RF Pulse Design

Multi-dimensional Excitation I

M229 Advanced Topics in MRI

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Class Business

- Office hours
  - Instructors: Fri 10-12pm
    TAs: Xinran Zhong and Zhaohuan Zhang (time: TBD)
  - Emails beforehand would be helpful
- Homework 1 will be out on 4/12 (due on 4/26)
- Papers and Slides

Today's Topics

- Recap of adiabatic pulses
- Small tip approximation
- Excitation k-space interpretation
- Design of 2D excitation pulses
  - Spiral pulse design

Recap of Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B1 field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Does not follow the small tip approximation

Adiabatic Pulses

$$\theta \neq \int_0^T \gamma B_1(s)ds$$

- Amplitude and frequency modulation
- Long duration (8-12 ms)
- High B1 amplitude (>12 μT)
- Generally NOT multi-purpose (inversion pulses cannot be used for refocusing, etc.)

Non-adiabatic Pulses

$$\theta = \int_0^T \gamma B_1(s)ds$$

- Amplitude modulation
- Short duration (0.3-1 ms)
- Low B1 amplitude
- Generally multi-purpose (inversion pulses can be used for refocusing, etc.)

Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = A(t)e^{-i\omega_1(t)t}$$

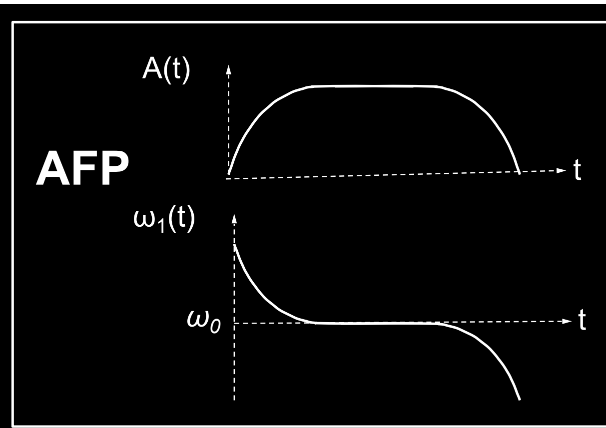
envelop

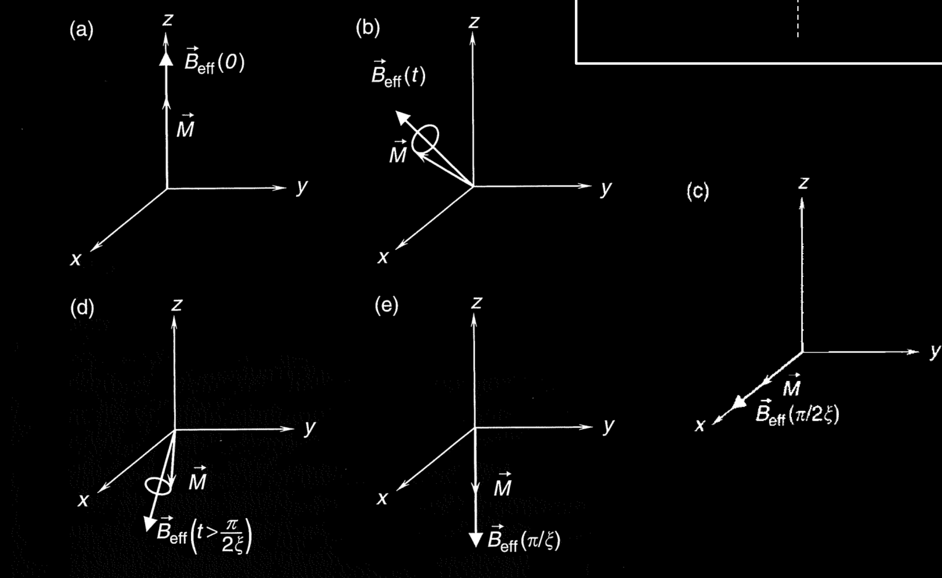
frequency sweep

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

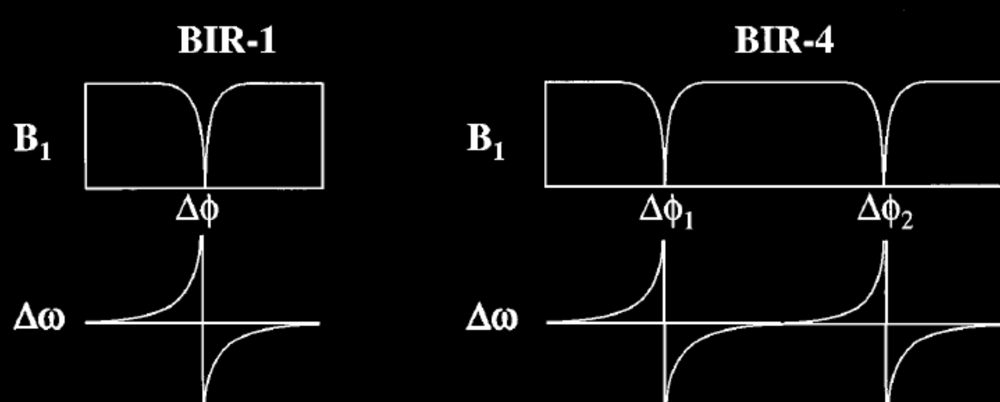
Adiabatic Inversion



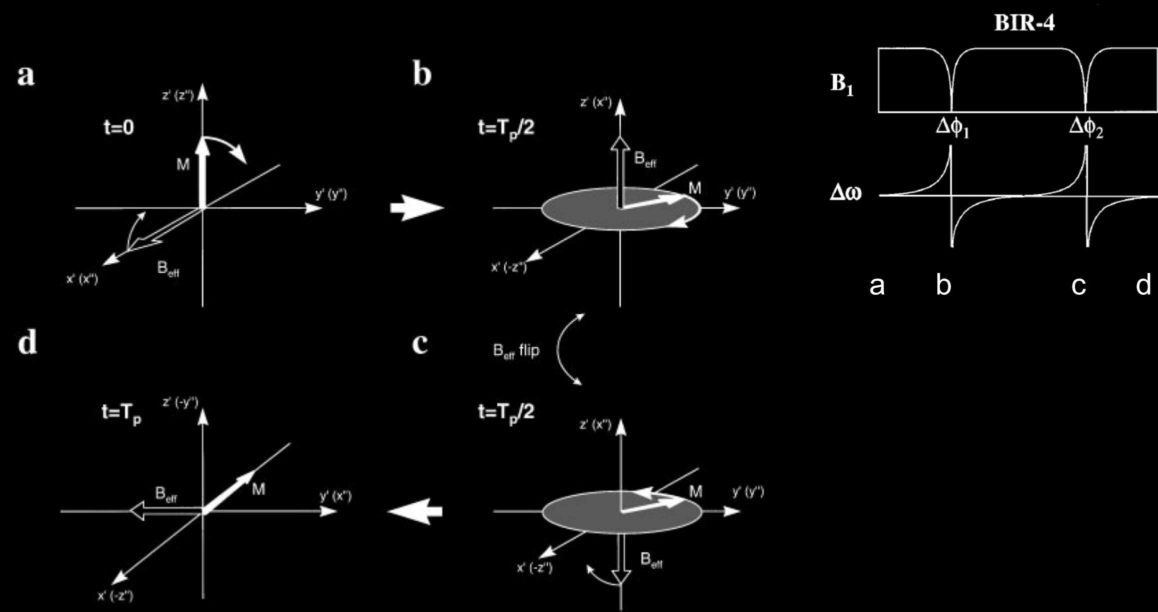


Adiabatic Excitation: BIR Pulses

- BIR: B1 Insensitive Rotation
- Most popular: BIR-4 Pulse



Adiabatic Excitation: BIR4 Pulse



Bloch Simulator

- http://mrsrl.stanford.edu/~brian/blochsim/

```
[mx,my,mz] = bloch(b1,gr,tp,t1,t2,df,dp,mode,mx,my,mz)

Bloch simulation of rotations due to B1, gradient and
off-resonance, including relaxation effects.  At each time
point, the rotation matrix and decay matrix are calculated.
Simulation can simulate the steady-state if the sequence
is applied repeatedly, or the magnetization starting at m0.

INPUT:
        b1 = (Mx1) RF pulse in G.  Can be complex.
        gr = (Mx1,2,or 3) 1,2 or 3-dimensional gradient in G/cm.
        tp = (Mx1) time duration of each b1 and gr point, in seconds,
                        or 1x1 time step if constant for all points
                        or monotonically INCREASING endtime of each
                        interval..
        t1 = T1 relaxation time in seconds.
        t2 = T2 relaxation time in seconds.
        df = (Nx1) Array of off-resonance frequencies (Hz)
        dp = (Px1,2,or 3) Array of spatial positions (cm).
                Width should match width of gr.
        mode= Bitmask mode:
                Bit 0:  0-Simulate from start or M0, 1-Steady State
                Bit 1:  1-Record m at time points.  0-just end time.
```

```
%%% User inputs:
mu = 5;    % Phase modulation parameter [dimensionless]
beta1 = 672;    % Frequency modulation parameter [rad/s]
pulseWidth = 10.24;    % RF pulse duration [ms]
A0 = 0.12;    % Peak B1 amplitude [Gauss].

%%%%%%

nSamples = 512;          % number of samples in the RF pulse
dt = pulseWidth/nSamples/1000;   % time step, [seconds]
tim_sech = linspace(-pulseWidth/2,pulseWidth/2,nSamples)./1000';
% time scale to calculate the RF waveforms in seconds.

% Amplitude modulation function B1(t):
B1 = A0.* sech(beta1.*tim_sech);

% Carrier frequency modulation function w(t):
w = -mu.*beta1.*tanh(beta1.*tim_sech)./(2*pi);
% The 2*PI scaling factor at the end converts the unit from rad/s to Hz

% Phase modulation function phi(t):
phi = mu .* log(sech(beta1.*tim_sech));

% Put together complex RF pulse waveform:
rf_pulse = B1 .* exp(1i.*phi);

% Generate a time scale for the Bloch simulation:
tim_bloch = [0:(nSamples-1)]*dt;
```

```
%% The Bloch simulator requires a gradient input. For our simulation,
% gradient will be zero, as we are simulating a non-selective RF pulse.
T1_value   = 10000;      % [ms]
T2_value   = 10000;      % [ms]

f_max = 4000;   % off-resonance frequency range [Hz]
freq_range  = linspace(-f_max,f_max,1000);   % off-resonance frequency range [Hz]

grad_pulse = zeros(1,length(rf_pulse));
mod = 0;
[mx1,my1,mz1] = bloch(rf_pulse,  grad_pulse,  dt,  ...
    T1_value/1000,  T2_value/1000,  freq_range,  0,  mod,  0,  0,  0);
```

```
% Plotting the longitudinal magnetization to see the inversion profile as a
% function of resonance frequency
figure(2);
plot(freq_range, mz1,'k','LineWidth',LineWidthVal);
title('Inversion Profile'); xlabel('Frequency (Hz)'); ylabel('M_z'); grid on;
v = axis; axis([v(1) v(2) -1.05 1.05]);
```

Small Tip Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

sin θ ≈ θ
cos θ ≈ 1
$M_z \approx M_0 \rightarrow$ constant

$$\frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

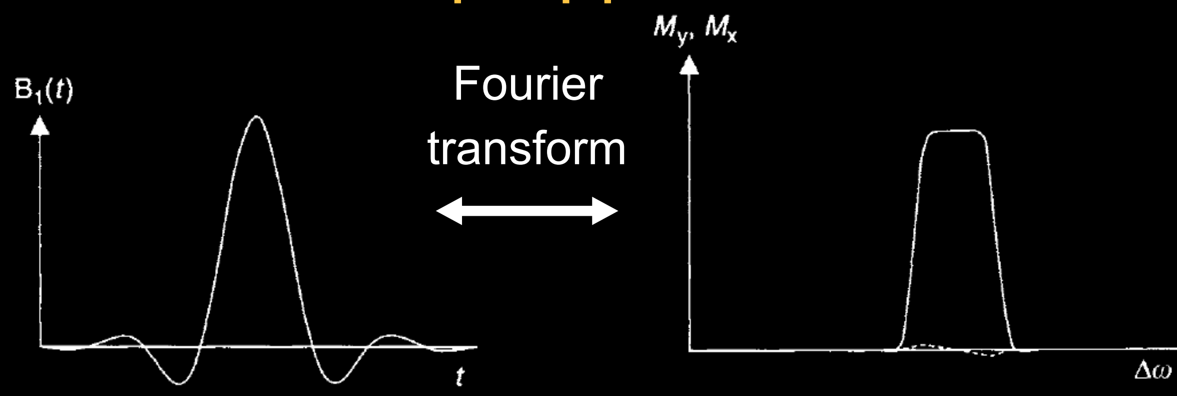
$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z\cdot(t-s)} ds$$

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \mid_{f=-(\gamma/2\pi)G_z z}$$

(See the note for complete derivation)

To the board ...

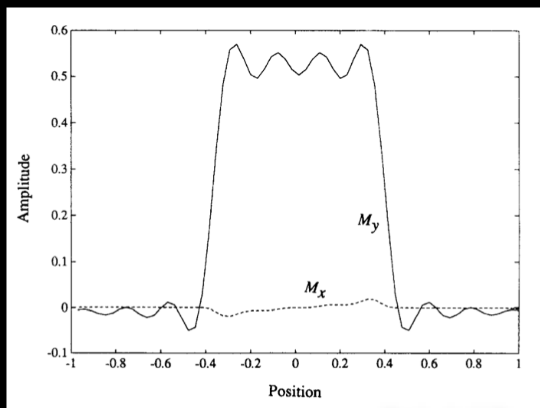
Small Tip Approximation



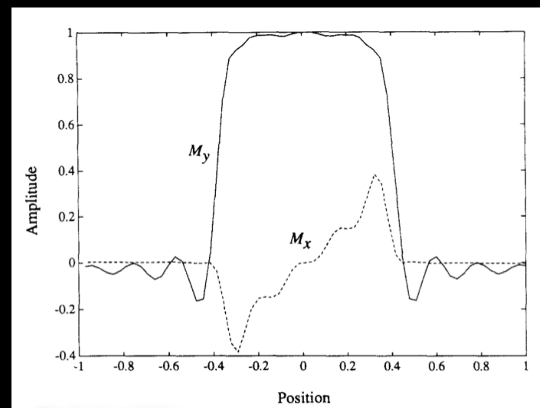
- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of B1(t)”
- The approximation works surprisingly well even for flip angles up to 90°

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

small-angle approximation still works reasonably well for flip angles that aren't necessarily "small"

Multi-Dimensional Excitation RF Pulses

- 1D vs. N-D RF pulses
- Small tip angle approximation revisited
- Excitation k-space interpretation
- 1D examples in excitation k-space
- Excitation k-space integrals
- 2D excitation pulse design steps
- 2D spiral pulse design example

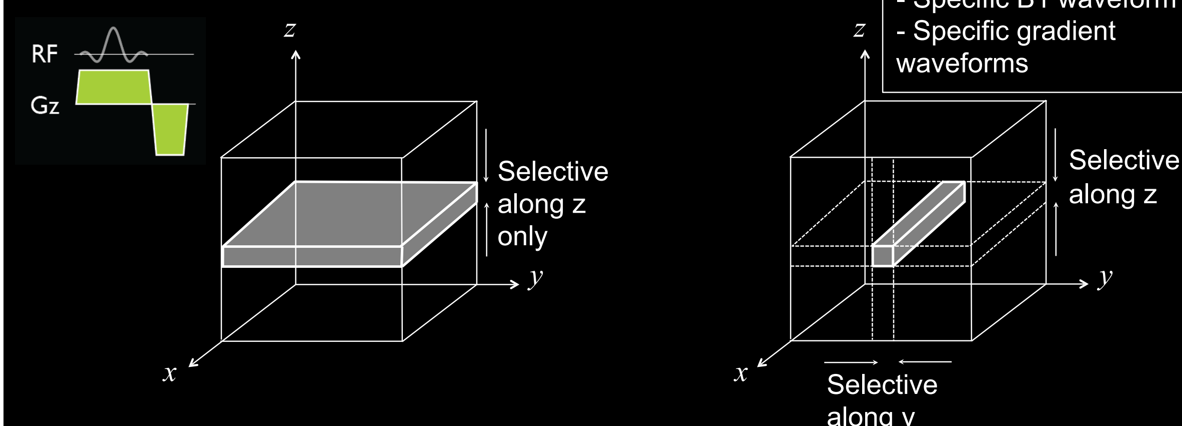
- EPI pulse design, spectral-spatial pulses (next lecture)

What is Multi-Dimensional Excitation?

Multi-dimensional excitation occurs when using multi-dimensional RF pulses in MRI/NMR, i.e. 2D or 3D RF pulses

1D vs. N-D RF Pulses

2D/N-D Pulse Design Requires:
- Specific B1 waveform
- Specific gradient waveforms



- 1D pulses are selective along 1 dimension, typically the slice select dimension
- 2D pulses are selective along 2 dimensions
  - So, a 2D pulse would select a long cylinder instead of a slice
  - The shape of the cross section depends on the 2D RF pulse

Excitation k-space Interpretation

Small Tip Approximation

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\omega(z)(t-s)} ds$$

$$\omega(z) = \gamma G_z z \quad \Rightarrow \quad \omega(\vec{r},t) = \gamma \vec{G}(t)\vec{r}$$

$$\Downarrow$$

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma \int_s^t \vec{G}(\tau)d\tau \cdot \vec{r}} ds$$

Small Tip Approximation

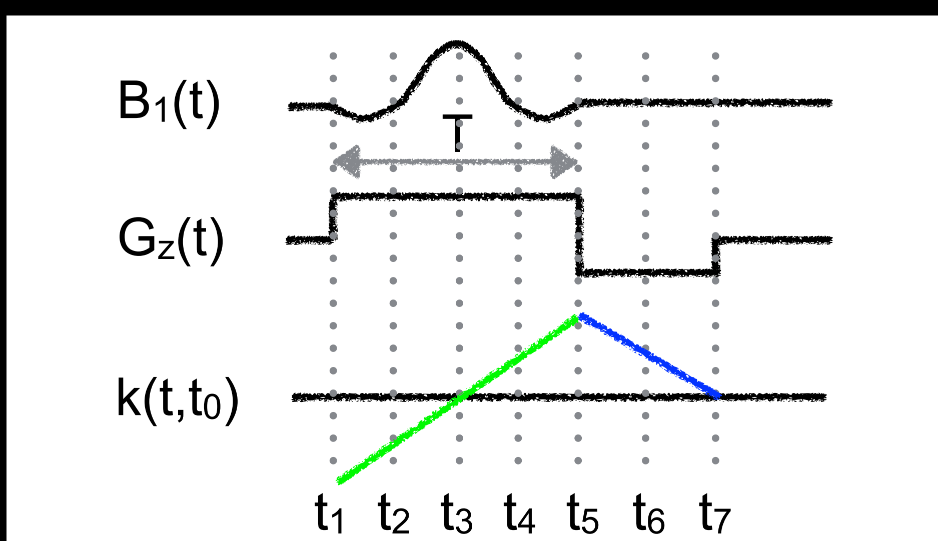
$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma \int_s^t \vec{G}(\tau)d\tau \cdot \vec{r}} ds$$

Let us define: $\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{i2\pi \vec{k}(s,t)\cdot\vec{r}} ds$$

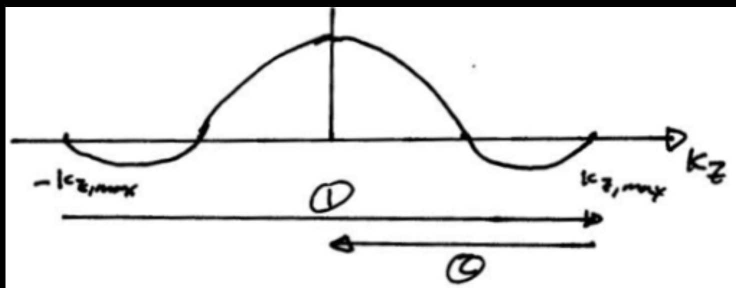
One-Dimensional Example

$$\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$$



Consider the value of **k** at s = t_1, t_2, … t_7

One-Dimensional Example



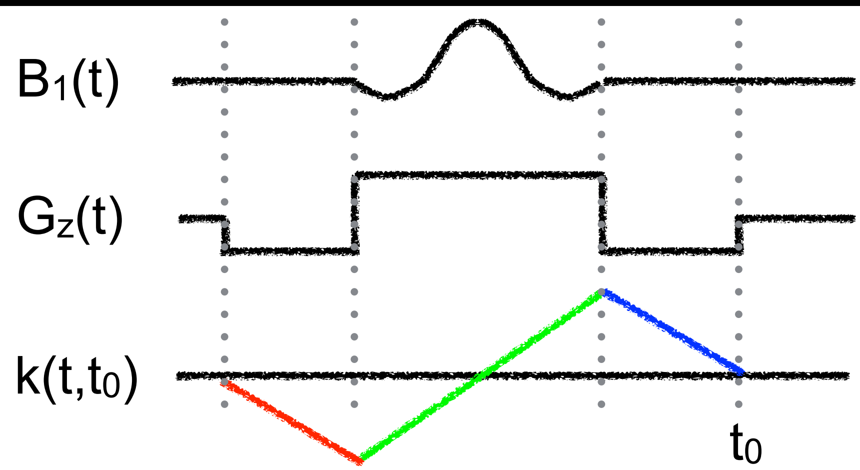
$$k_{z,max} = \frac{T}{2}\frac{\gamma}{2\pi}G_z$$

- This gives magnetization at t = t_0, the end of the pulse
- Looks like you scan across k-space, then return to origin

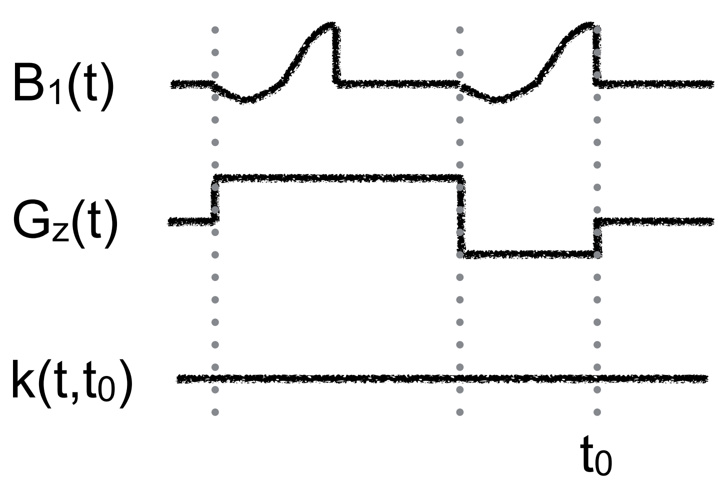
Evolution of Magnetization During Pulse

- RF pulse goes in at DC (k_z = 0)
- Gradients move previously applied weighting around
- Think of the RF as "writing" an analog waveform in k-space
- Same idea applies to reception

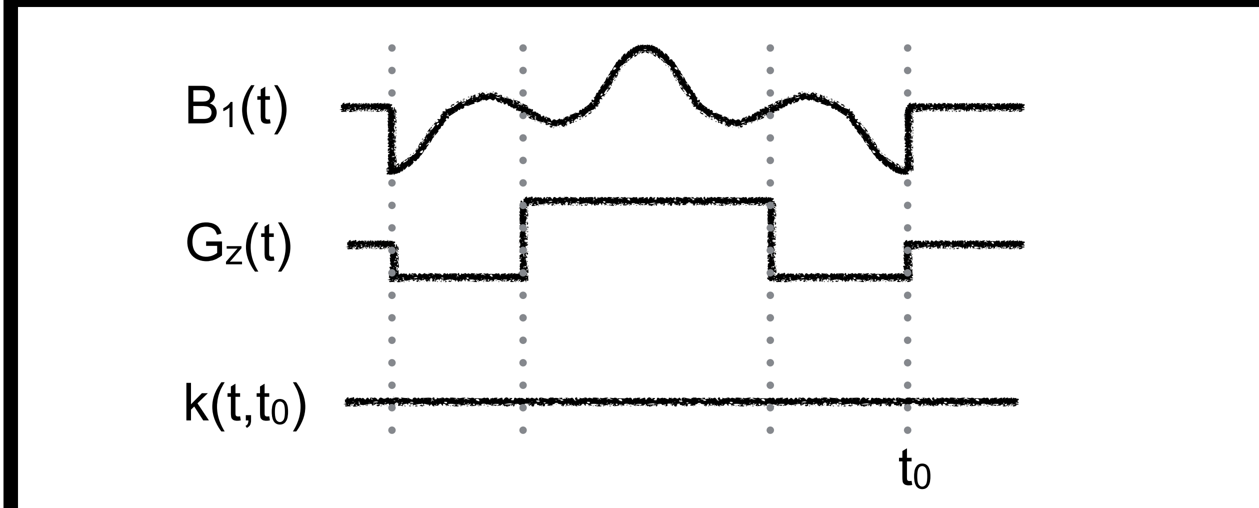
Other 1D Examples



Other 1D Examples



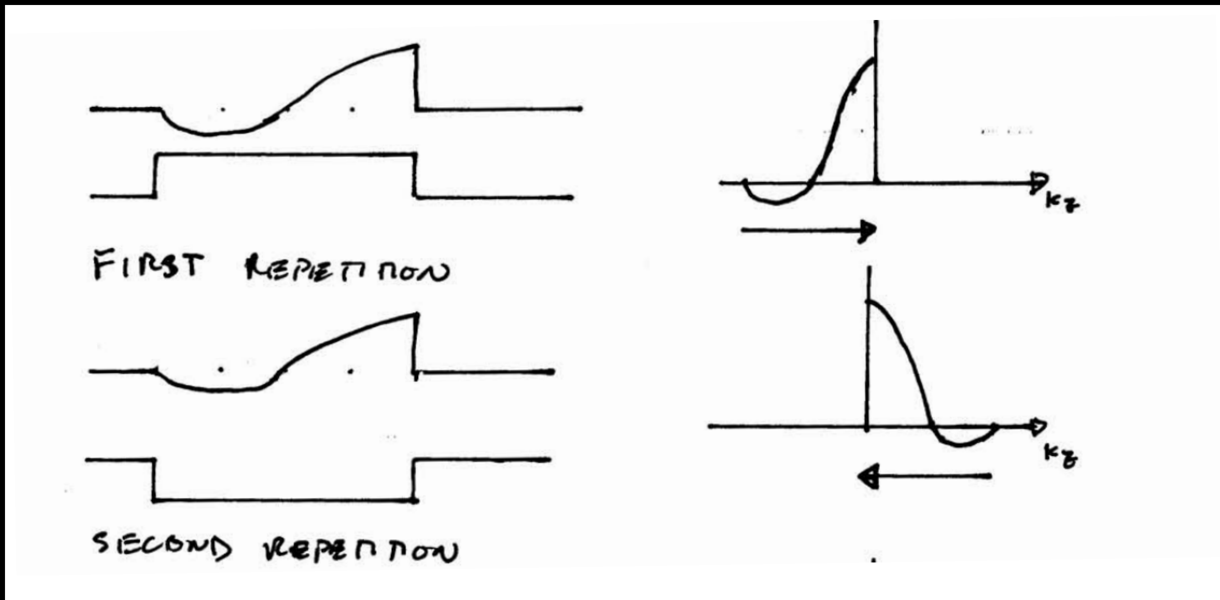
Other 1D Examples



Multiple Excitations

- Most acquisition methods require several repetitions to make an image
  - e.g., 128 phase encodes
- Data is combined to reconstruct an image
- Same idea works for excitation!

Simple 1D Example



Sum the data from two acquisitions

Same profile as slice selective pulse, but zero echo time

Small Tip Approximation Solution as k-space Integral

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{i2\pi \vec{k}(s,t)\cdot\vec{r}} ds$$

$$e^{i2\pi \vec{k}(s,t)\cdot\vec{r}} = \int_{\vec{k}} {}^3\delta(\vec{k}(s,t) - \vec{k}) e^{i2\pi \vec{k}\cdot\vec{r}} d\vec{k}$$

Substituting and changing the order of integration:

$$M_{xy}(t,\vec{r}) = iM_0 \int_{\vec{k}} \underbrace{\left[\int_{-\infty}^t \gamma B_1(s) {}^3\delta(\vec{k}(s,t) - \vec{k}) ds\right]}_{p(\vec{k})} e^{i2\pi \vec{k}\cdot\vec{r}} d\vec{k}$$

$$M_{xy}(t,\vec{r}) = iM_0 \int_{\vec{k}} \left[\int_{-\infty}^{t} \gamma B_1(s)\, ^3\delta(\vec{k}(s,t) - \vec{k})ds \right] e^{i2\pi\vec{k}\cdot\vec{r}} d\vec{k}$$

$$M_{xy}(t,\vec{r}) = iM_0 \int_{\vec{k}} p(\vec{k}) e^{i2\pi\vec{k}\cdot\vec{r}} d\vec{k}$$

where $p(\vec{k}) = \int_{-\infty}^{t} \gamma B_1(s)\, ^3\delta(\vec{k}(s,t) - \vec{k})ds$

- The magnetization is the inverse transform of *p(k)*
- We want this to be a unit delta, multiplied by a weighting function

Small Tip Approximation Solution as k-space Integral

Multiply and divide by |*k'(s,t)*|:

$$p(\vec{k}) = \int_{-\infty}^{t} \underbrace{\frac{\gamma B_1(s)}{|k'(s,t)|}}_{W(k(s,t))} \underbrace{\, ^3\delta(\vec{k}(s,t) - \vec{k})|k'(s,t)|ds}_{\text{Unit Delta}}$$

If we assume W(k) is single-valued

$$p(\vec{k}) = W(\vec{k}) \int_{-\infty}^{t} \, ^3\delta(\vec{k}(s,t) - \vec{k})|k'(s,t)|ds$$

$$p(\vec{k}) = W(\vec{k}) S(\vec{k})$$

Small Tip Approximation Solution as k-space Integral

$$M_{xy}(t,\vec{r}) = iM_0 \int_{\vec{k}} p(\vec{k}) e^{i2\pi\vec{k}\cdot\vec{r}} d\vec{k}$$

$$p(\vec{k}) = W(\vec{k})S(\vec{k})$$

$$W(\vec{k}) = \frac{\gamma B_1(s)}{|k'(s,t)|}$$

k-space weighting

$$S(\vec{k}) = \int_{-\infty}^{t} {}^3\delta(\vec{k}(s,t) - \vec{k})|k'(s,t)|ds$$

k-space sampling

Small Tip Approximation Solution as k-space Integral

$$M_{xy}(t,\vec{r}) = iM_0 \int_{\vec{k}} W(\vec{k})S(\vec{k}) e^{i2\pi\vec{k}\cdot\vec{r}} d\vec{k}$$

So, the inverse Fourier transform of the k-space weighting will give us the excitation profile!

$$W(\vec{k}) = \frac{\gamma B_1(s)}{|k'(s,t)|}$$

k-space weighting

Design of 2D Excitation Pulses

2D Pulse Design

1. Choose a k-space trajectory

$$\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$$

2. Choose a weighting function

$$W(\vec{k})$$

3. Design the RF pulse

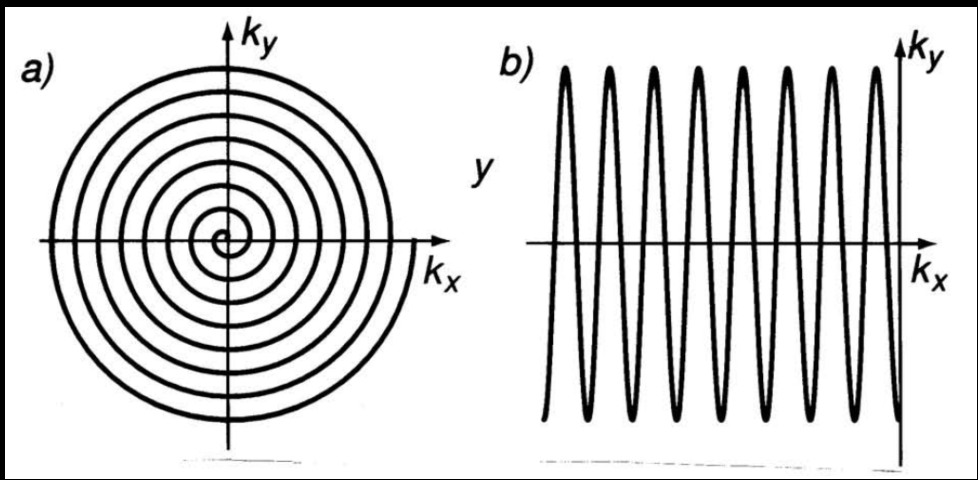
$$B_1(s) = \frac{1}{\gamma}W(\vec{k}) \cdot |k'(s,t)|$$

1. Choose a k-space trajectory

- Select a k-space trajectory that uniformly covers k-space
  - k-space extent ($-k_{max}$, k_{max}) $\Rightarrow$ spatial resolution
  - sampling density (Δk) $\Rightarrow$ spatial FOV

1. Choose a k-space trajectory

- Two most common choices:



- Spiral is common for pencil beams
- EPI is common for spectral-spatial pulses

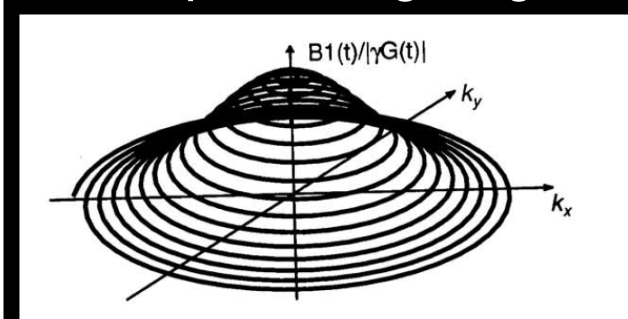
2. Choose a weighting function

$$M_{xy}(t, \vec{r}) = iM_0 \int_{\vec{k}} W(\vec{k}) S(\vec{k}) e^{i2\pi \vec{k}\cdot\vec{r}} d\vec{k}$$

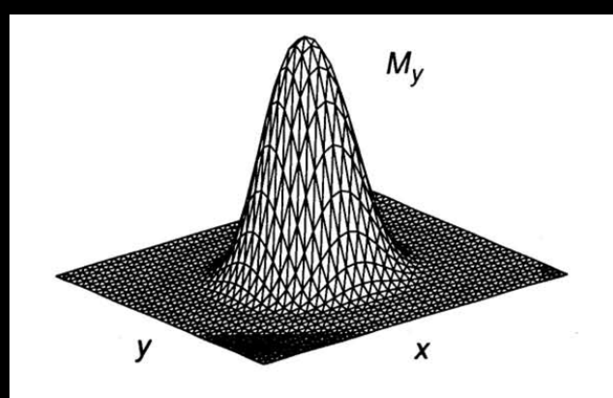
- An excitation profile is the inverse Fourier transform of the weighting function
- If you know what excitation profile you want, its Fourier transform will be the weighting function
- Localized excitation ⇒ low-pass k-space weighting

2. Choose a weighting function

k-space weighting



Excitation profile



3. Design the RF pulse

$$\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$$

$$k'(s,t) = -\frac{\gamma}{2\pi}\vec{G}(s)$$

$$B_1(s) = \frac{1}{2\pi}|\vec{G}(s)|W(\vec{k})$$

B_1 needs to be scaled for flip angle

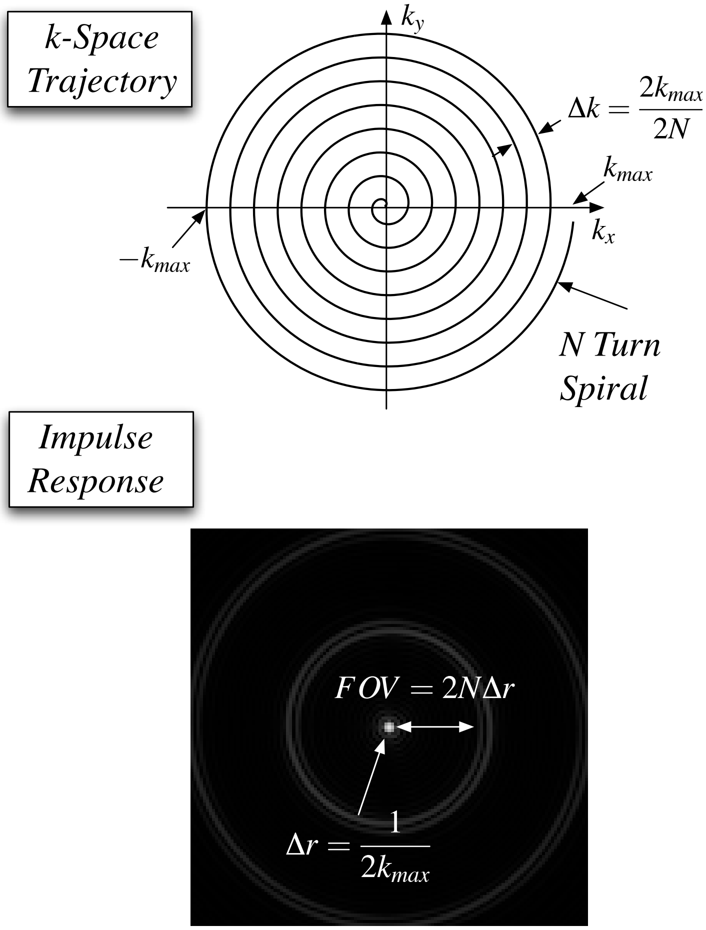
2D Spiral Pulse Design

- Two major choices:
  - Resolution Δr
    - Smallest volume / minimum transition width

$$\Delta r = \frac{1}{2k_{max}}$$

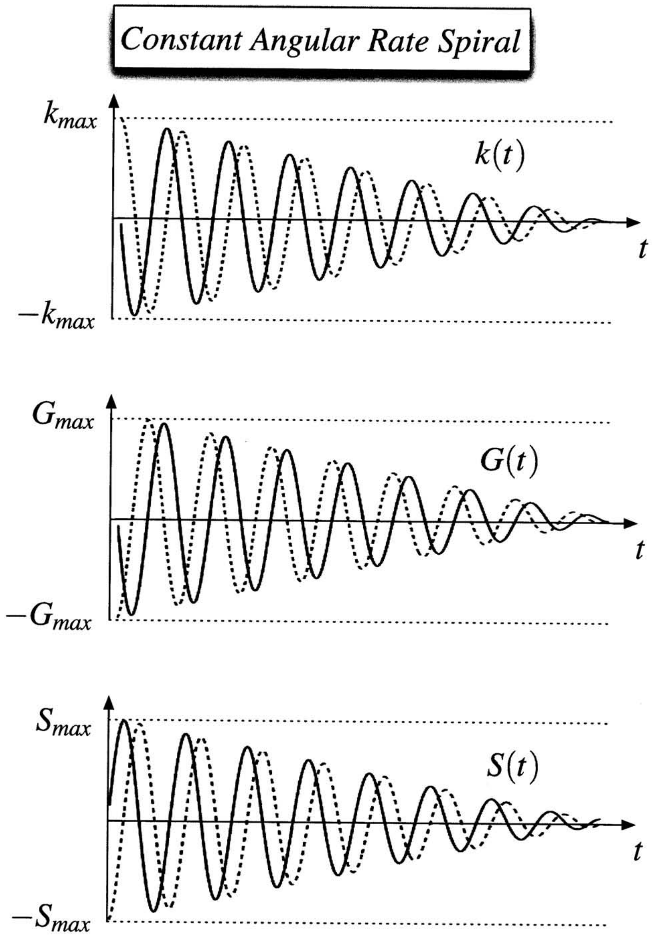
  - Field-of-View (FOV)
    - Distance to center of first sidelobe

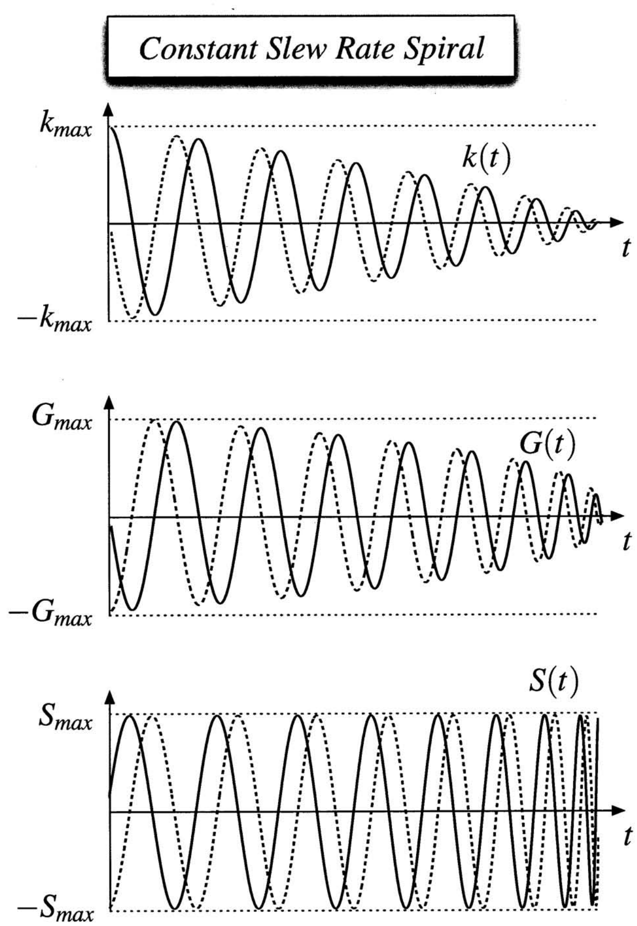
$$FOV = \frac{1}{\Delta k} = \frac{2N}{2k_{max}}$$



2D Spiral Pulse Design

- Spiral Gradient Design
  - Constant angular rate spiral
  - Constant slew rate spiral

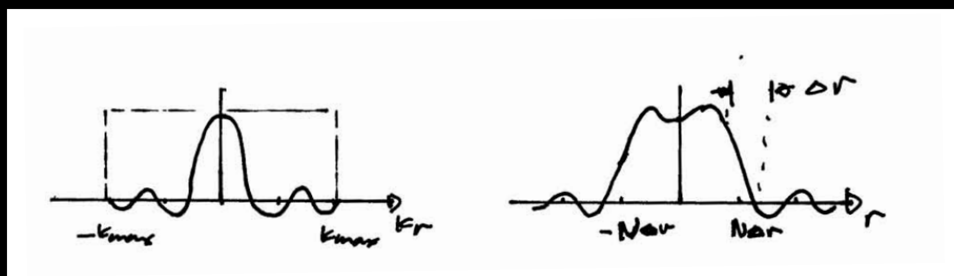

Constant Angular Rate Spiral
k_{max}
$-k_{max}$
$k(t)$
t
G_{max}
$-G_{max}$
$G(t)$
t
S_{max}
$-S_{max}$
$S(t)$
t


Constant Slew Rate Spiral
k_{max}
$-k_{max}$
$k(t)$
t
G_{max}
$-G_{max}$
$G(t)$
t
$S(t)$
S_{max}
$-S_{max}$
t

2D Spiral Pulse Design

- Truncated Jinc Weighting

$$W(\vec{k}) = jinc(N\frac{k_r}{k_{max}}) \cdot rect(\frac{k_r}{2k_{max}})$$



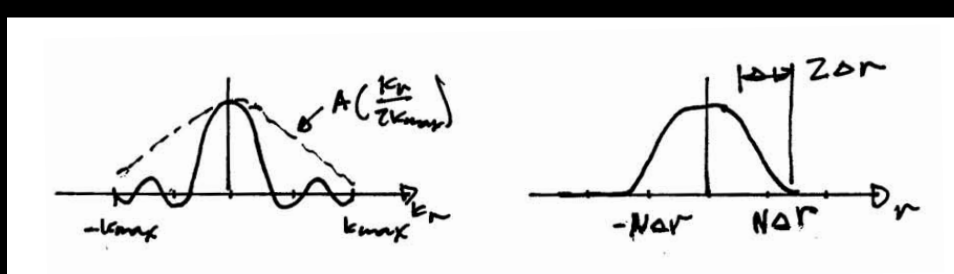
Minimum transition width,
but residual ripples

2D Spiral Pulse Design

- Windowed Jinc Weighting

$$W(\vec{k}) = jinc(N\frac{k_r}{k_{max}}) \cdot A(\frac{k_r}{2k_{max}})$$

Window Function



Doubled transition width,
but smoother response

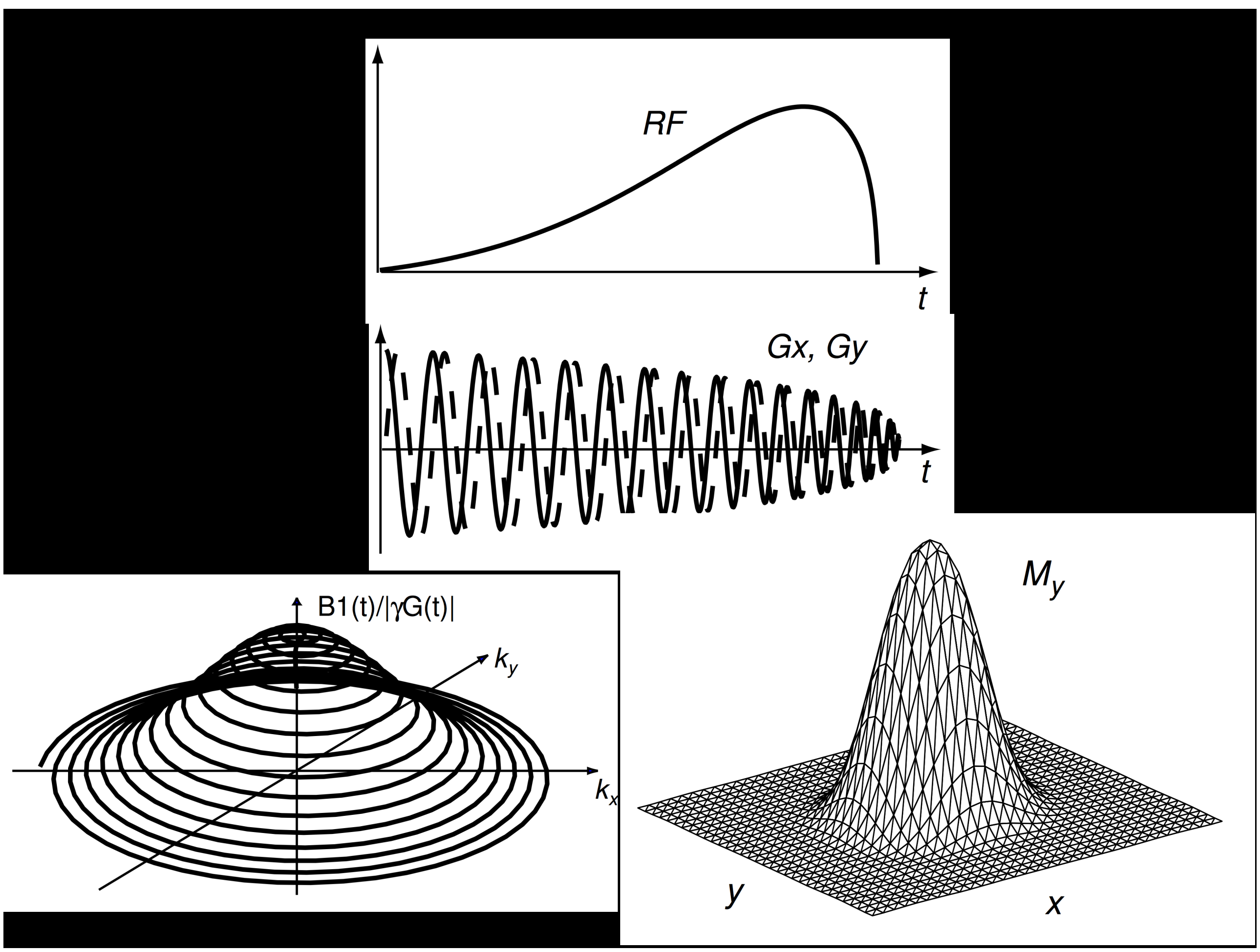
2D Spiral Pulse Design

- Calculation of the RF pulse
  - given W(k) and k(s,t)

$$W(\vec{k}) = \frac{\gamma B_1(s)}{|k'(s,t)|}$$

$$B_1(s) = \frac{1}{2\pi}|\vec{G}(s)|W(\vec{k})$$

  - needs to be scaled for flip angle
  - |k'(s,t)| is an estimate of the density compensation function d(t)



Conclusions

- N-D RF pulses are selective in N-dimensions
- The small tip approximation can be extended to describe the excitation k-space
- The small tip approximation solution can be used to show that the excitation profile of an N-D pulse is given by the inverse Fourier transform of the excitation k-space weighting

Conclusions

- An N-D RF pulse can be designed by:
  - Choosing a k-space trajectory
  - Choosing a k-space weighting function
  - Then calculating the $B_1(t)$ and $G(t)$ functions

Next Class

- N-D pulses with EPI trajectory
- Spatial-spectral pulses
- Matlab demo of N-D pulses

Thank You!

- Further reading
  - Read "Spatial-Spectral Pulses" p.153-163

- Acknowledgments
  - John Pauly's EE469b (RF Pulse Design for MRI)
  - Shams Rashid, Ph.D.

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