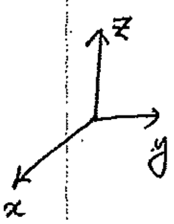
bloch equation ⟵ governs behavior of $\overline{m}$

$$\frac{d\overline{m}}{dt} = \underbrace{\overline{m} \times \gamma \overline{B}}_{} - \underbrace{\frac{m_x \hat{i} + m_y \hat{j}}{T_2}}_{T_2 \text{ relaxation}} - \underbrace{\frac{(m_z - m_0)\hat{k}}{T_1}}_{T_1 \text{ relaxation}}$$

✓ precession
✓ RF excitation
$|\overline{m}|$ unchanged

relaxation changes $|\overline{M}|$



$\hat{i}, \hat{j}, \hat{k}$, $\overline{B}$ includes B_0, B_1, & G

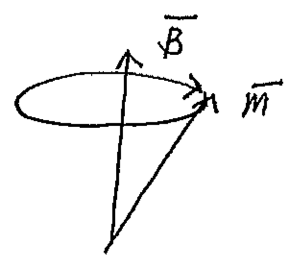
B_0 ← static field
B_1 ← time-varying field
G ← linear gradient fie

$$\overline{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

<u>solution of Bloch equations</u>

Start with <u>just precession</u> $\dfrac{d\overline{m}}{dt} = \overline{m} \times \gamma \overline{B}$

let $\overline{B} = B_0 \hat{k}$



precession
$\overline{m}$ is precessing about $\overline{B}$

frequency is $\gamma |\overline{B}|$

rotation freq. is w_0

$$\dot{\overline{m}} = \begin{bmatrix} 0 & w_0 & 0 \\ -w_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \overline{m}$$

⇒ rotation in x, y plane

solution

$$\bar{m}(t) = \underbrace{\bar{R}_z(\omega_0 t)}\ \bar{m}(0)$$

$$\underset{\substack{\uparrow \\ \text{axis of} \\ \text{rotation}}}{\bar{R}_z}\,(\underset{\substack{\uparrow \\ \text{angle of} \\ \text{rotation}}}{\alpha}) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for rotation about $\hat{z}$

$\Rightarrow$ Lefthanded rotation

$\bar{R}_z(\omega_0 t)$ angle increased linearly with time

with T_2 & T_1 :

$T_2 \longrightarrow$ exponential decay in m_x m_y

$T_1 \longrightarrow$ exponential recovery in m_z

$$\bar{m}(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix} \bar{R}_z(\omega_0 t)\, m(0)$$

$\rightarrow$ decay x, y, z

$$+ \begin{bmatrix} 0 \\ 0 \\ M_0(1-e^{-t/T_1}) \end{bmatrix}$$

$\rightarrow$ z recovery term

$$m_z(t) = m_z(0)\, e^{-t/T_1} + m_0\,(1-e^{-t/T_1})$$

Will revisit <u>Bloch equation & solution</u> Matlab