

Spatial Localization II

M219 - Principles and Applications of MRI

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2/8/2023

Course Overview

- 2023 course schedule
 - https://mrrl.ucla.edu/pages/m219_2023
- Assignments
 - Homework #2 is due on 2/15
- Office hours, Fridays 10-12pm
 - In-person (Ueberroth, 1417B)
 - Zoom is also available (<https://uclahs.zoom.us/j/98066349714?pwd=cnVmV1J5QjR1d3l3cmJkQnVLSFZVZz09>)

MRI Sampling Requirements

k-space Sampling

Remember that the collected data in MRI is discrete

Discrete sampling can lead to artifacts if not careful

Sampling considerations

- Field of View
- Spatial Resolution

Sampling Considerations

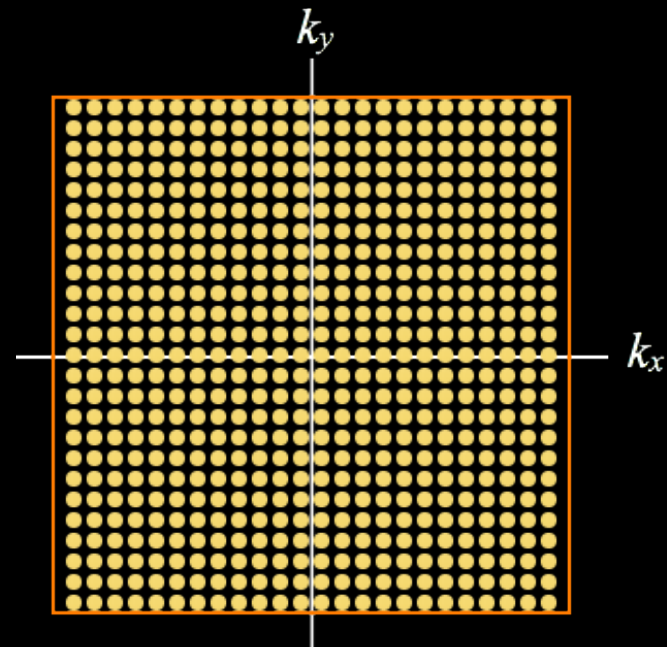
$$s(t) = m(k_x(t), k_y(t))$$

$$s(n\Delta t) = M(k_x(n\Delta t), k_y(n\Delta t))$$

Index

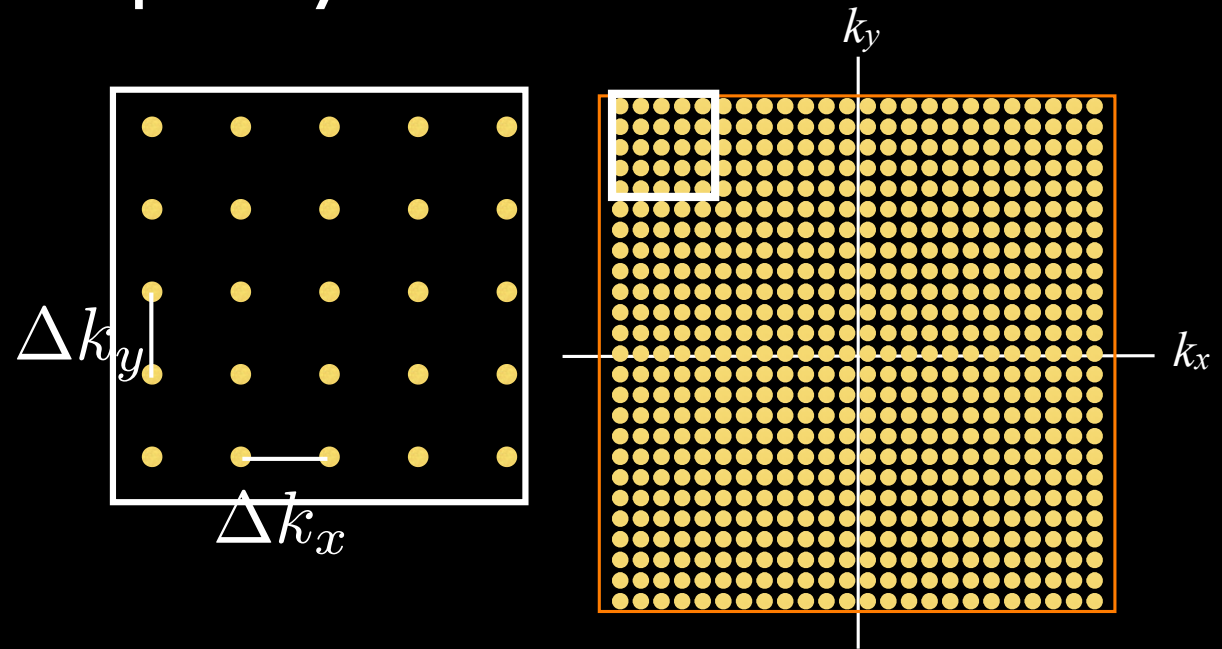
Sampling period

discrete sampling in spatial
frequency domain



Sampling Considerations

discrete sampling in spatial
frequency domain



$$w_{k_x} = N_{read} \times \Delta k_x$$

$$w_{k_y} = N_{PE} \times \Delta k_y$$

Review: Properties of DFT

Convolution

$$f(x) * h(x) \longleftrightarrow F(k_x) H(k_x)$$

Similarity (scaling)

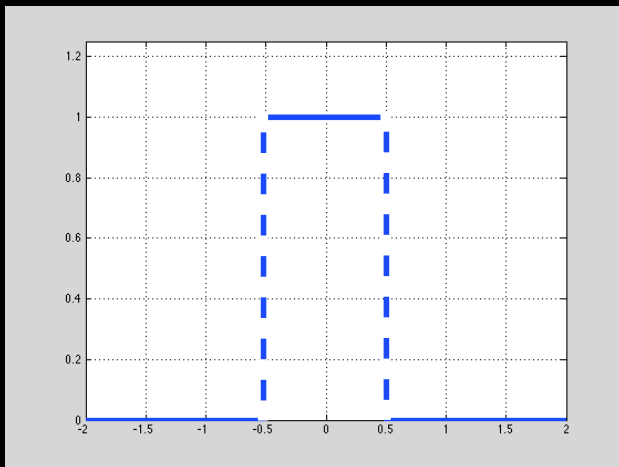
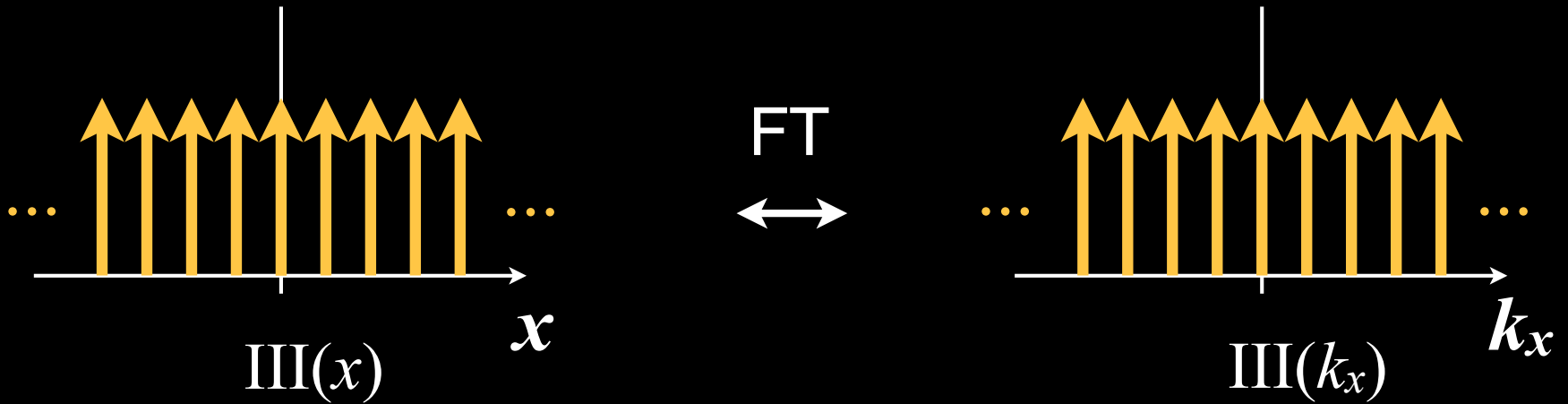
$$f(ax) \longleftrightarrow \frac{1}{|a|} F\left(\frac{k_x}{a}\right)$$

Shift

$$f(x - a) \longleftrightarrow \exp(-i2\pi(ak_x)) \cdot F(k_x)$$

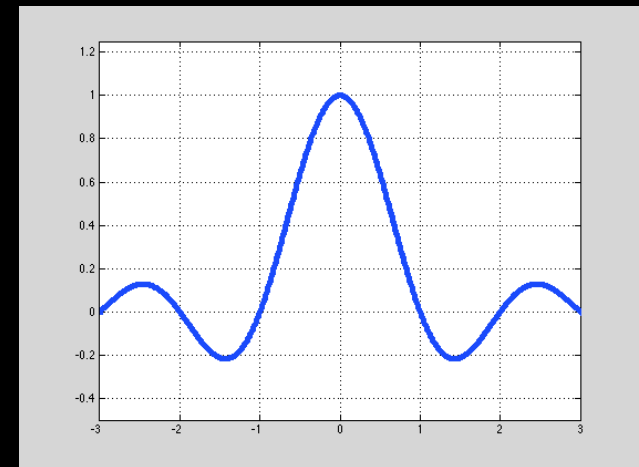
Review: Properties of DFT

comb or “Shah”



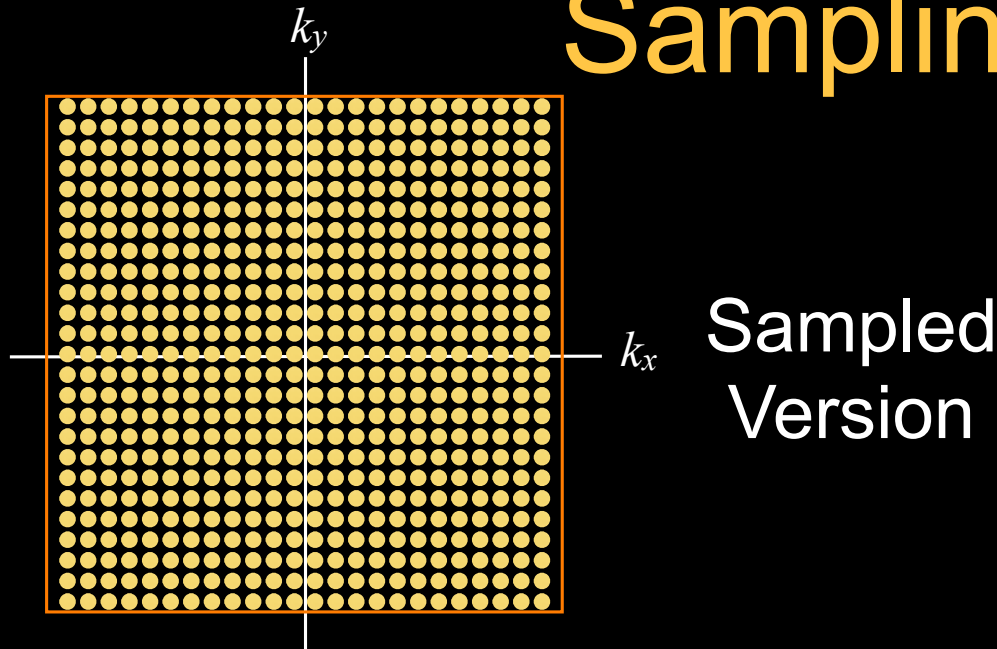
rect

FT



$$\text{sinc}(k_x) = \frac{\sin(\pi k_x)}{\pi k_x}$$

Sampling Model



$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \frac{1}{\Delta k_x \Delta k_y} \text{rect}\left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling
Extent

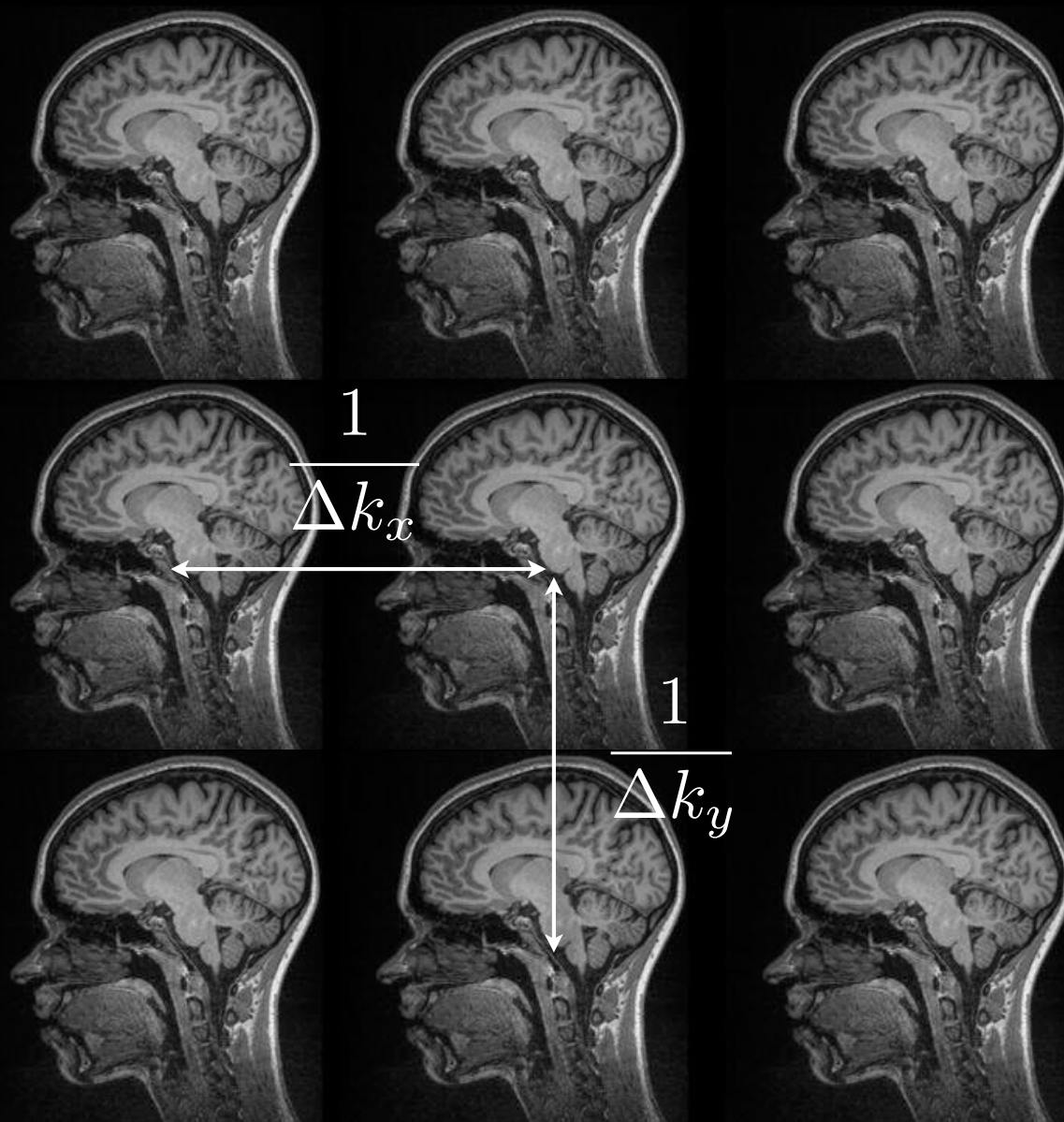
FT \updownarrow

$$\hat{m}(x, y) = m(x, y) * \text{III}(\Delta k_x x, \Delta k_y y) * \text{sinc}(w_{k_x} x) \text{sinc}(w_{k_y} y)$$

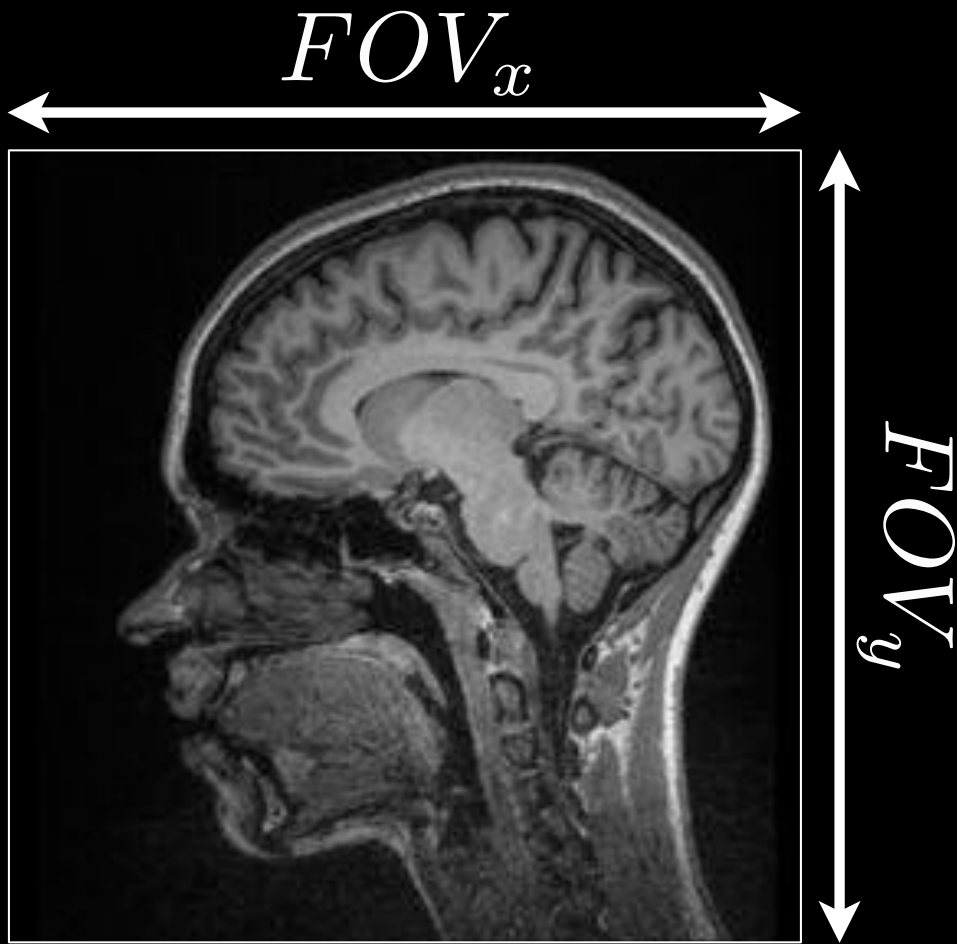
Field of View
Spatial Resolution

Field of View

$$m(x, y) * \text{III}(\Delta k_x x, \Delta k_y y)$$



Field of View



Eq. 5.76

$$\Delta k_x = \frac{1}{FOV_x} = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$\Delta k_y = \frac{1}{FOV_y} = \frac{\gamma}{2\pi} G_{yi} \tau_y$$



Field of View

To avoid any aliasing artifacts:

In phase encoding,

- Reduce Δk_y

Either lose spatial resolution

or

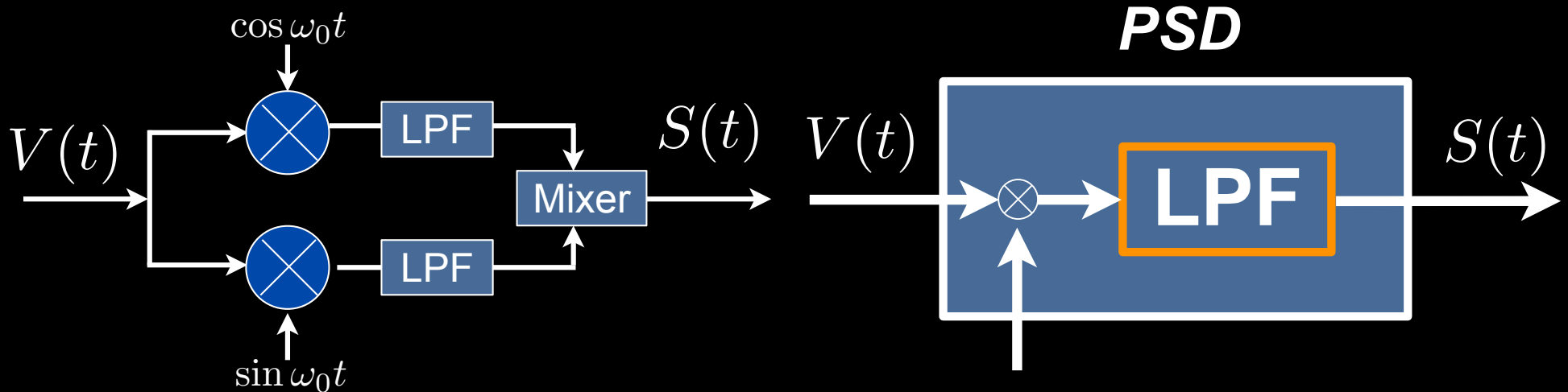
increase scan time

Field of View

To avoid any aliasing artifacts:

In frequency encoding,

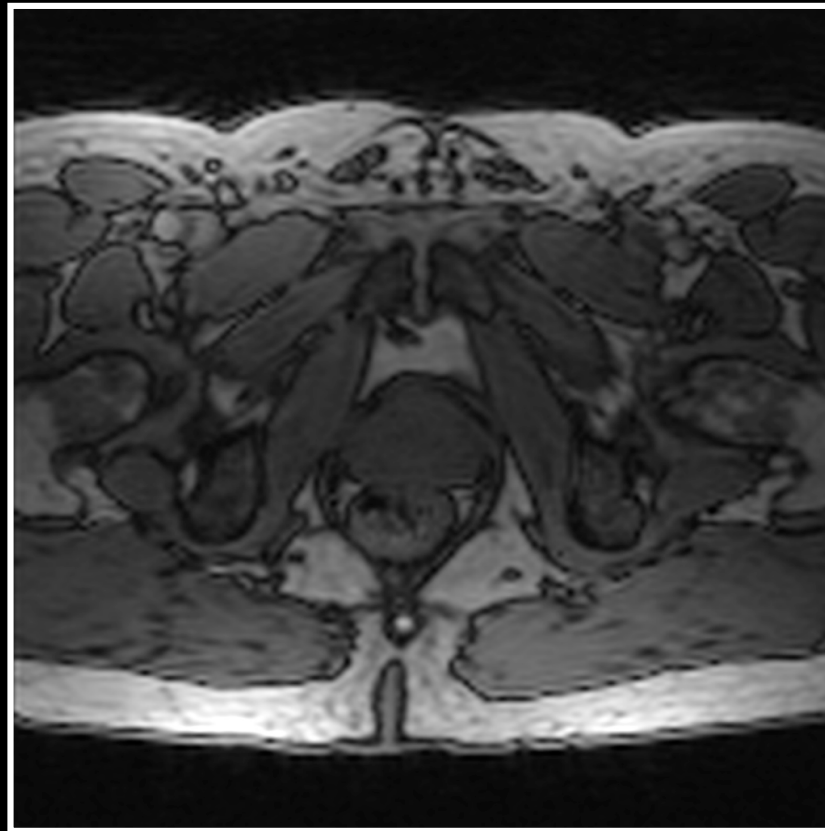
- Reduce Δk_x
- Utilize LPF (low pass filter)



Typically, put long axis of object
in readout direction

Field of View

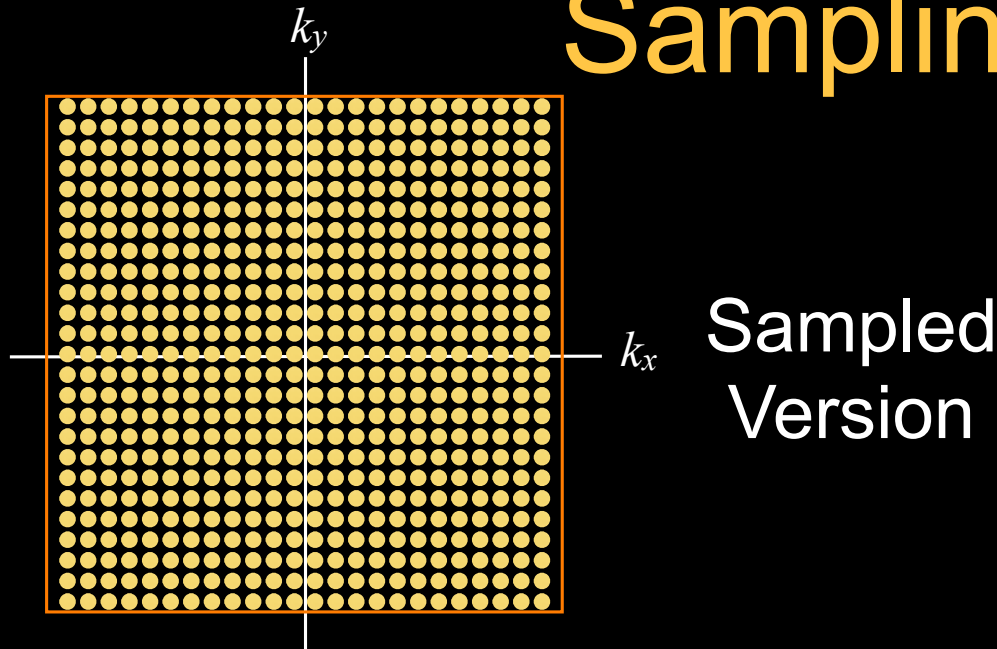
Prostate Imaging Example



Which direction will be
readout direction?

Spatial Resolution

Sampling Model



$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \frac{1}{\Delta k_x \Delta k_y} \text{rect}\left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling
Extent

FT \updownarrow

$$\hat{m}(x, y) = m(x, y) * \text{III}(\Delta k_x x, \Delta k_y y) * \text{sinc}(w_{k_x} x) \text{sinc}(w_{k_y} y)$$

Field of View
Spatial Resolution

Point Spread Function (PSF)

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \frac{1}{\Delta k_x \Delta k_y} \underbrace{\square\left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)}_{\text{Extent}}$$

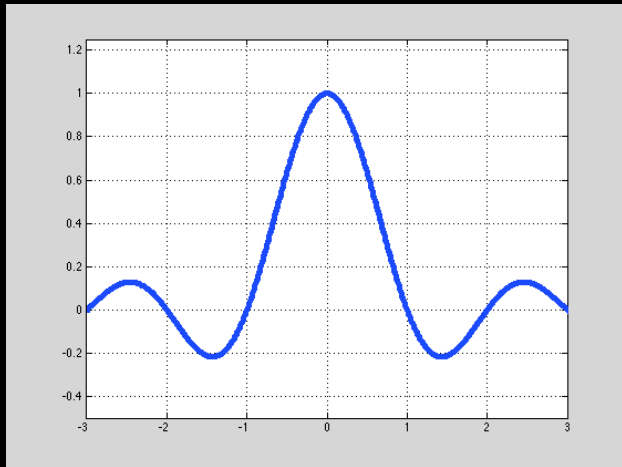
$$\hat{M}'(k_x, k_y) = \hat{M}(k_x, k_y) \cdot \text{window}$$

$$\text{PSF} = \text{FT}(\text{window})$$

Point spread function can show
the extent of blurring of the image

Spatial Resolution

$$m(x, y) * \text{sinc}(w_{k_x} x) \text{sinc}(w_{k_y} y) w_{k_x} w_{k_y}$$



Main lobe causes blurring!
(spatial resolution)

Spatial resolution: δ_x, δ_y

$$\delta_x = \frac{1}{w_{k_x}} \quad \delta_y = \frac{1}{w_{k_y}}$$

Spatial Resolution

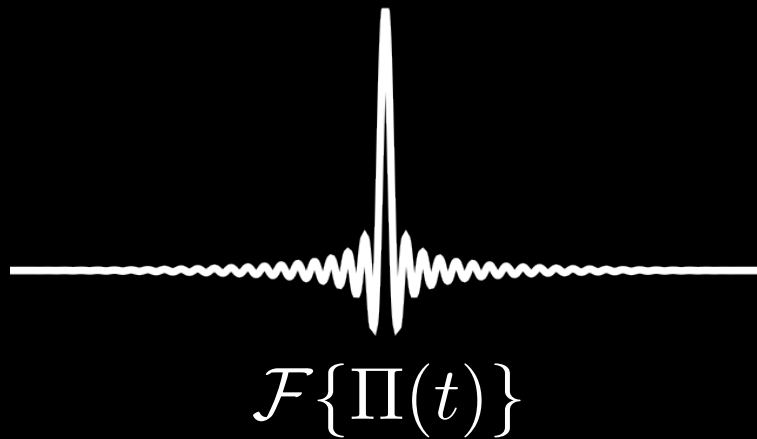
- Spatial resolution of an imaging system is the smallest separation δx of two point sources necessary for them to remain resolvable in the resultant image.

$$\hat{I}(x) = I(x) * h(x)$$

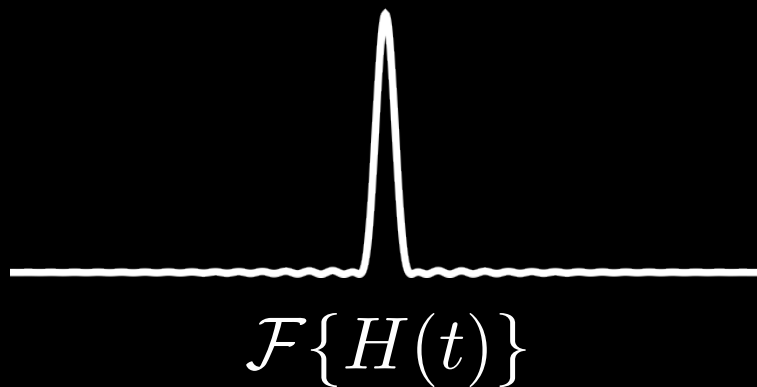
The diagram illustrates the relationship between the terms in the equation above. Three white arrows point upwards from the labels below to the corresponding terms in the equation. The label 'Image' is positioned below the $\hat{I}(x)$ term, 'Object' is below the $I(x)$ term, and 'Point Spread Function' is below the $h(x)$ term.

Image Object Point
Spread
Function

PSFs



Narrower central peak,
but lots of ringing



Reduced ringing, but
broader central peak

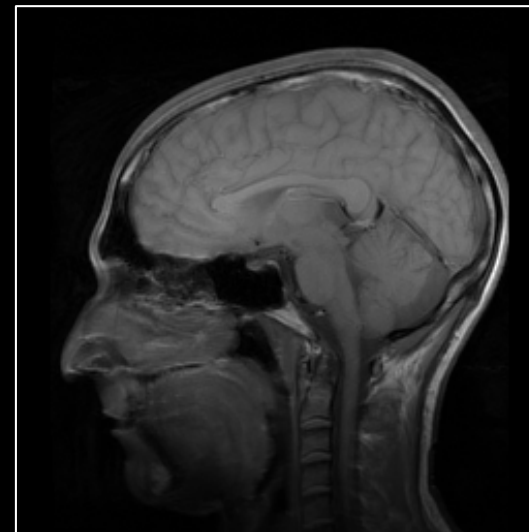
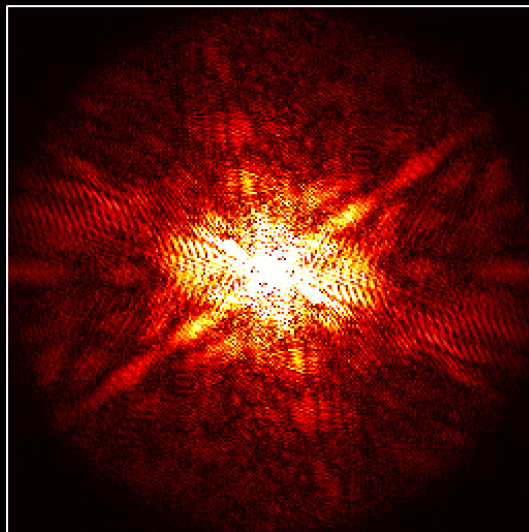
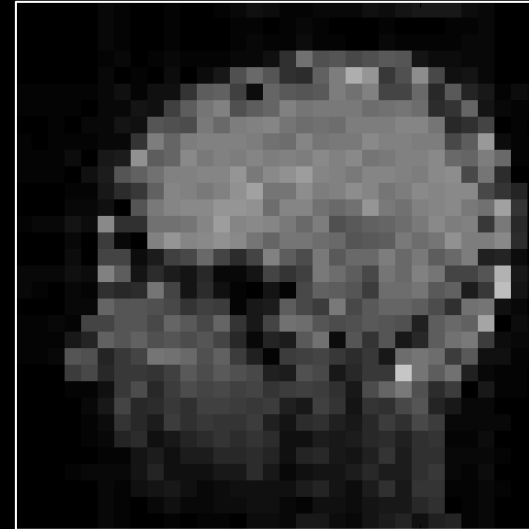
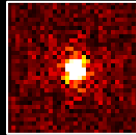
PSFs

Filters can be used to reduce ringing artifacts but often at the expense of spatial resolution

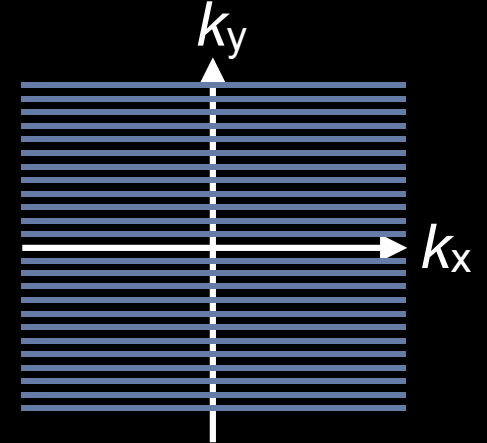
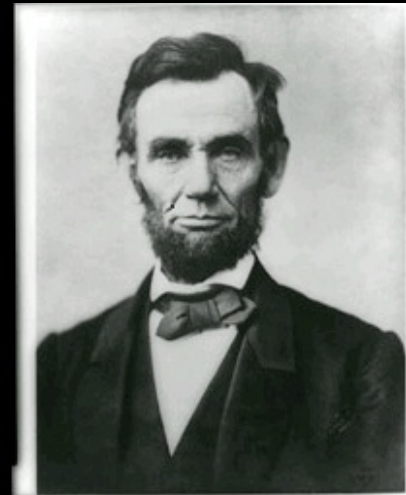
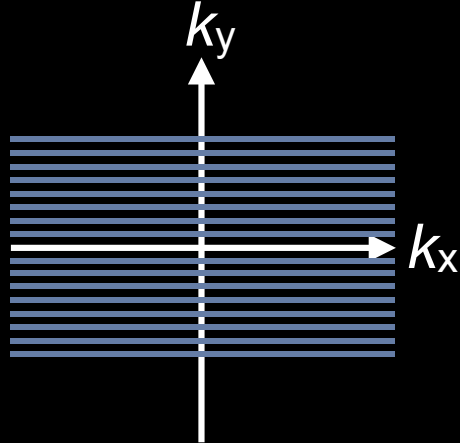
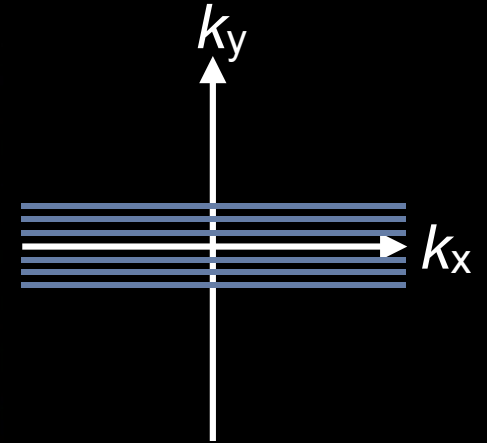
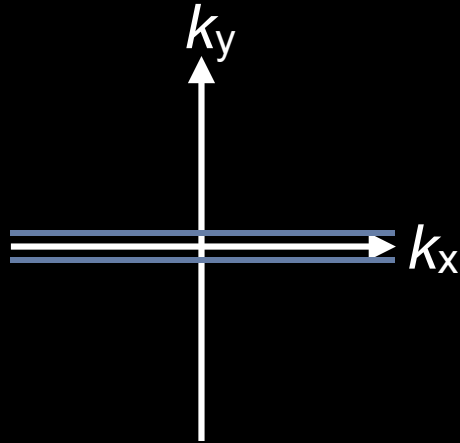
Hamming window seems to have good balance in reducing ringing

Finite Sampling

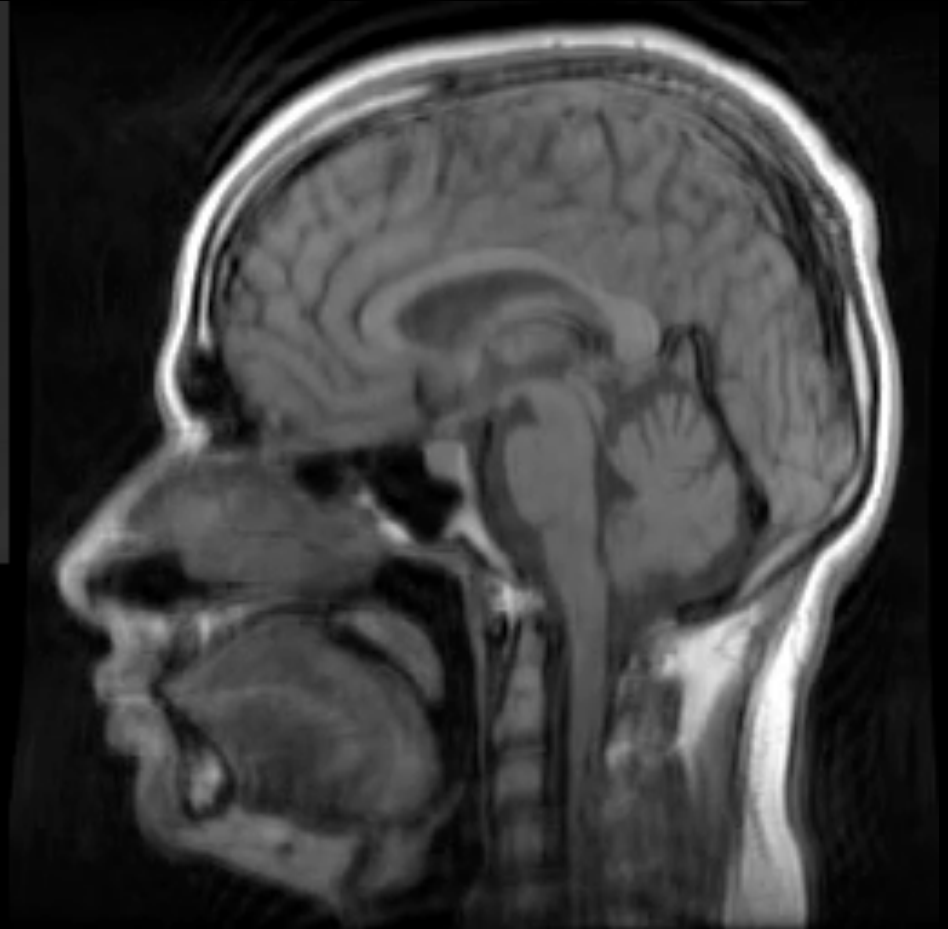
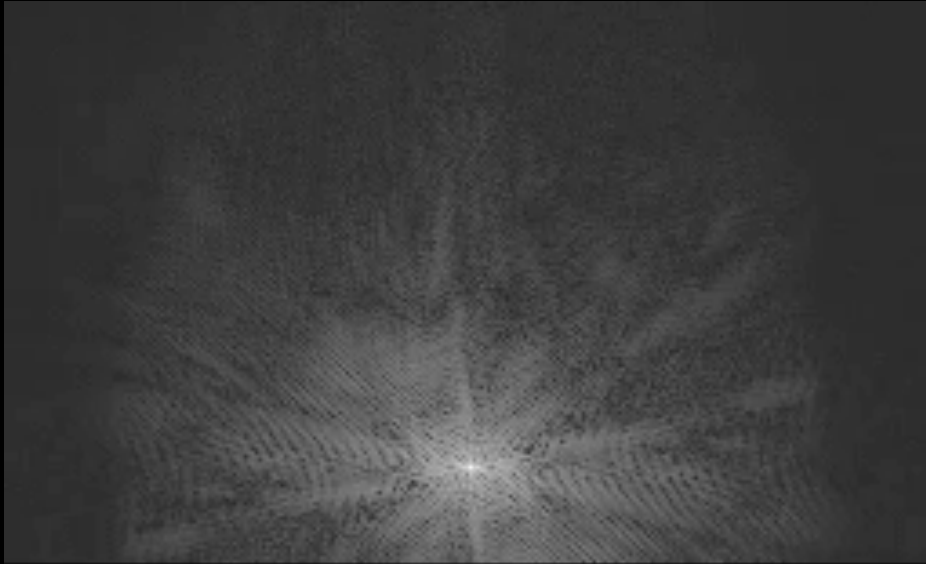
$$W_h = \frac{1}{N\Delta k} = \frac{FOV}{N}$$



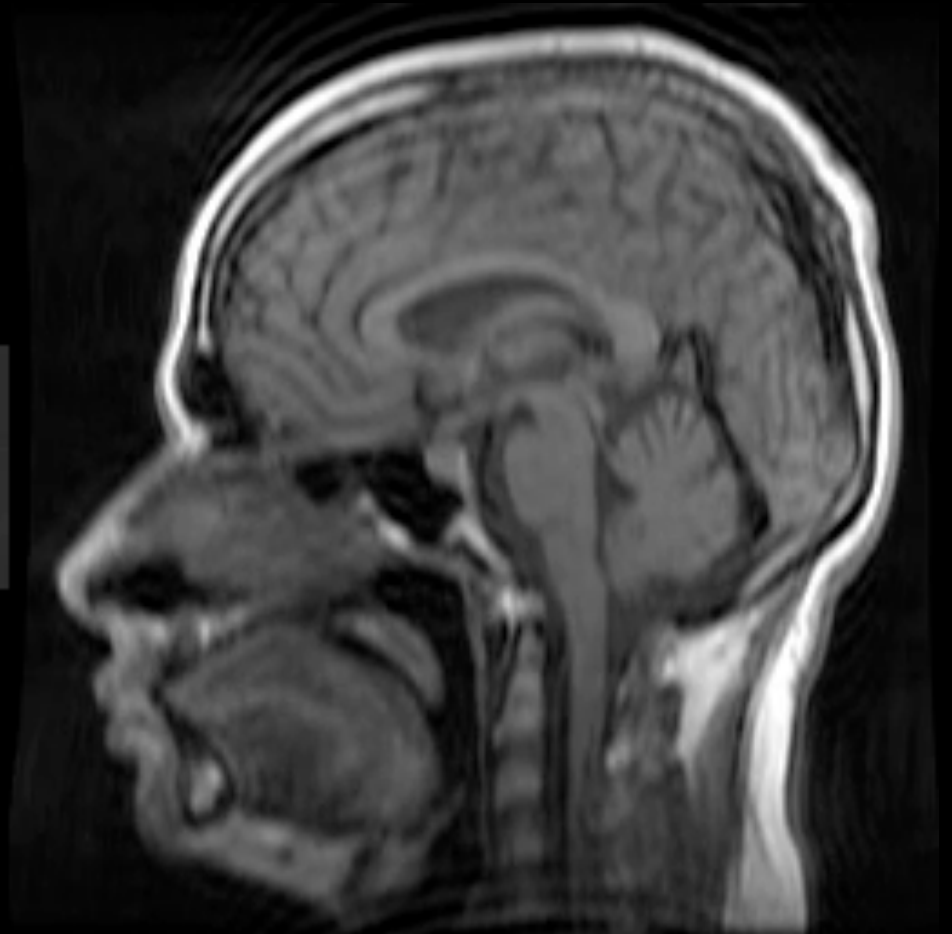
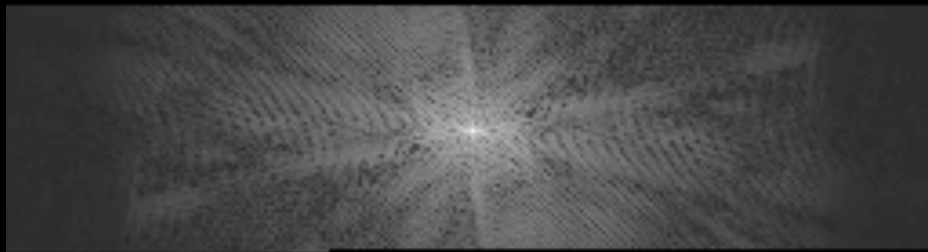
k-space Sampling



k-space Sampling

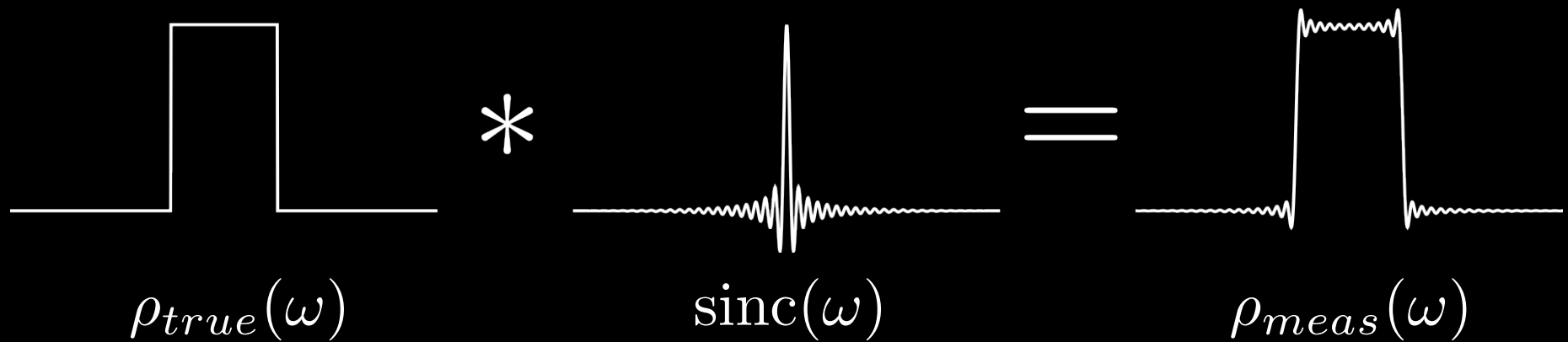


k-space Sampling



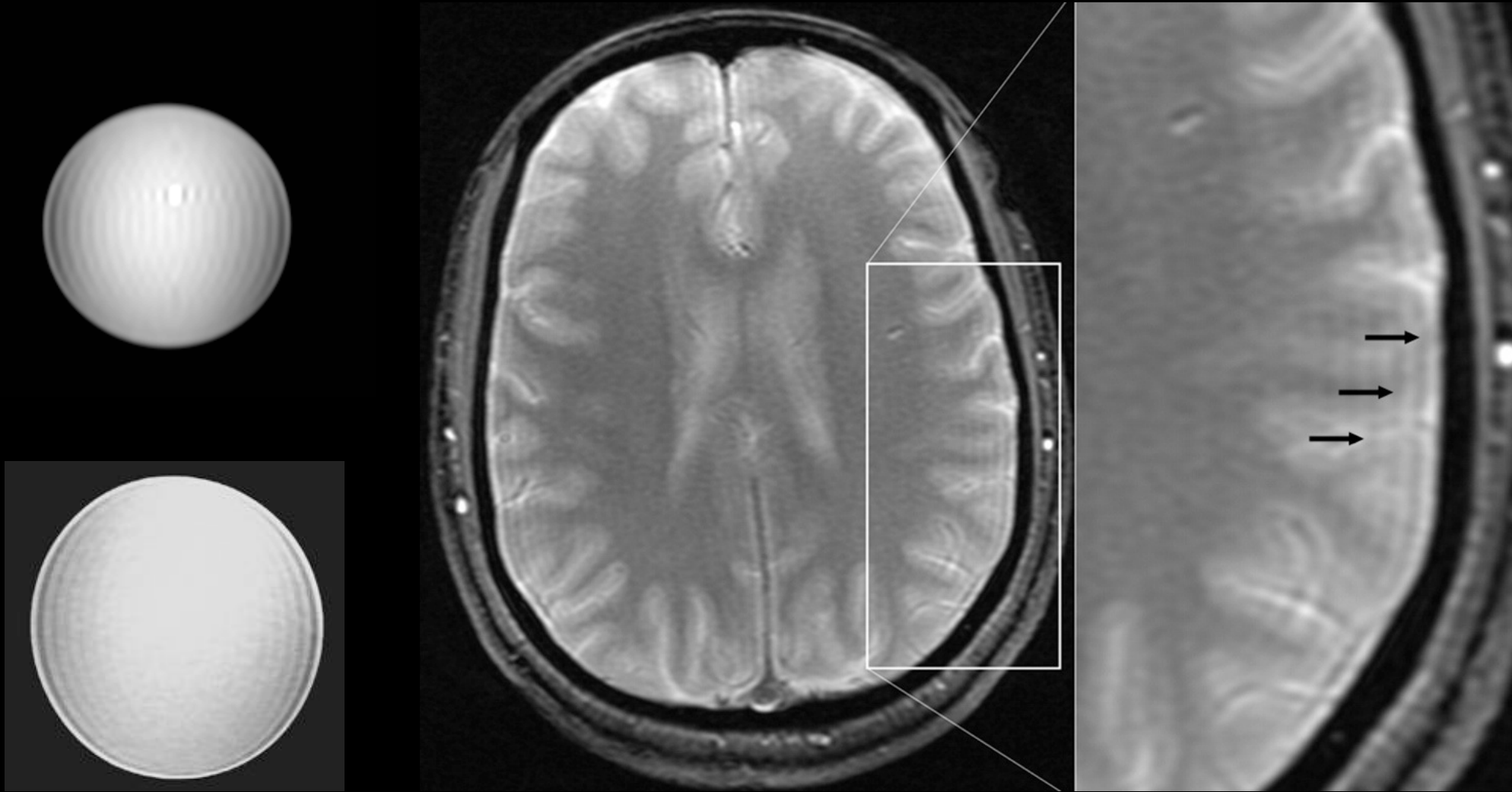
Gibb's Ringing

Distortions in the profile arising from the finite sampling of the data



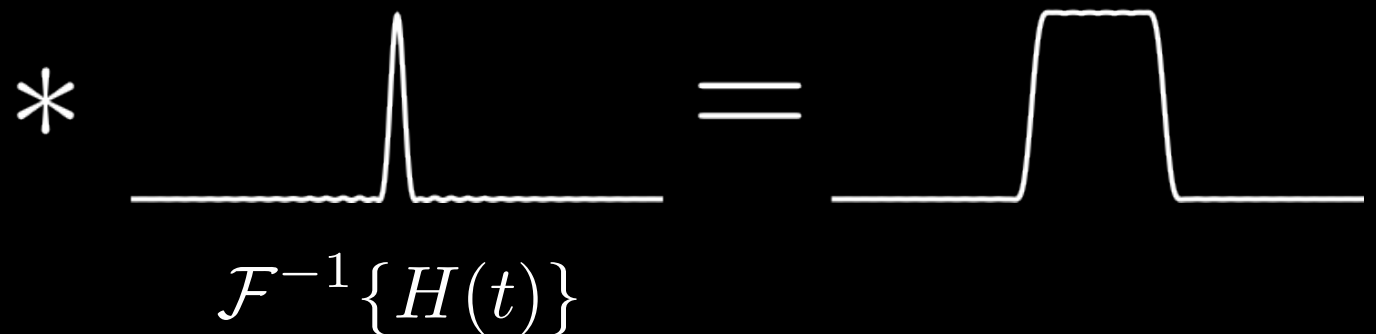
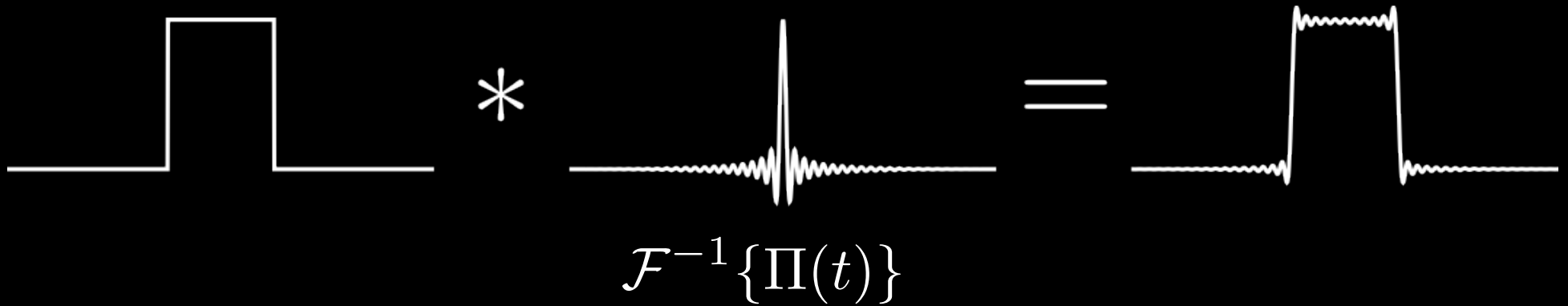
This type of distortion is most commonly referred to as Gibbs' ringing

Examples of Gibb's Ringing



Gibb's Ringing

how to reduce ringing



Hamming window can be used to reduce ringing

Questions?

- Related reading materials
 - Nishimura - Chap 6.1, 6.2, 6.4

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<http://mrri.ucla.edu/sunglab>

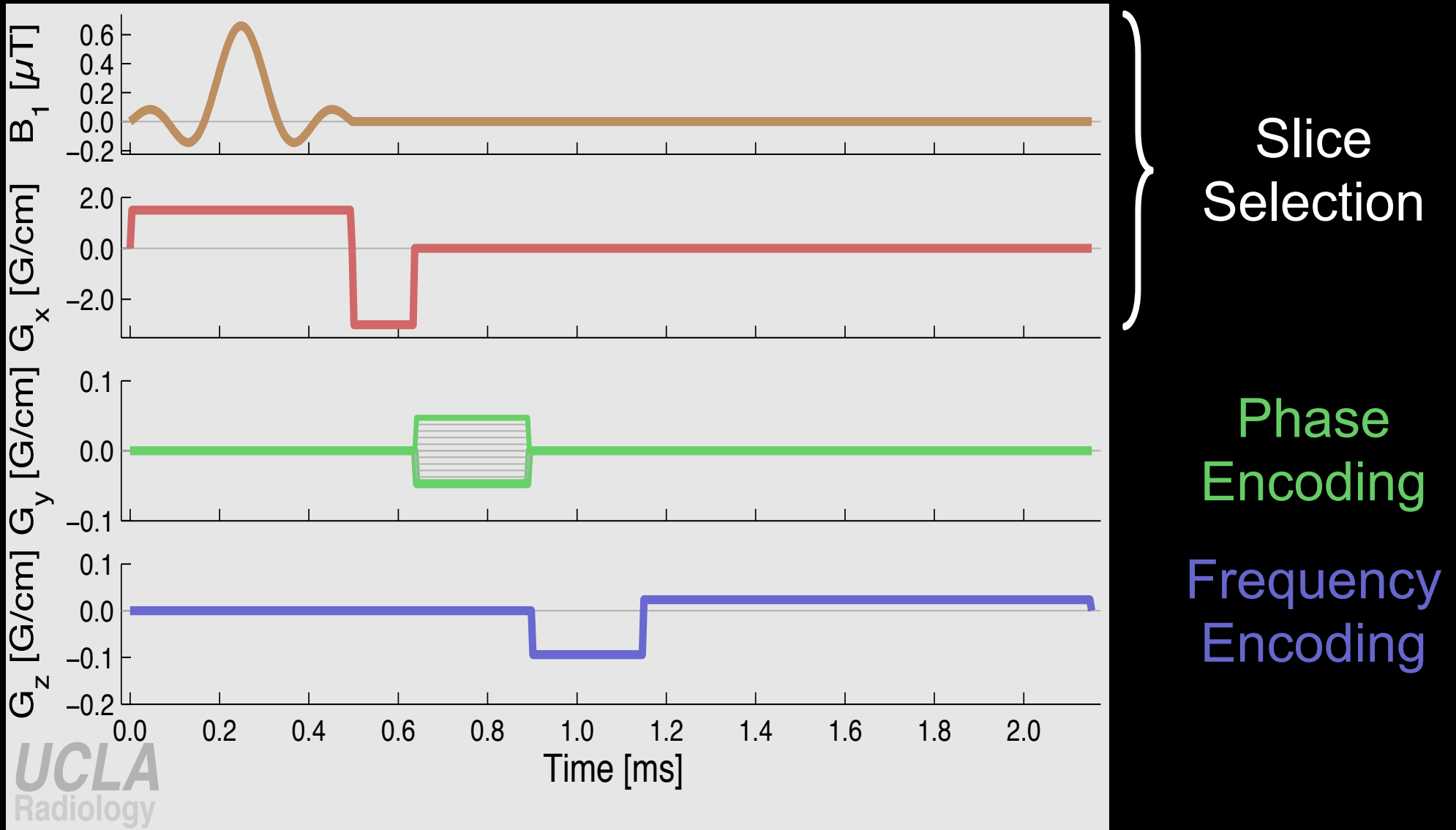
Spatial Localization

Spatial Encoding

- Three key steps:
 - **Slice selection**
 - You have to pick slice!
 - **Phase Encoding**
 - You have to encode 1 of 2 dimensions within the slice.
 - **Frequency Encoding (aka *readout*)**
 - You have to encode the other dimension within the slice.



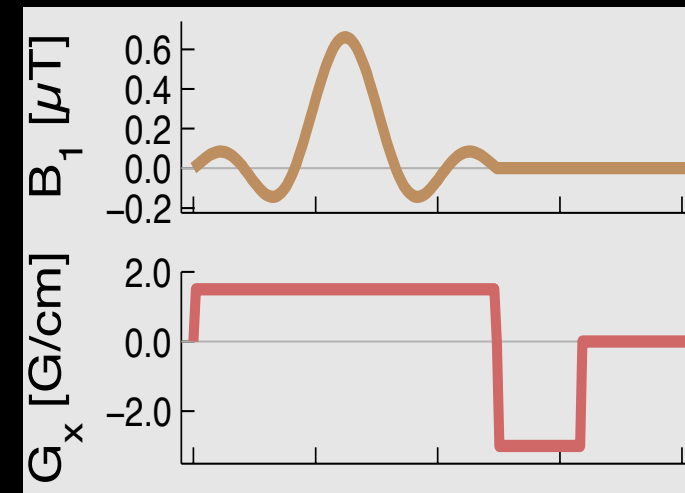
3 Steps for Spatial Localization



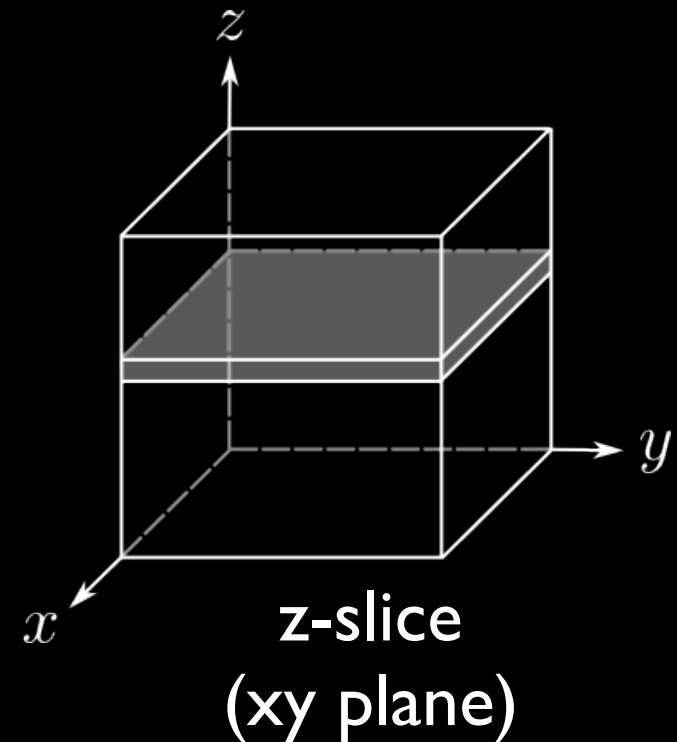
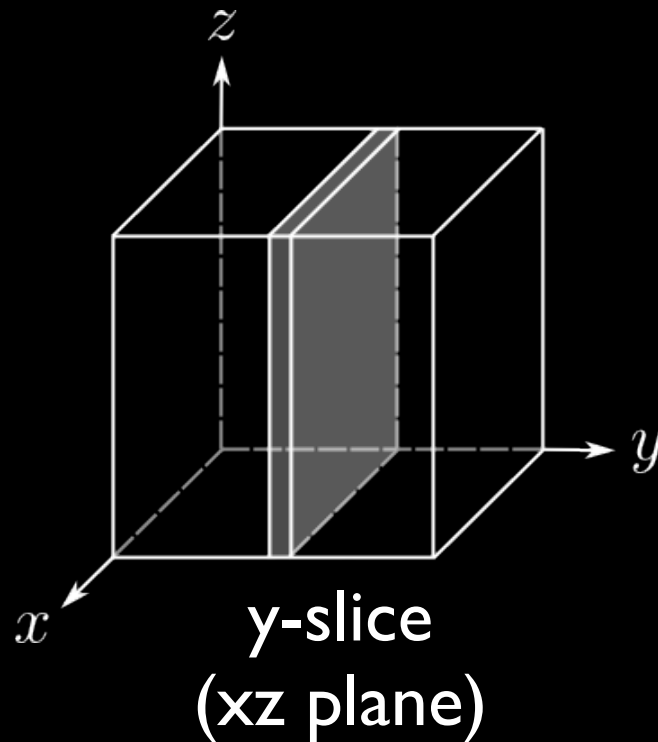
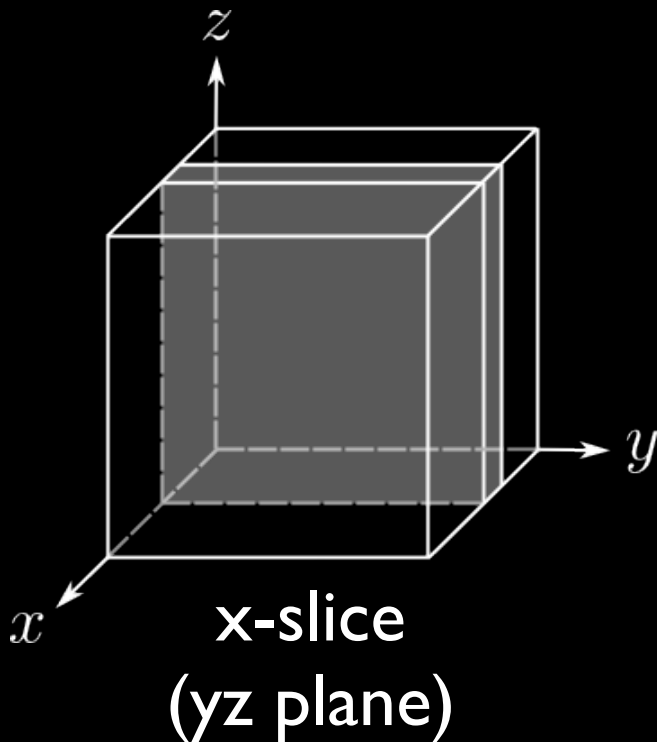
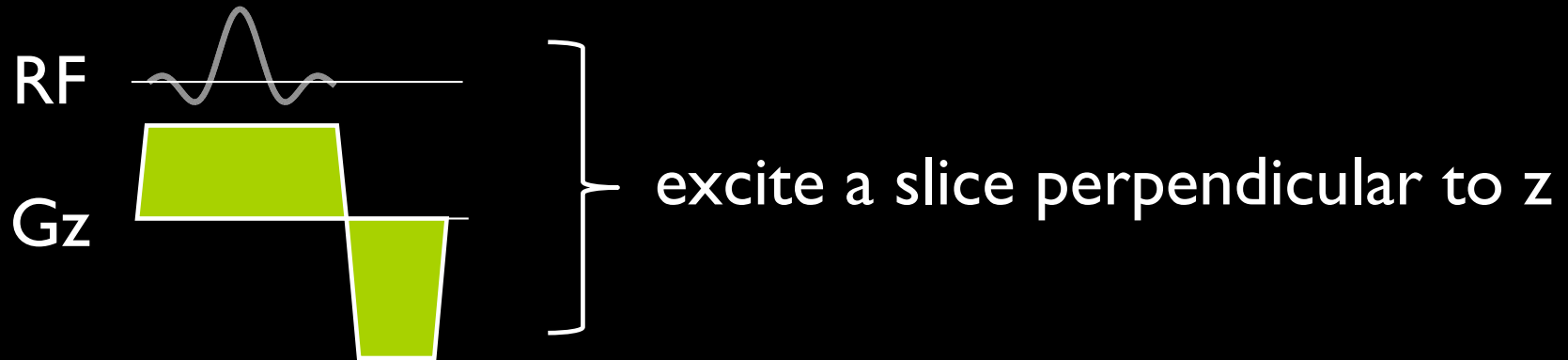
Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

Slice Selection

- Consists of:
 - RF (B_1) Pulse
 - Contains frequencies matched to slice of interest
 - Slice selection gradient
 - Constant magnitude
 - Slice re-phasing gradient
 - Increases SNR
 - Re-phases spins within slice
 - AKA “slice refocusing gradient”
- **Permits exciting the slice of interest.**



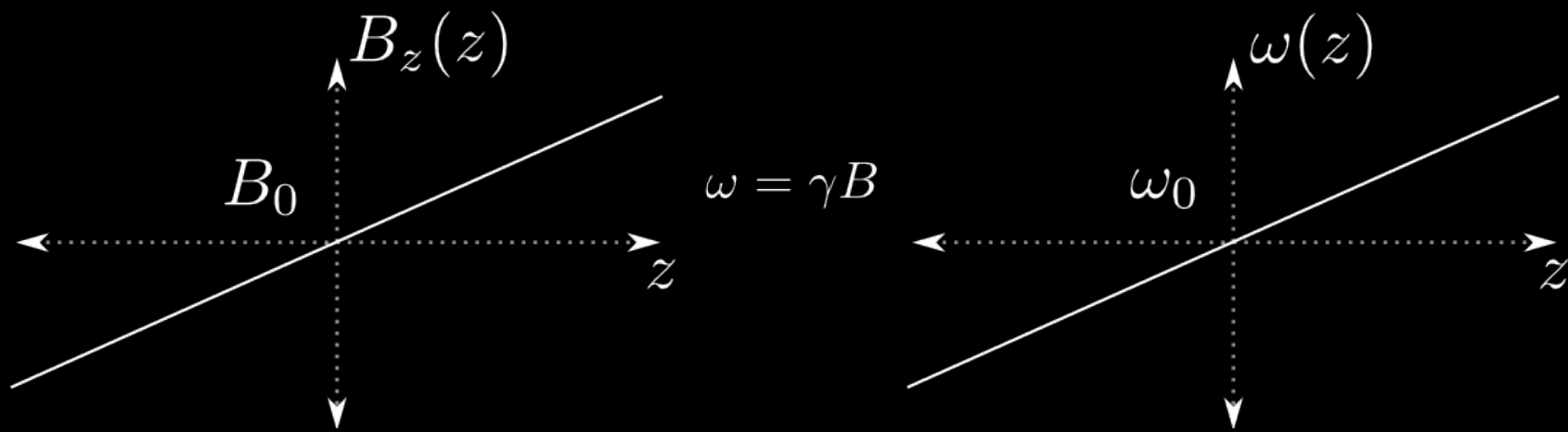
Selective Excitation



Gradients?

gradients produce a spatial distribution of frequencies

$$B_z(z) = B_0 + G_z \cdot z$$



$$\omega(z) = \omega_0 + \gamma G_z \cdot z$$

there is a direct correspondence between
frequency and spatial position

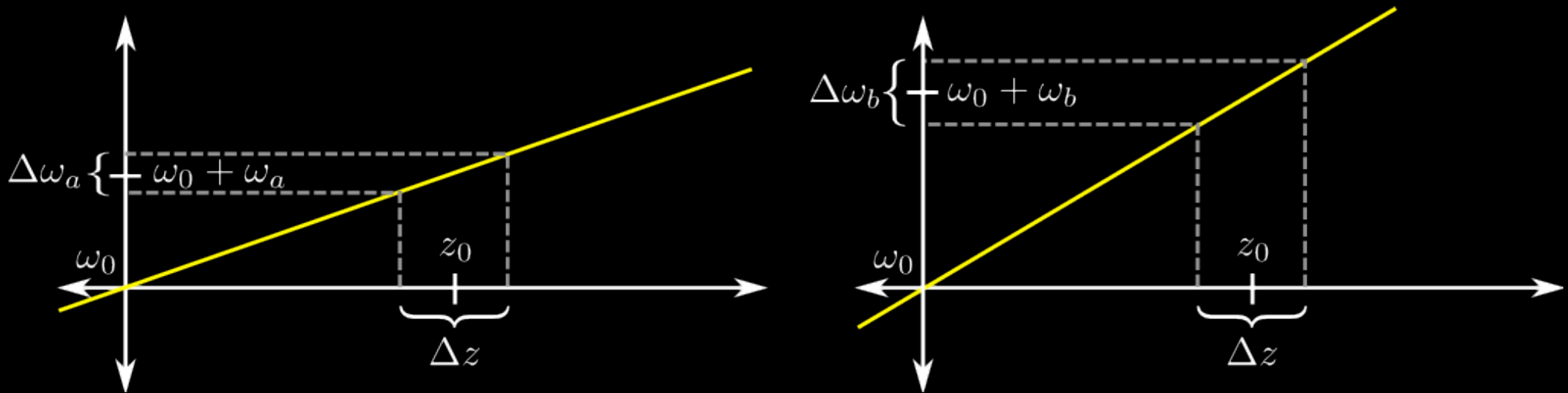
Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{pmatrix} \rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} + G_z z \end{pmatrix}$$

Slice Selection

how do we “excite” a certain slice?



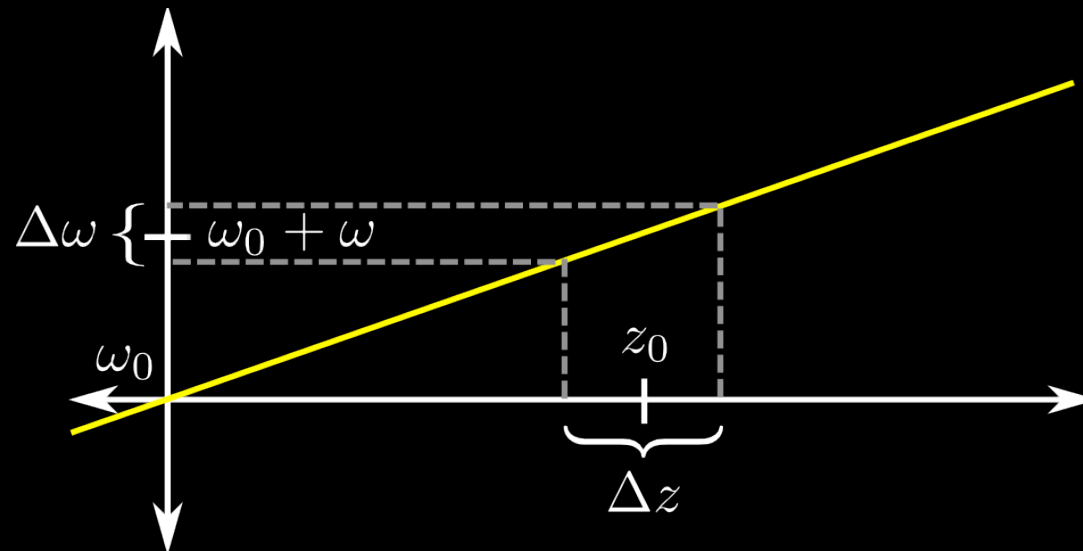
the strength of the gradient affects all parameters for the same spatial location

$$\Delta\omega_a < \Delta\omega_b$$

$$\omega_a < \omega_b$$

Slice Selection

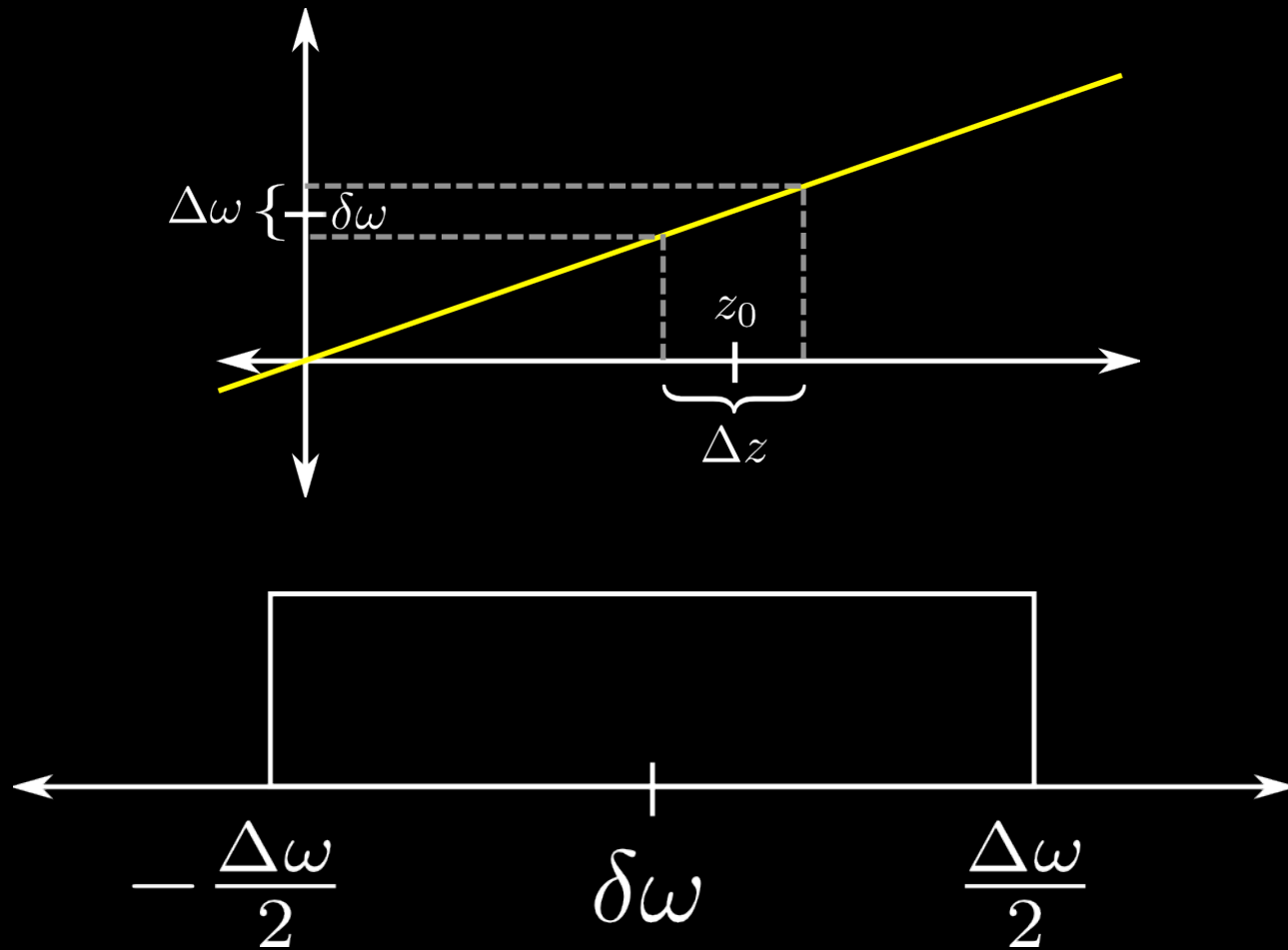
how do we physically set the parameters?



ω - the carrier frequency of the RF pulse

$\Delta\omega$ - frequency bandwidth of the RF pulse

Slice Selection



we want a pulse with as rectangular of an slice profile as possible

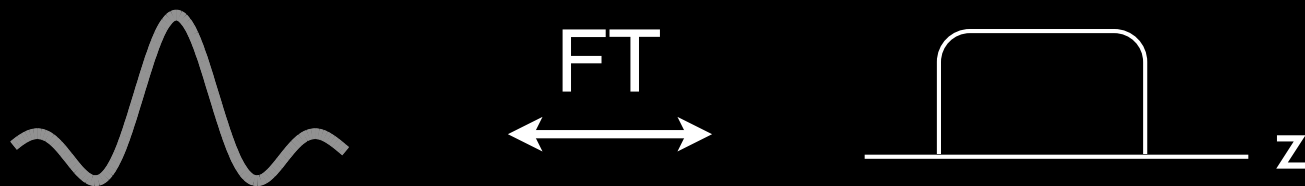
Selective Excitation

changing the shape of the pulse affects
the bandwidth of excitation

how do we know which shape to use?

small angle approximation

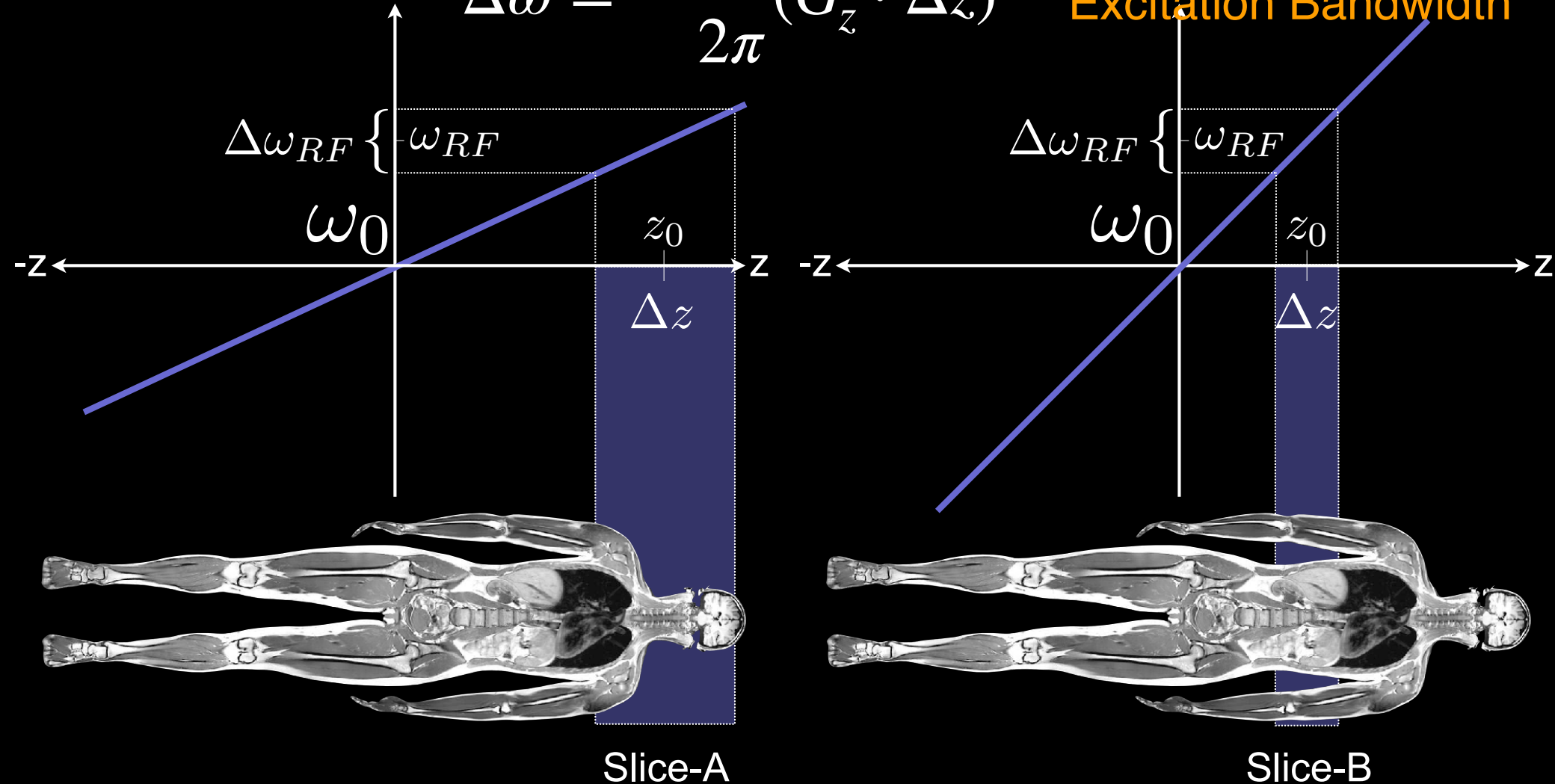
We will show the slice profile depends on



Slice Selective Excitation

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z)$$

Excitation Bandwidth



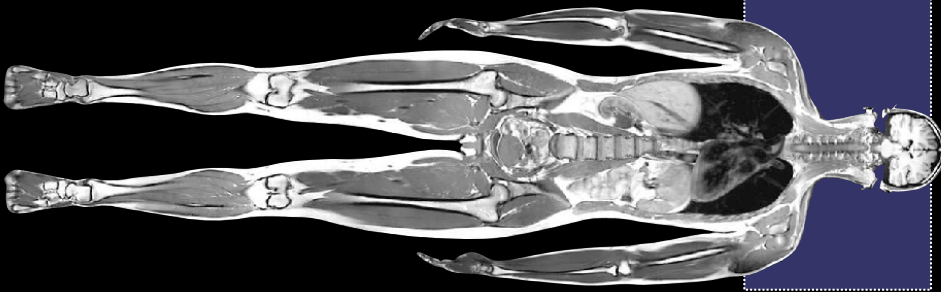
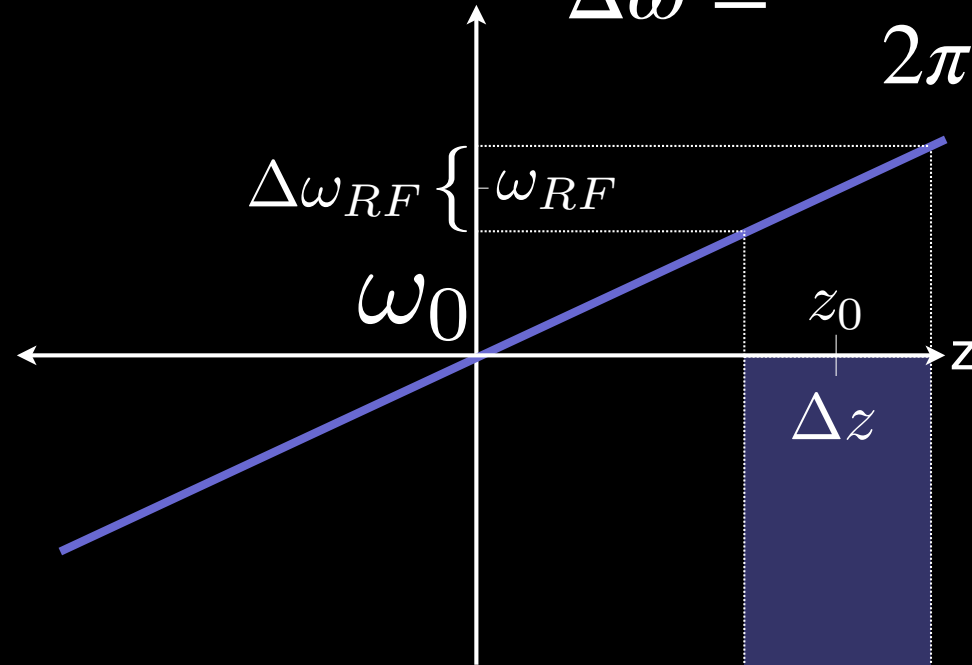
How do you move the slice along $\pm z$?
 Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.
 Do we usually acquire $\omega_{RF} > \omega_0$?

Time Bandwidth Product (TBW)

- **Time bandwidth (TBW) product:**
 - **Pulse Duration [s] x Pulse Bandwidth [Hz]**
 - **Unitless**
 - **# of zero crossings**
 - **High TBW**
 - Large # of zero crossings \therefore fewer truncation artifacts
 - Longer duration pulse
- **Examples:**
 - **TBW = 4, RF = 1ms**
 - Excitation (RF) bandwidth?
 - Required G_z for 1cm slice?
 - **TBW = 16, RF = 1ms**
 - Excitation (RF) bandwidth?
 - Required G_z for 1cm slice?

Slice Selective Excitation - Example

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z) \quad \text{Excitation Bandwidth}$$



Slice-A

$$TBW = \tau_{RF} \cdot \Delta\omega_{RF}$$

$$\begin{aligned} \Delta\omega_{RF} &= \frac{TBW}{\tau_{RF}} \\ &= \frac{4}{1\text{ms}} \\ &= 4\text{kHz} \end{aligned}$$

$$G_z = \frac{\Delta\omega_{RF}}{\gamma\Delta z}$$

$$\begin{aligned} &= \frac{4000\text{Hz}}{42.57\text{e6} \frac{\text{Hz}}{\text{T}} \frac{1\text{T}}{10000\text{G}} \cdot 10\text{mm}} \\ &= 0.94 \frac{\text{G}}{\text{cm}} \end{aligned}$$

Selective Excitation

- What factors control slice selection?

$$B_1^e(t)$$

Pulse envelope function

(e.g. $B_{1,\max}$ and $\Delta\omega$)

$$\omega_{RF}$$

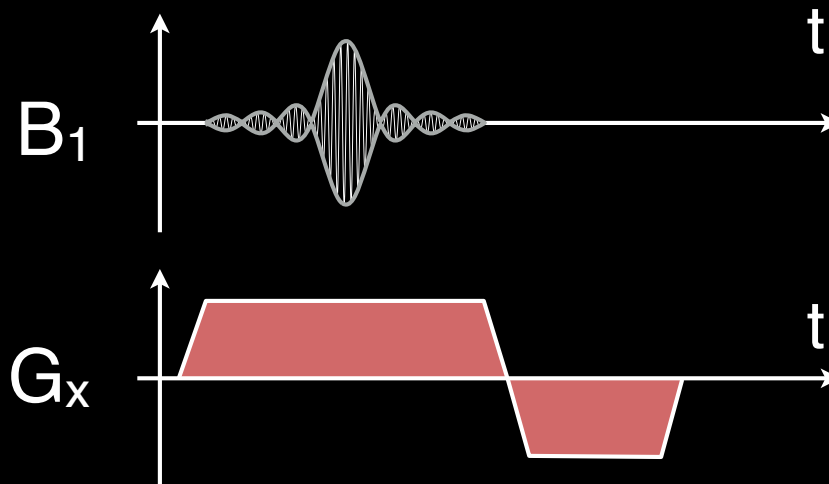
Excitation carrier frequency

BW

RF pulse bandwidth

$$\vec{G}$$

Gradient amplitude





RF Pulse Bandwidth and Slice
Profile:
Small Tip Angle Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0} - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \quad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1$$

$$M_z \approx M_0 \rightarrow \text{constant}$$

$$\left. \begin{array}{l} \sin \theta \approx \theta \\ \cos \theta \approx 1 \\ M_z \approx M_0 \rightarrow \text{constant} \end{array} \right\} \frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

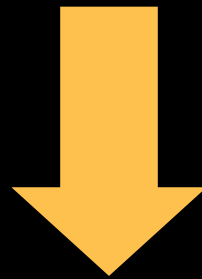
$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

$$M_{xy}(t, z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z \cdot (t-s)} ds$$



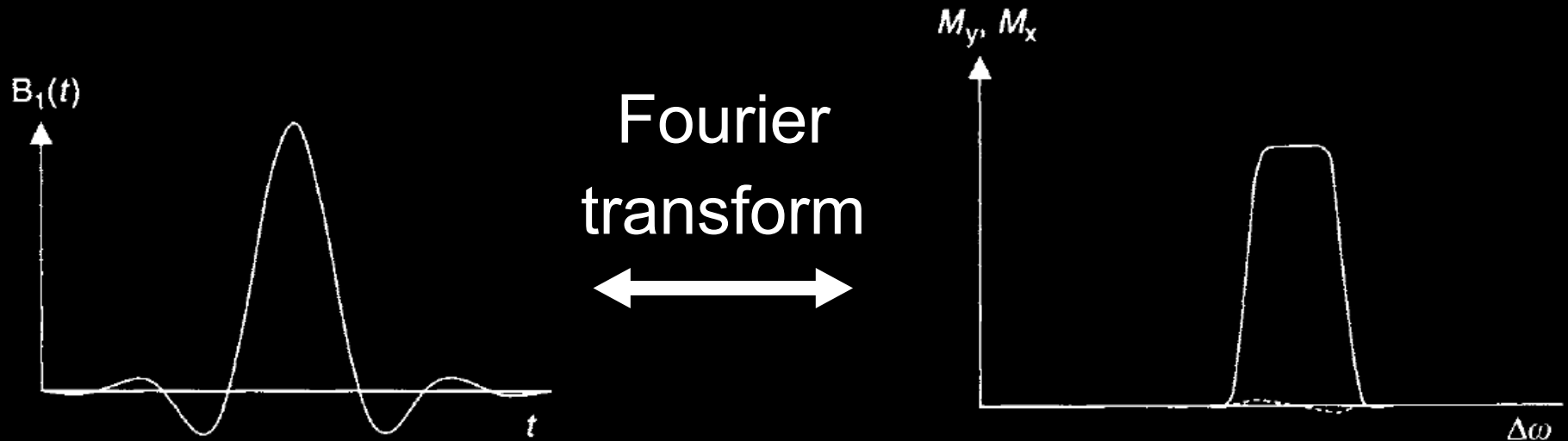
$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \Big|_{f=-(\gamma/2\pi)G_z z}$$

(See the note for complete derivation)

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\left\{\omega_1\left(t + \frac{\tau}{2}\right)\right\} \Big|_{f=-(\gamma/2\pi)G_z z}$$

Small Tip Approximation

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\left\{\omega_1\left(t + \frac{\tau}{2}\right)\right\} \Big|_{f = -(\gamma/2\pi)G_z z}$$



- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of $B_1(t)$ ”
- The approximation works surprisingly well even for flip angles up to 90°

Small Tip Approximation

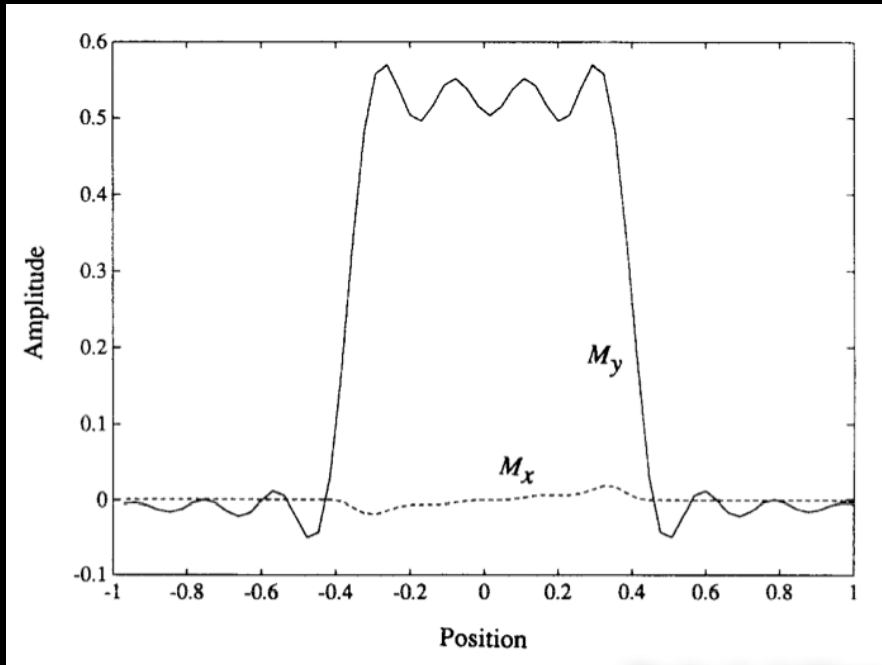
the excitation profile, within the small angle approximation, is just the Fourier transform of the pulse

remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly

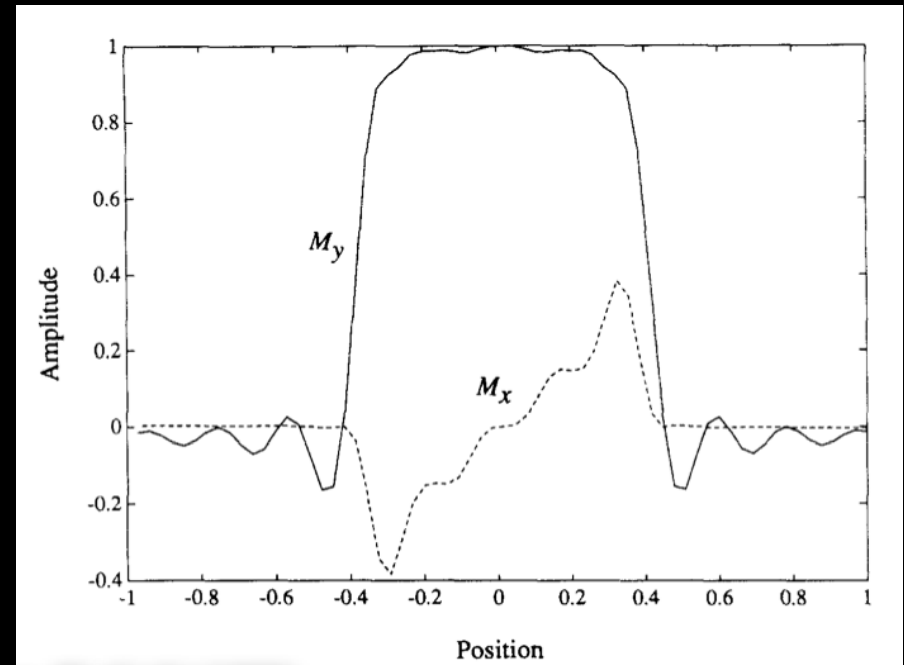
the approximation works surprisingly well even for flip angles up to 90°

Shaped Pulses

30°



90°



Pauly, J. J. *Magn. Reson.* 81 43-56 (1989)

small-angle approximation still works reasonably well for flip angles that aren't necessarily "small"

Truncation Artifacts

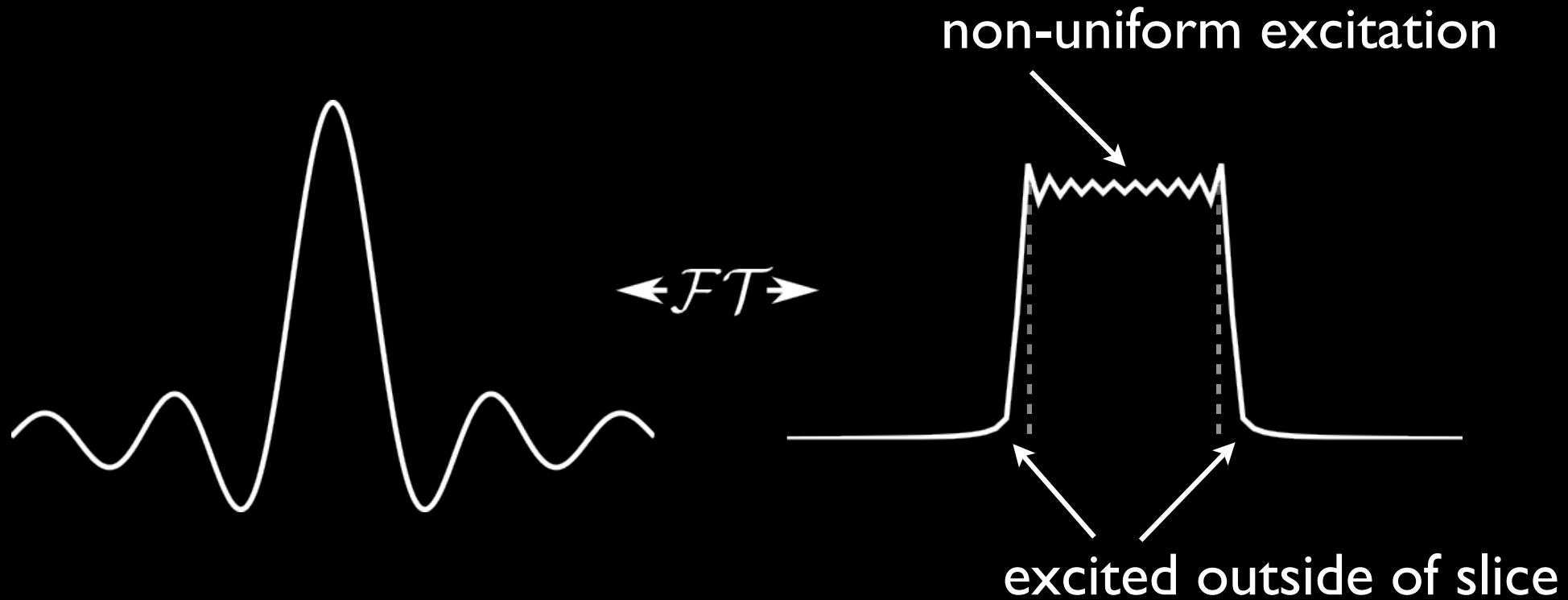
in MRI we want pulses to be as short as possible
to avoid relaxation effects

the sinc function is defined over all time
which is impractical in any experiment

the sinc pulse needs to be truncated to be
appropriate for clinical scans

Truncation Artifacts

what happens when we truncate our pulses?



these deviations from the ideal are known
as truncation artifacts

Truncation Artifacts

alternative Pulse Shapes

gaussian

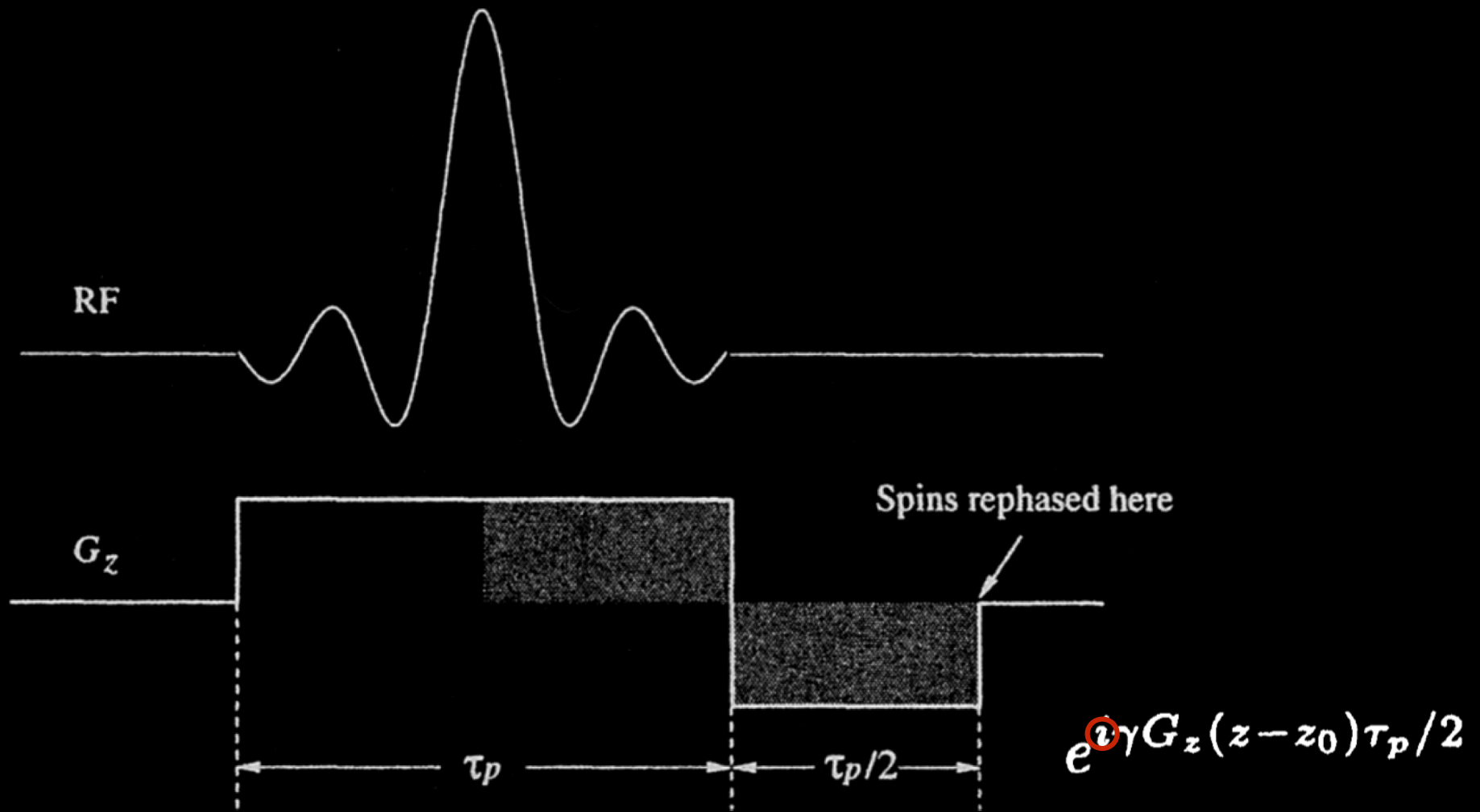
$$B_x(t) = A \exp \left[-a(t - \tau/2)^2 \right]$$

reduced side-lobes, but not as flat of a profile

Window Functions

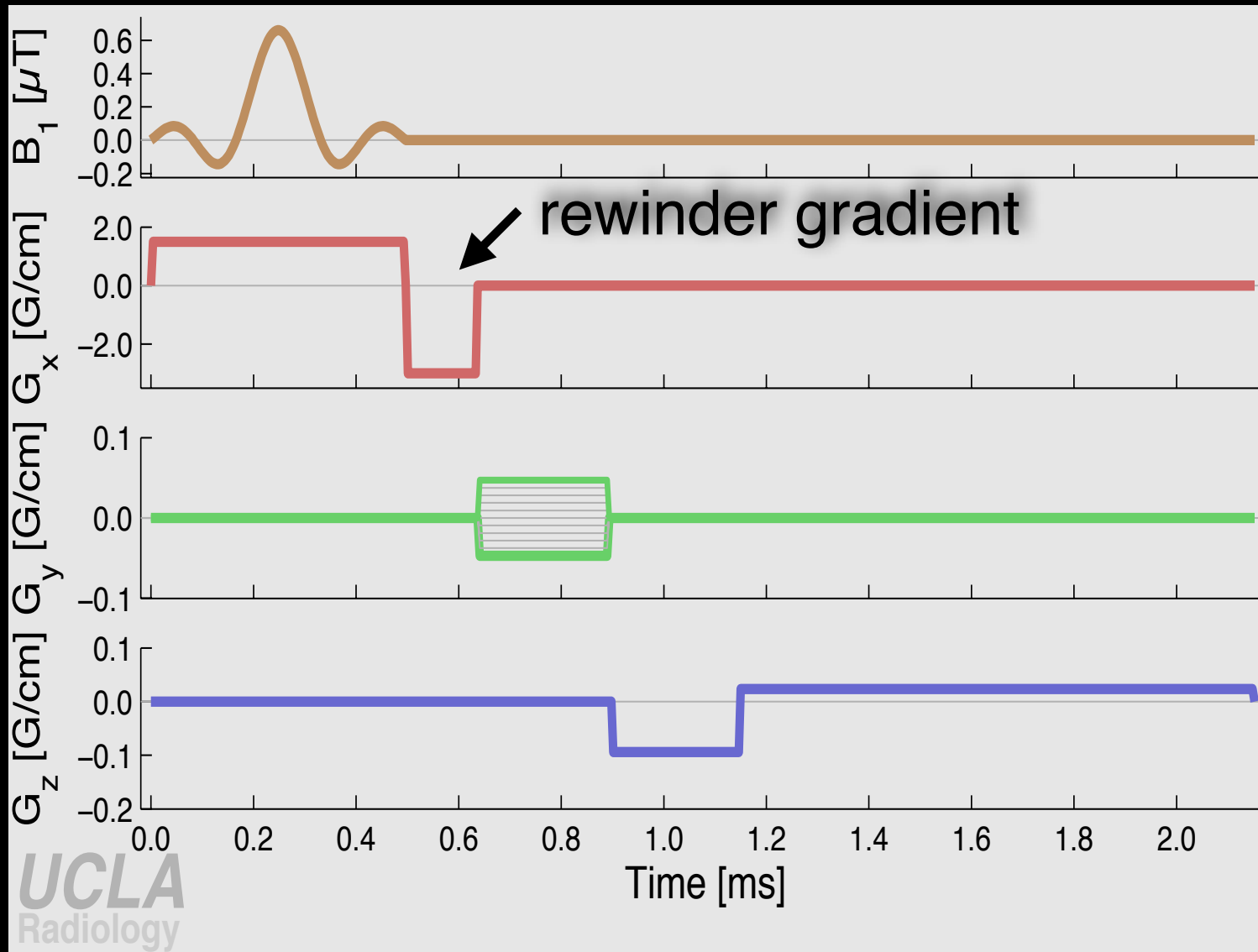
Hamming, Hanning, ...

Slice Rewinder



Opposite Polarity

Slice Selective Excitation Example



slice select gradient rewinder eliminates the linear phase ramp

Selective Excitation: Conclusion

B1 amplitude

-> flip angle

B1 amplitude profile

-> bandwidth, slice profile

B1 carrier frequency

-> slice location

B1 phase profile

-> slice location, etc.

Small Tip Approximation

-> slice profile = FT of B1 envelope function

MATLAB Demo

```
%% Design of Windowed Sinc RF Pulses
```

```
tbw = 4;  
samples = 512;  
rf = wsinc(tbw, samples);
```

```
%% Plot RF Amplitude
```

```
flip_angle = pi/2;  
rf = flip_angle*rf;
```

```
pulseduration = 1;      % in msec  
dt = pulseduration/samples;  
rfs = rf/(gamma*dt);    % Scaled to Gauss
```

```
bw = tbw/pulseduration; % in kHz  
gmax = bw/gamma_2pi;
```

```
b1      = [rfs zeros(1,samples/2)];           % in Gauss  
g       = [ones(1,samples) -ones(1,samples/2)]*gmax; % in G/cm  
t_all   = (1:length(g))*dt; % in msec
```

MATLAB Demo

```
%% Simulate Slice Profile using Bloch Simulation
x = (-2:.01:2);           % in cm
f = 0;                   % in Hz
dt = pulseduration/samples/1e3;
t = (1:length(bl))*dt;   % in usec

% Bloch Simulation
[mx,my,mz] = bloch(bl(:),g(:),t(:),1,.2,f(:),x(:),0);

% Transverse Magnetization
mxy_bloch = mx+1i*my;
```

```
%% Simulate Slice Profile using Small Tip Approximation
samples_st = 4096;
f_st = linspace(-0.5/dt,0.5/dt,samples_st)/1e3;
x_st = -f_st/(gamma_2pi*gmax);

rfs_zp = zeros(1,samples_st);
rfs_zp(1:samples) = rfs;

mxy_st = fftshift(fftn(fftshift(rfs_zp)))/30;
```