MRI Systems II – Nuclear Precession and B1

M219 - Principles and Applications of MRI

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1/13/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #1 due on 1/29
- TA office hours, Mondays 4-6pm
- Office hours, Fridays 10-11am
  - In-person (Ueberroth, 1417B) or Zoom

Main Field (B_0) - Principles

- B_0 is a strong magnetic field
  - >1.5T
  - Z-oriented

$$\vec{B}_0 = B_0\vec{k}$$

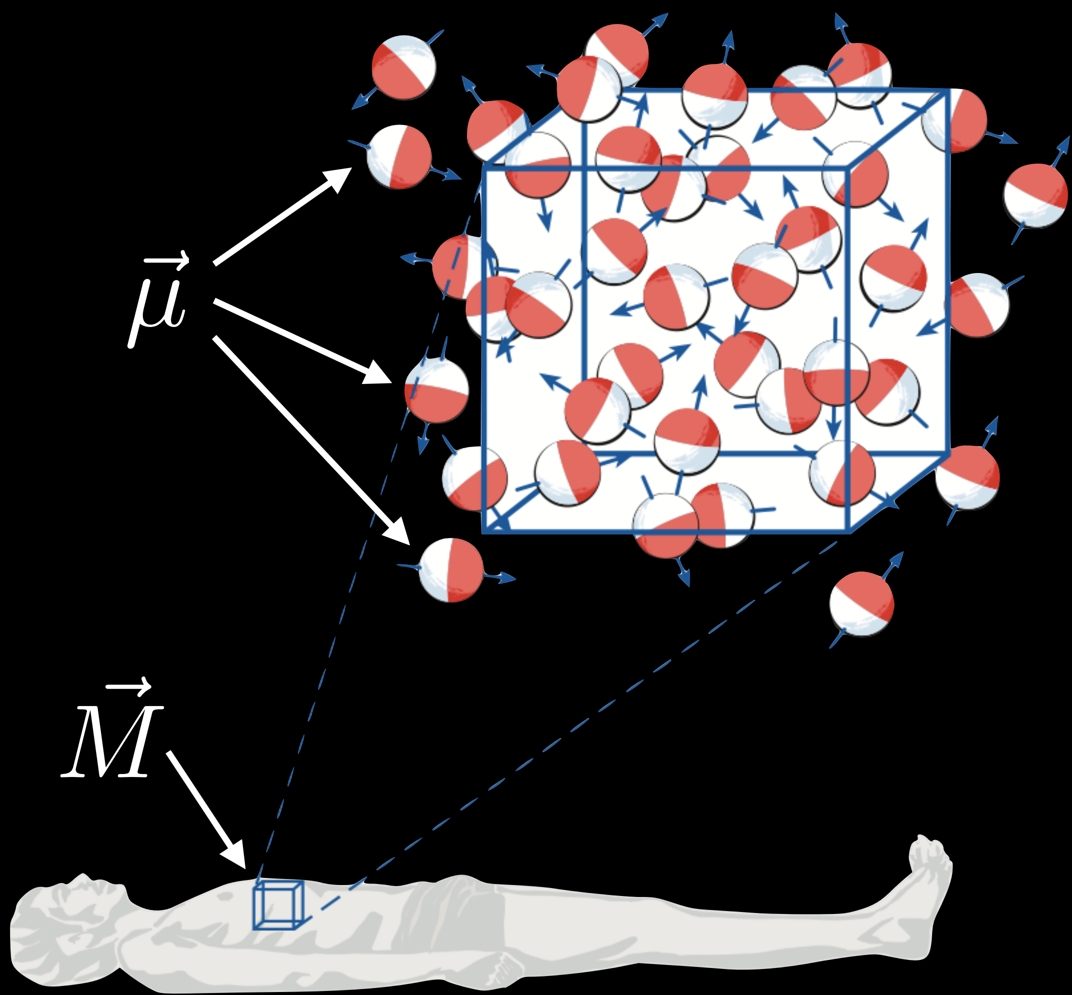
- B_0 generates bulk magnetization ($\vec{M}$)
  - More B_0, more

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

- B_0 forces $\vec{M}$ to precess
  - Larmor Equation

$$\omega = \gamma B$$

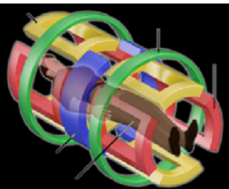
Bulk Magnetization



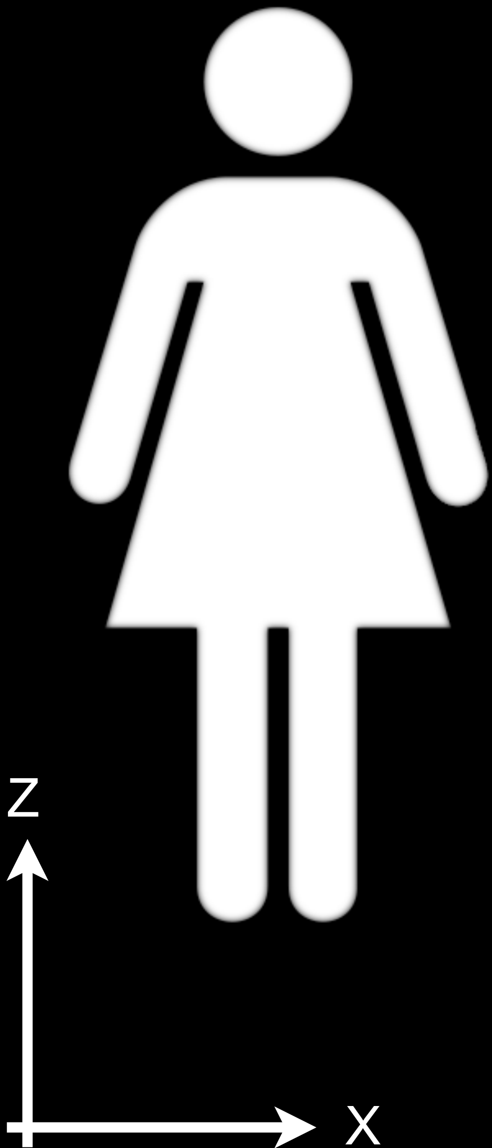
$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

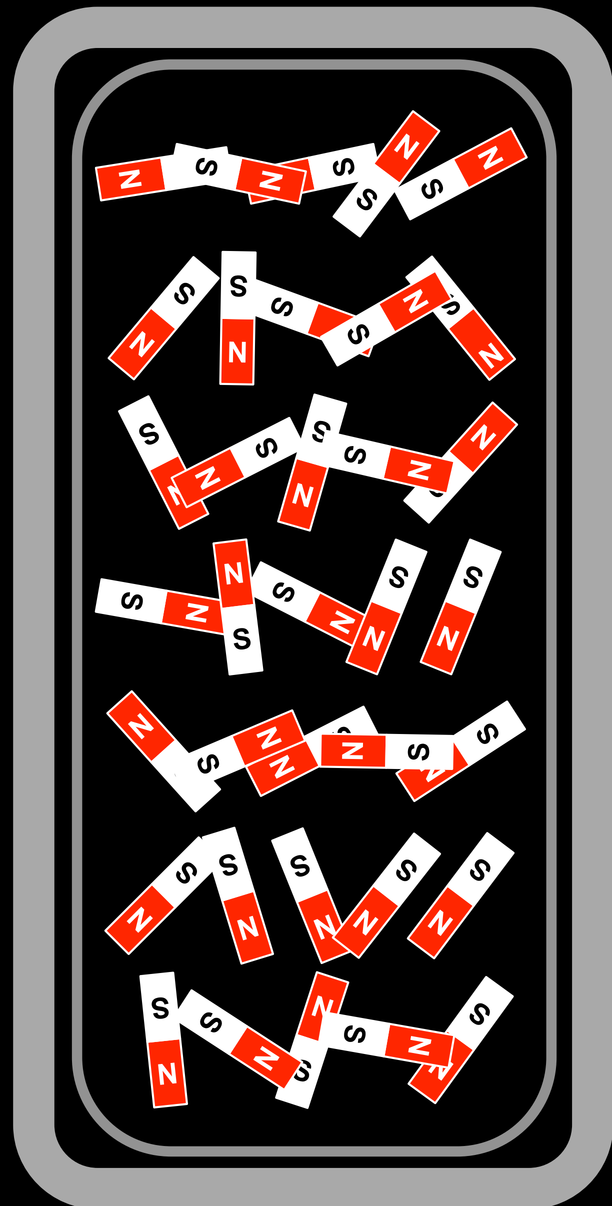
N_{total}=0.24x10^{23} spins in a 2x2x10mm voxel

But not all spins contribute to our measured signal...



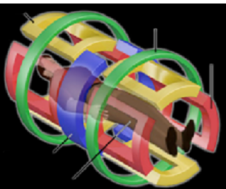
B_0 Field OFF



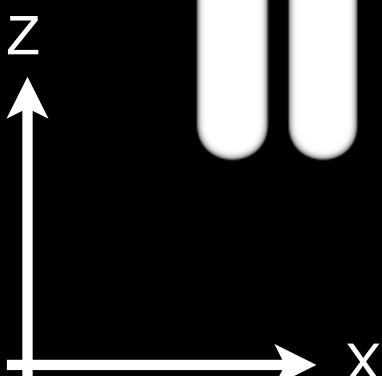


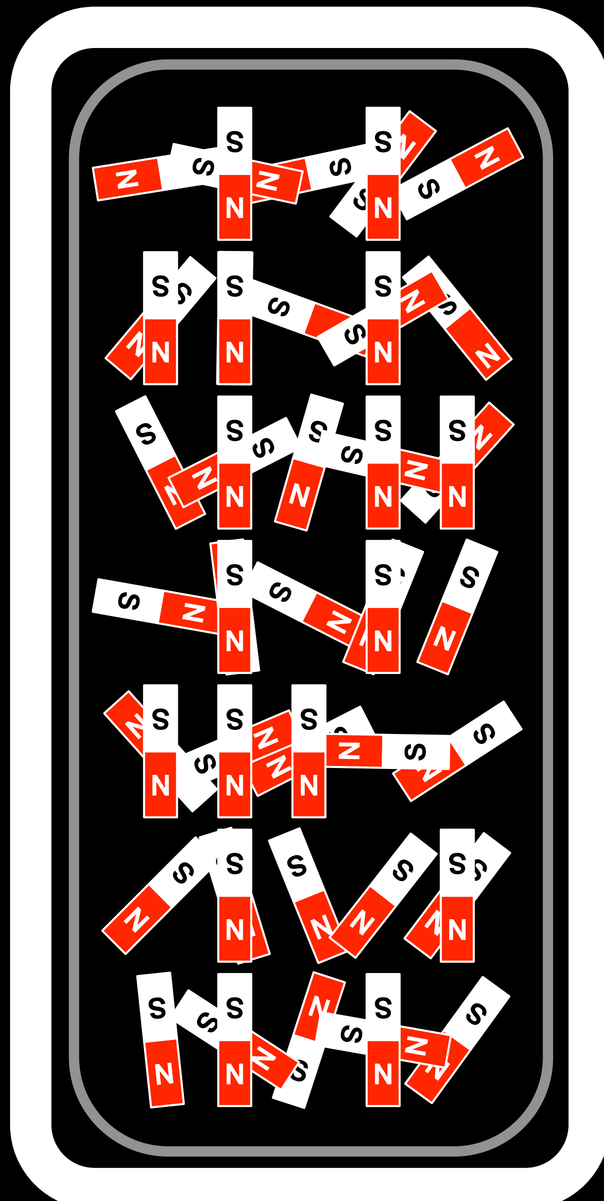
$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = 0$$

Spins point in all directions.



B_0 Field ON

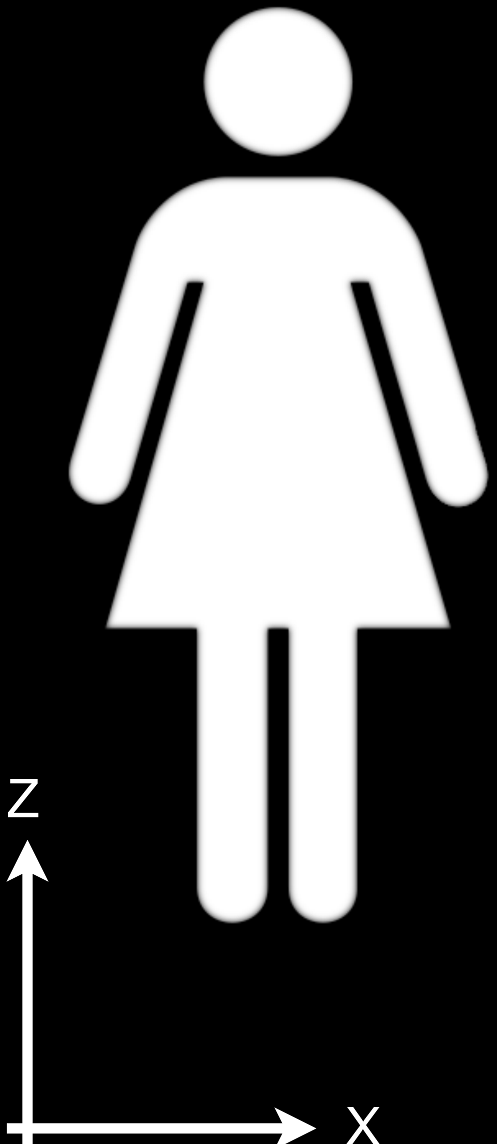


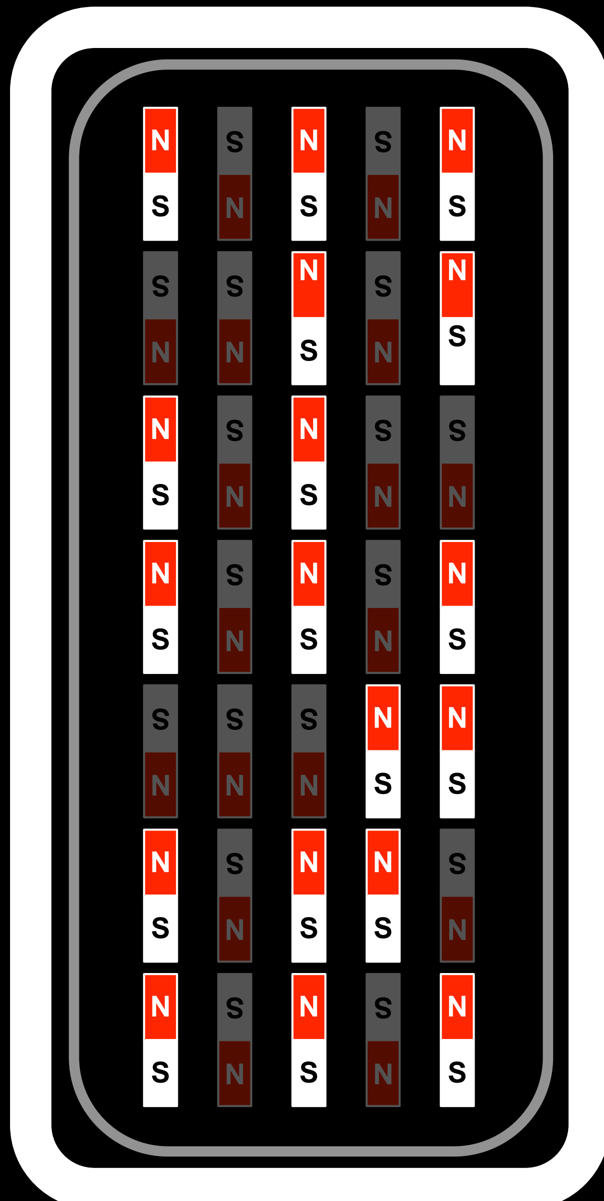


$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = M_z$$

B_0 polarizes the spins and generates bulk magnetization.

B_0 Field ON





$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = M_z$$

Only a very small number are spin-up relative to spin-down.

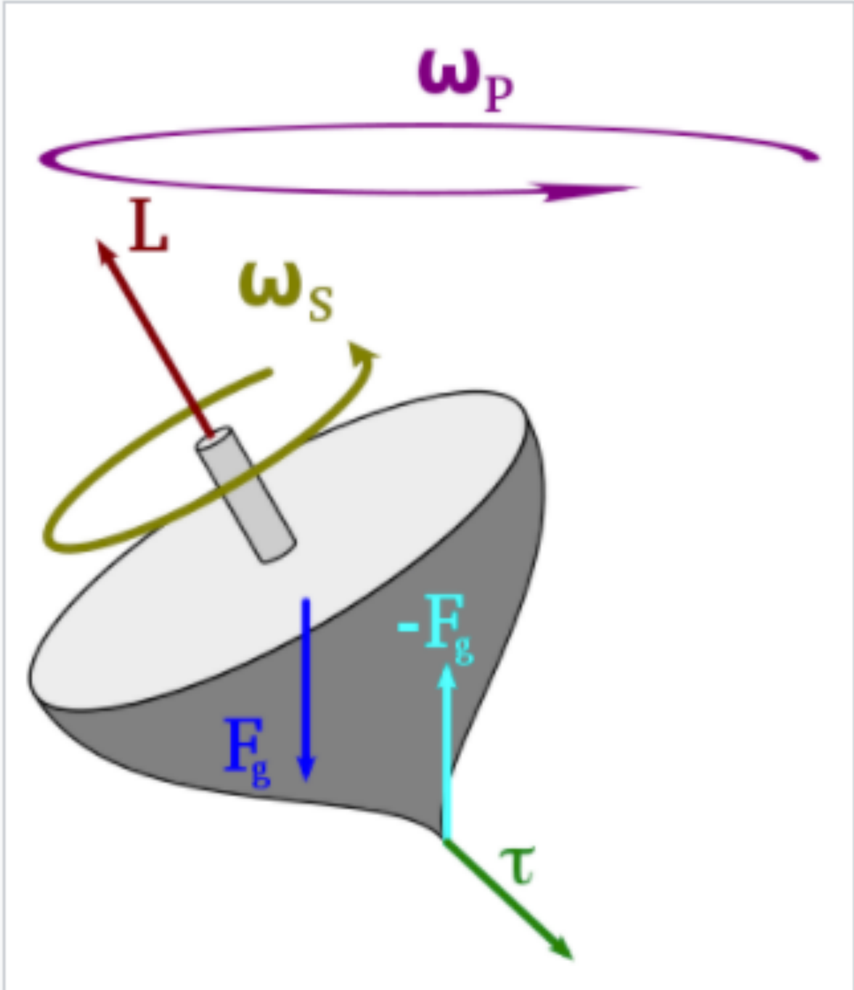
To the board

Spin vs. Precession

- **Spin**
  - Intrinsic form of angular momentum
  - Quantum mechanical phenomena
  - No classical physics counterpart
    - Except by hand-waving analogy…

- **Precession**
  - Spin+Mass+Charge give rise to precession

Precession



The torque caused by the normal force $-\mathbf{F}_g$ and the weight of the top causes a change in the angular momentum $\mathbf{L}$ in the direction of that torque. This causes the top to precess.

https://en.wikipedia.org/wiki/Precession

Larmor Equation

- Spin≠Precession
  - Protons *intrinsically* have spin
  - Protons *precess* in the presence of a B-field
- Larmor frequency increases with:
  - Larger B_0
  - Higher gyromagnetic ratio
  - Higher frequencies produce stronger signals...

$$\omega = \gamma B_0$$

$$\gamma = 267.52 \times 10^6 \text{ rad} \cdot \text{s}^{-1} \cdot \text{T}^{-1}$$

$$\gamma / 2\pi = 42.577 \text{ MHz/T}$$

Equation of Motion for the Bulk Magnetization

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

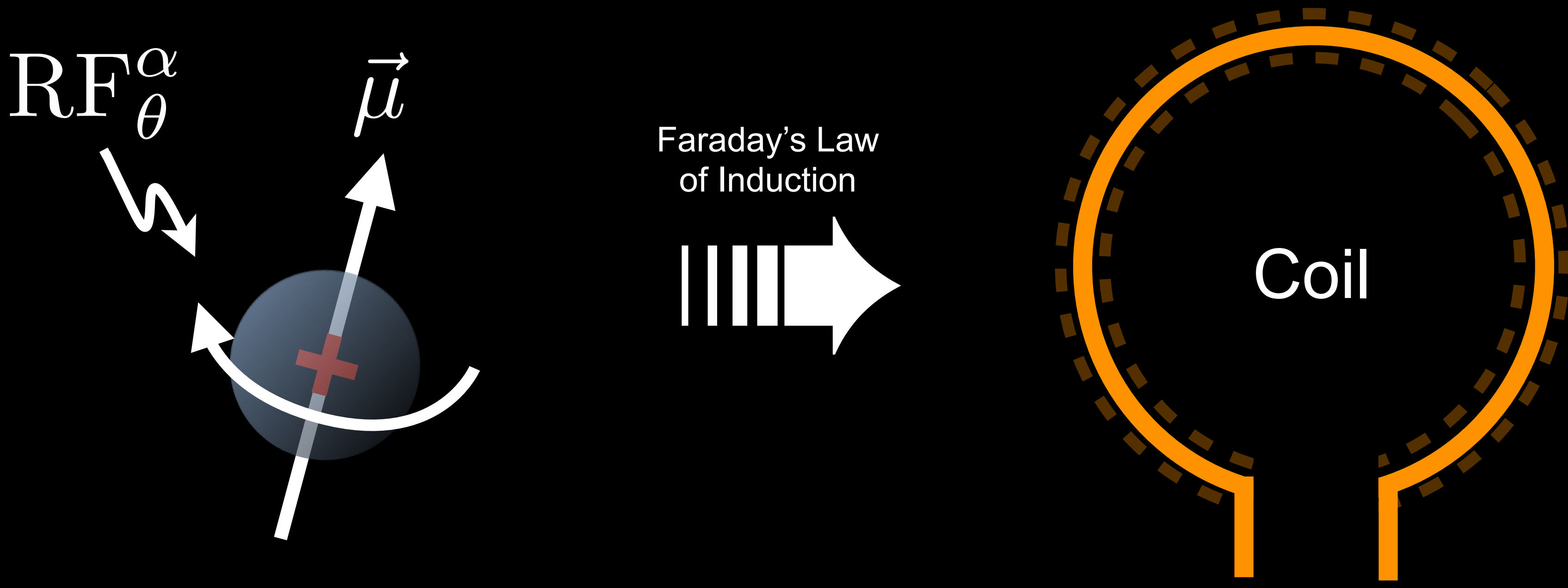
Equation of motion for an ensemble of spins [Classical Description]

What is a general solution?

The *equation of motion* describes the bulk magnetization "behavior" in the presence of a B-field.

To the board

Signal Reception

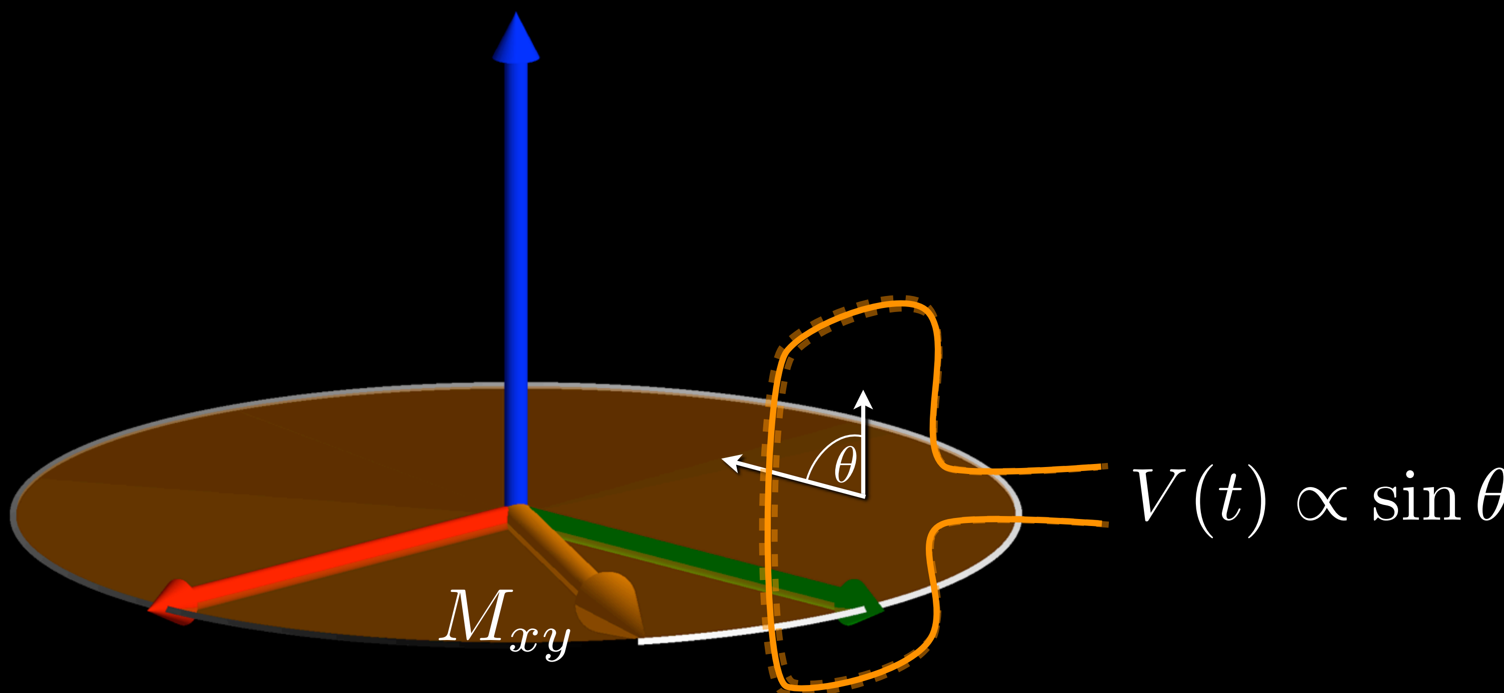


$M_{xy}(\vec{r}, t)$

$V(t)$

NMR Signal Detection

- **Coil only detects M_{xy}**
- **Coil does *not* detect M_z**
- **Coil must be properly oriented**



How does RF alter $\vec{M}$?

$$\vec{B}_1(t)$$

Generating B_1-Fields

MRI Hardware

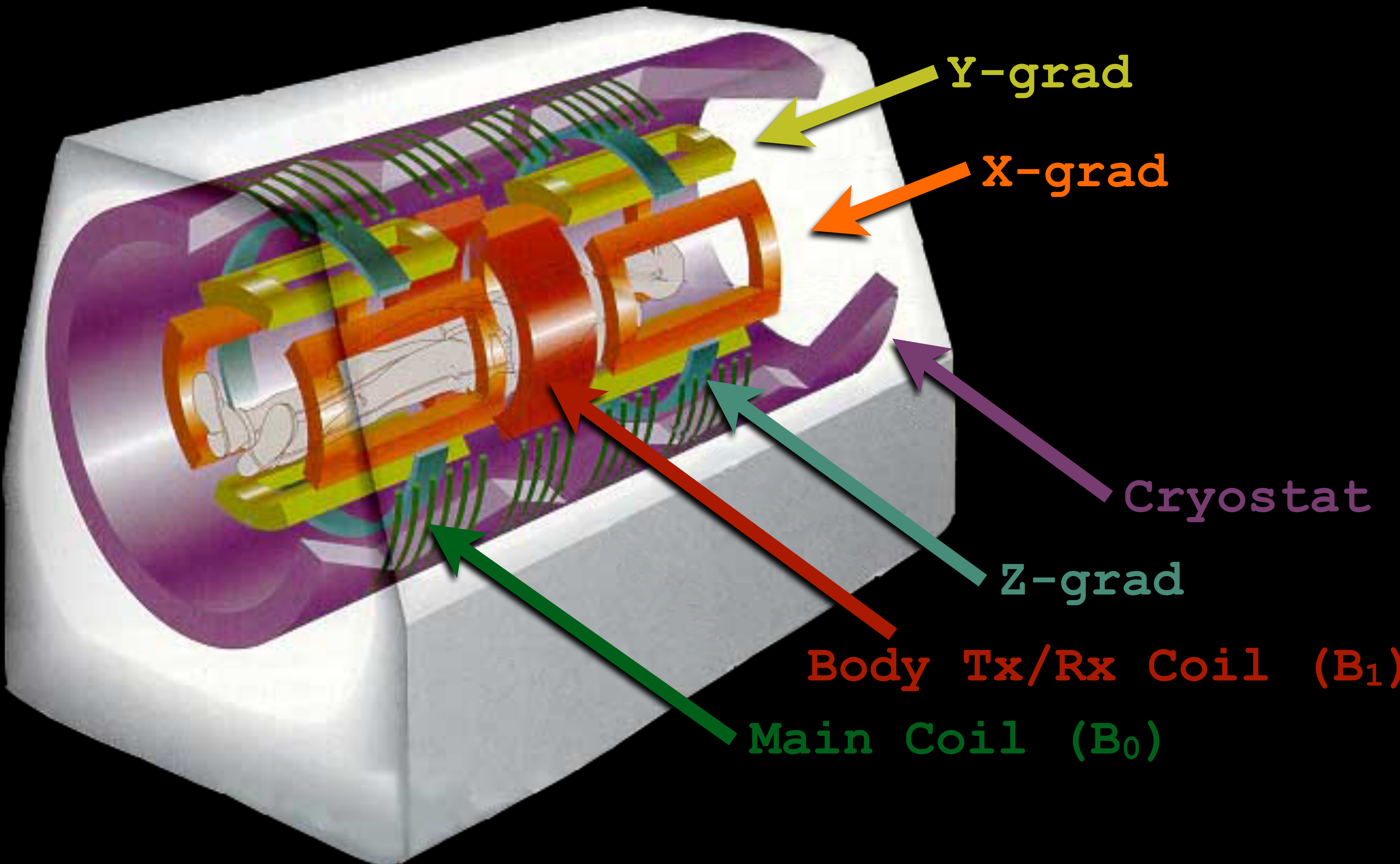


Image Adapted From: http://www.ee.duke.edu/~jshorey

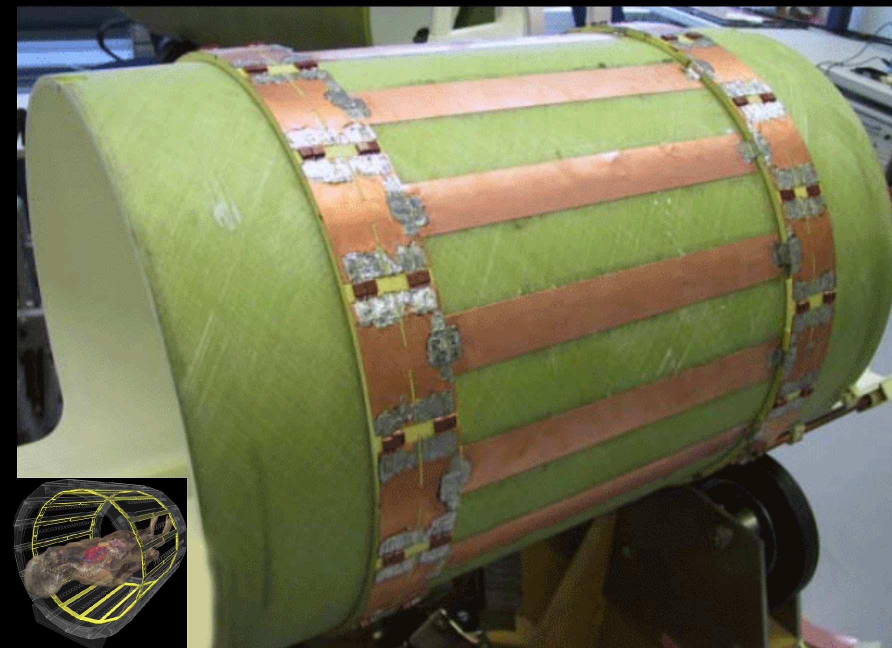
RF Shielding

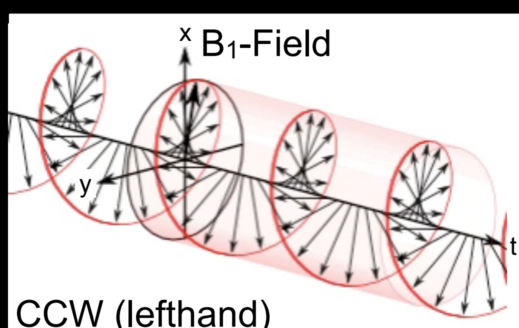
- RF fields are close to FM radio
  - ^1H @ 1.5T ⇒ 63.85 MHz
  - ^1H @ 3.0T ⇒ 127.71 MHz
  - KROQ ⇒ 106.7 MHz
- Need to shield local sources from interfering
- Copper room shielding required

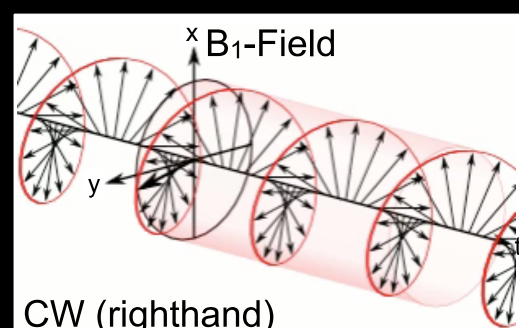
RF Birdcage Coil

- **Most common design**
- **Highly efficient**
  - Nearly all of the fields produced contribute to imaging
- **Very uniform field**
  - Especially radially
  - Decays axially
  - Uniform sphere if L≈D
- **Generates a "quadrature" field**
  - Circular polarization

Body Tx/Rx Coil (B_1)







B_1 Field - RF Pulse

- B_1 is a
  - radiofrequency (RF)
    - 42.58MHz/T (63MHz at 1.5T)
  - short duration pulse (~0.1 to 5ms)
  - small amplitude
    - $<30\ \mu T$
  - circularly polarized
    - rotates at Larmor frequency
  - magnetic field
  - perpendicular to B_0

Basic RF Pulse

$$\overrightarrow{B} = \overrightarrow{B}_0 + \overrightarrow{B}_1(t)$$

$$\overrightarrow{B}_1(t) = B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$B_1^e(t)$ **pulse envelope function**

ω_{RF} **excitation carrier frequency**

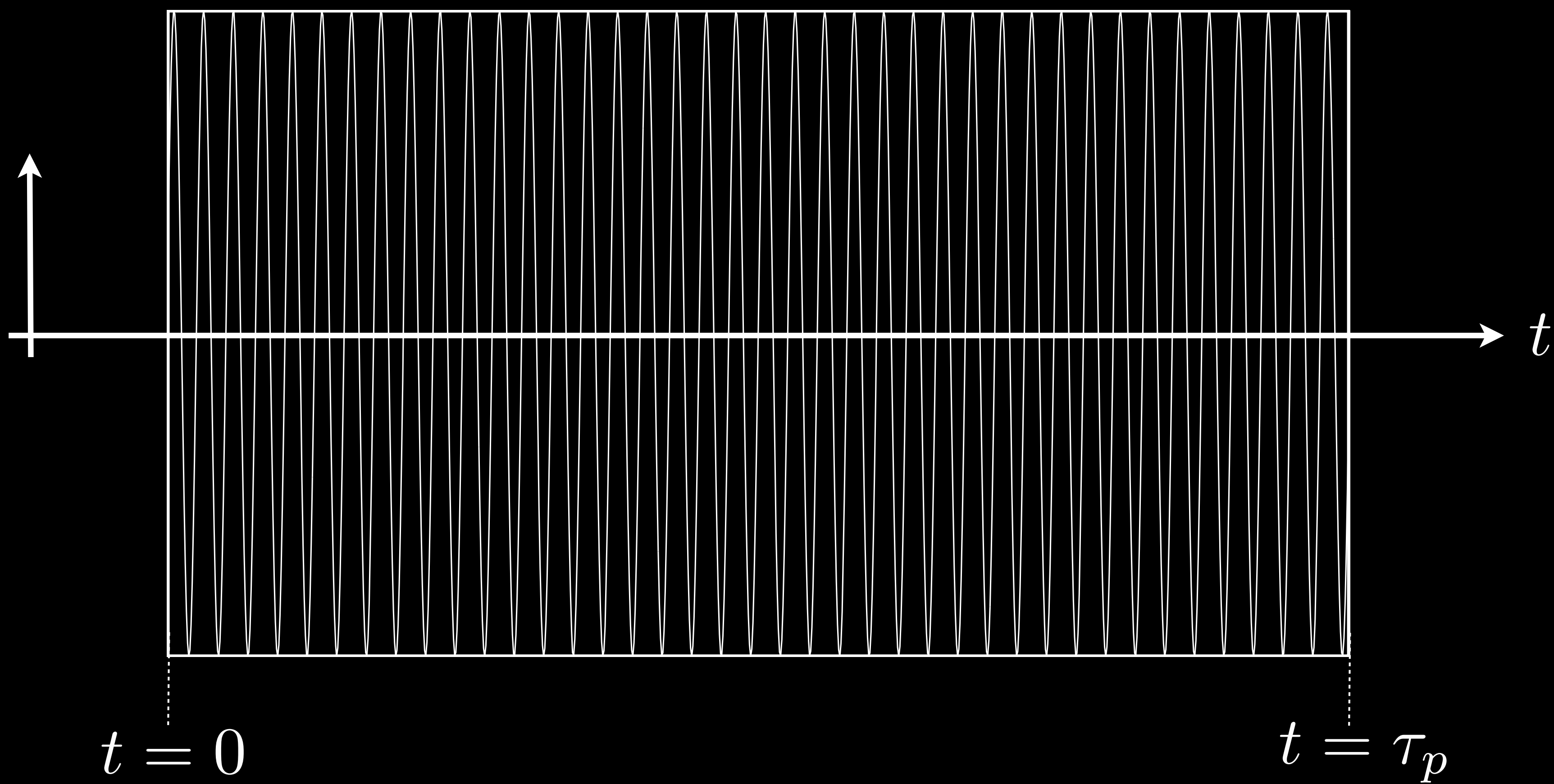
θ **initial phase angle**

B_1 is perpendicular to B_0.

$$\overrightarrow{B}_0 = B_0\hat{k}$$

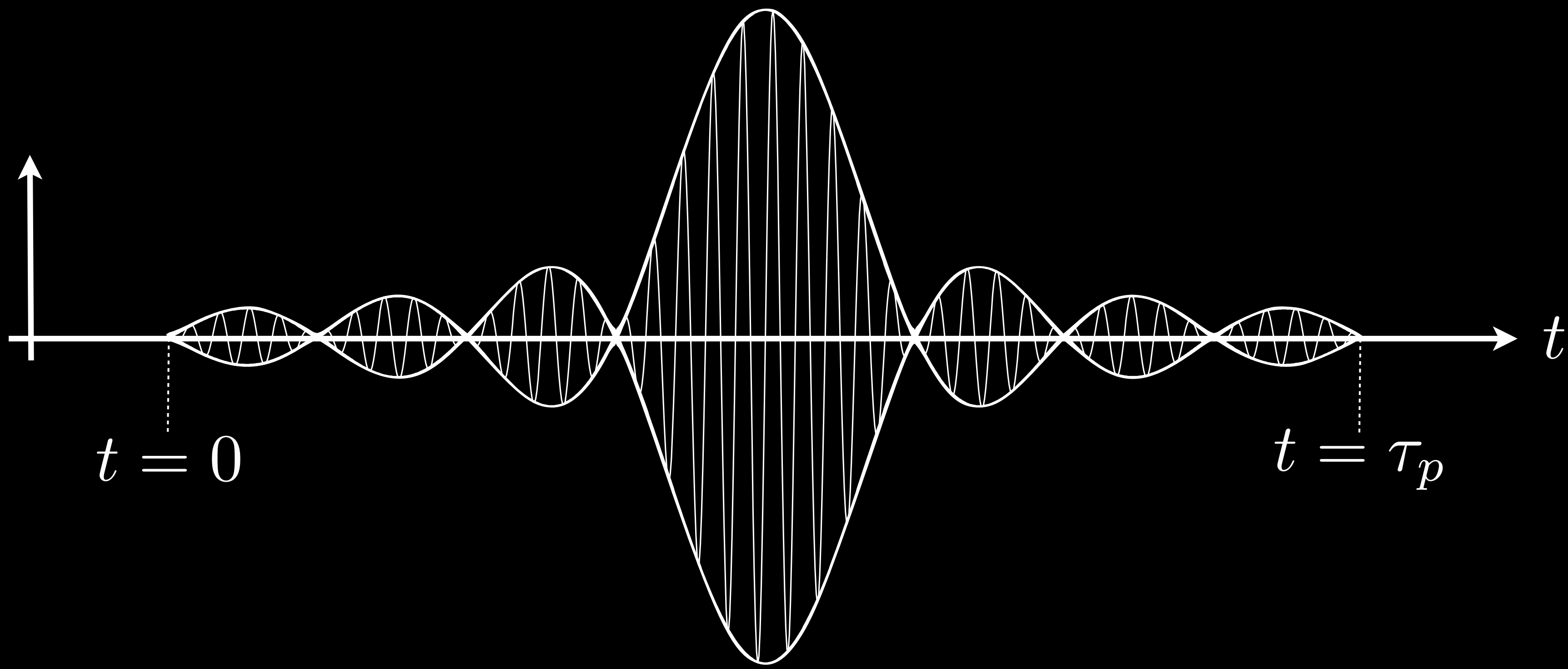
Rect Envelope Function

$$B_1^e(t) = B_1 \sqcap \left(\frac{t - \tau_p/2}{\tau_p} \right) = \begin{cases} B_1, & 0 \leq t \leq \tau_p \\ 0, & otherwise \end{cases}$$



Sinc Envelope Function

$$B_1^e(t) = \begin{cases} B_1 \text{sinc}\left[\pi f_\omega \left(t - \tau_p/2\right)\right], & 0 \leq t \leq \tau_p \\ 0, & otherwise \end{cases}$$

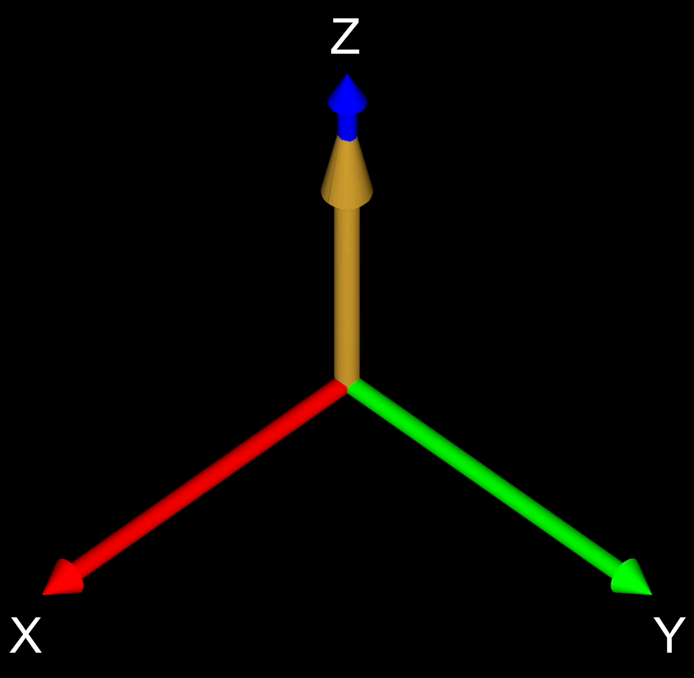


Rotating Frame

Lab vs. Rotating Frame

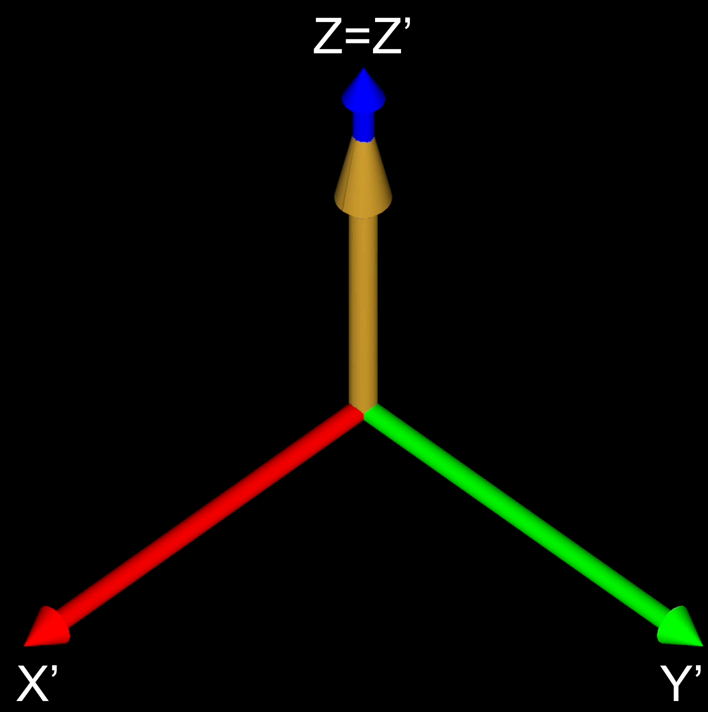
- **The rotating frame simplifies the mathematics and permits more intuitive understanding.**

90° RF (Laboratory Frame)



Spins Precess

90° RF (Rotating Frame)



Observer Precesses

Note: Both coordinate frames share the same z-axis.

Combined B_0 & B_1 Effects

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

$$= \vec{M} \times \gamma \left(\vec{B}_0 + \vec{B}_1 \right)$$

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats). [Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\overrightarrow{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$$\overrightarrow{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t + \theta) \\ -B_1^e(t)\sin(\omega_{RF} + \theta) \\ B_0 \end{pmatrix} \qquad \overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Assume no RF phase ($\theta = 0$)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{pmatrix}$$

Questions?

- Related reading materials
  - Nishimura Chap 3 and 4

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