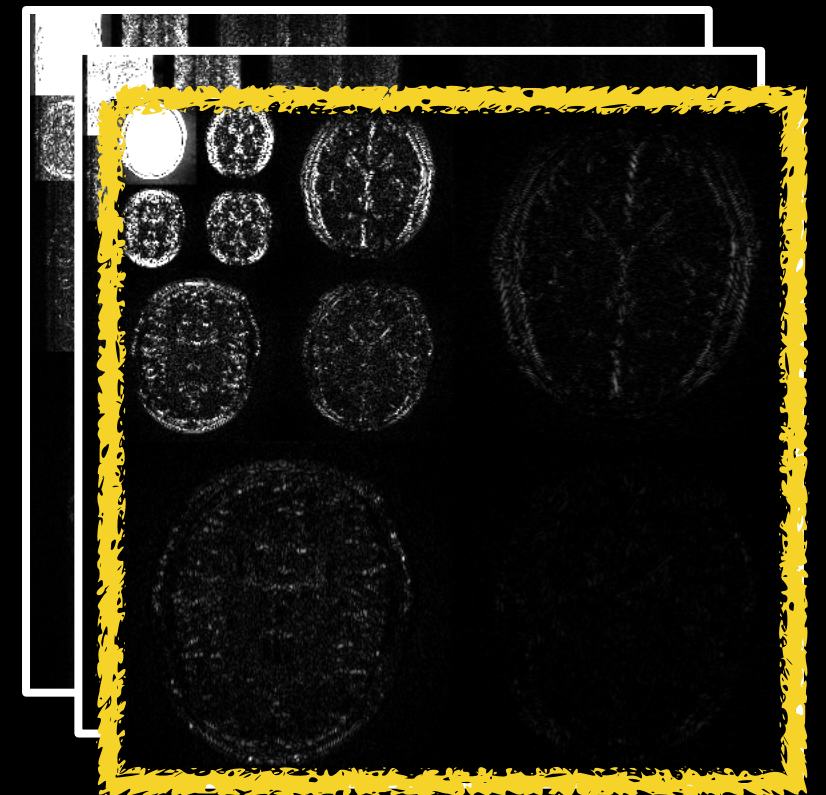
y: k-space



x: Image



w: Wavelet



$$\mathbf{y}' = \text{FT}(\mathbf{x})$$

$$\mathbf{x} = \Psi^{-1}\mathbf{w}$$

Three Tenets of CS

$$\text{minimize } F(\mathbf{x}): \underbrace{|\mathbf{y} - \Phi\mathbf{x}|_2^2}_{\text{Data Consistency}} + \underbrace{R(\mathbf{x})}_{\text{Compressibility Constraint}}$$

Data Consistency **Compressibility Constraint**

- Three key elements of Compressed Sensing:

Compressibility
Incoherence
Nonlinear Reconstruction

CS-MRI Reconstruction

$$\text{minimize } F(\mathbf{x}): \underbrace{\|\mathbf{y} - \Phi\mathbf{x}\|_2^2}_{\text{data fidelity}} + R(\mathbf{x})$$

- Minimizing $F(\mathbf{x})$ is non-trivial since $R(\mathbf{x})$ is not differentiable
 - Linear programming is challenging due to high computational complexity
- Simple gradient-based algorithms have been developed:
 - Re-weighted L1 / FOCUSS
 - IST / IHT / AMP / FISTA
 - Split Bregman / ADMM

*I.F. Gorodnitsky, et al., J. Electroencephalog. Clinical Neurophysiol. 1995 Daubechies I, et al. Commun. Pure Appl. Math. 2004
Elad M, et al. in Proc. SPIE 2007
T. Goldstein, S. Osher, SIAM J. Imaging Sci. 2009*

State-of-the-Art CS-MRI

- Reducing possible reconstruction failure
 - Improve sparse transformations
 - Develop k-space undersampling schemes
- Integrating CS with DL/parallel imaging
 - Develop compatible undersampling patterns
 - Develop reconstruction methods

State-of-the-Art CS-MRI

- Methods to evaluate CS reconstructed images
 - RMSE / SSIM / Mutual Information
- Reducing reconstruction time
 - Reduce computational complexity
 - Parallelize reconstruction problems
- Developing stable reconstruction algorithms
 - Minimize / avoid the number of regularization parameters

Thanks!

- Interested in more? M229 in Spring
 - Fast imaging sequences
 - Fast sampling trajectories
 - Parallel imaging
 - Constrained reconstruction
 - Deep learning-based methods

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