Spatial Localization I

M219 - Principles and Applications of MRI

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2/3/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #2 due on 2/12
- TA office hours, Mon 4-6pm
- Office hours, Fri 10-11am

3 Types of Magnetic Fields

B_0 - Large static field

e.g., 1.5 Tesla or 3 Tesla

B_1 - Radiofrequency field

e.g., 0.16 G

Selective Excitation

$G_{x,y,z}$ - Gradient fields

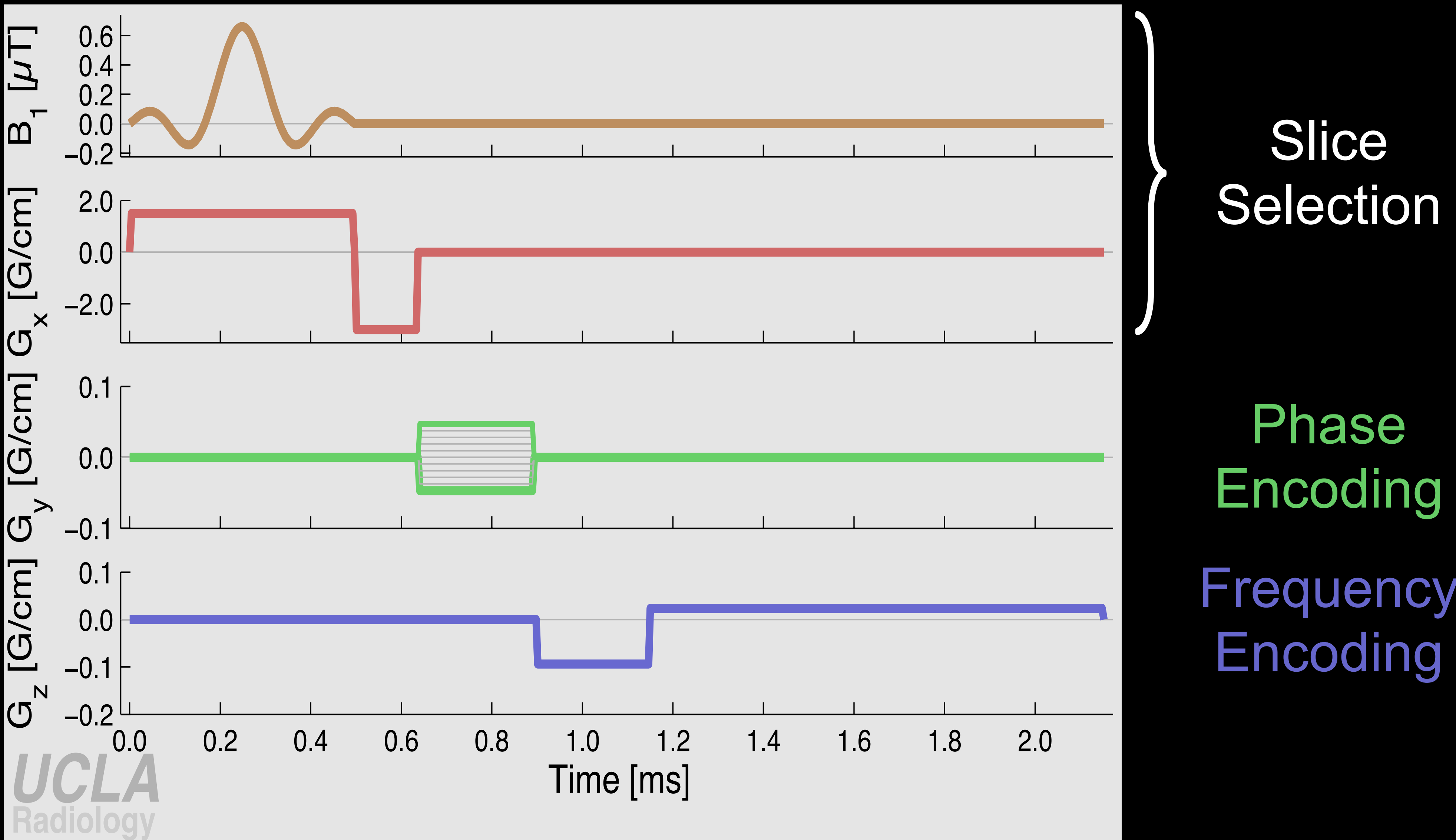
e.g., 4 G/cm

Frequency and Phase Encoding

Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

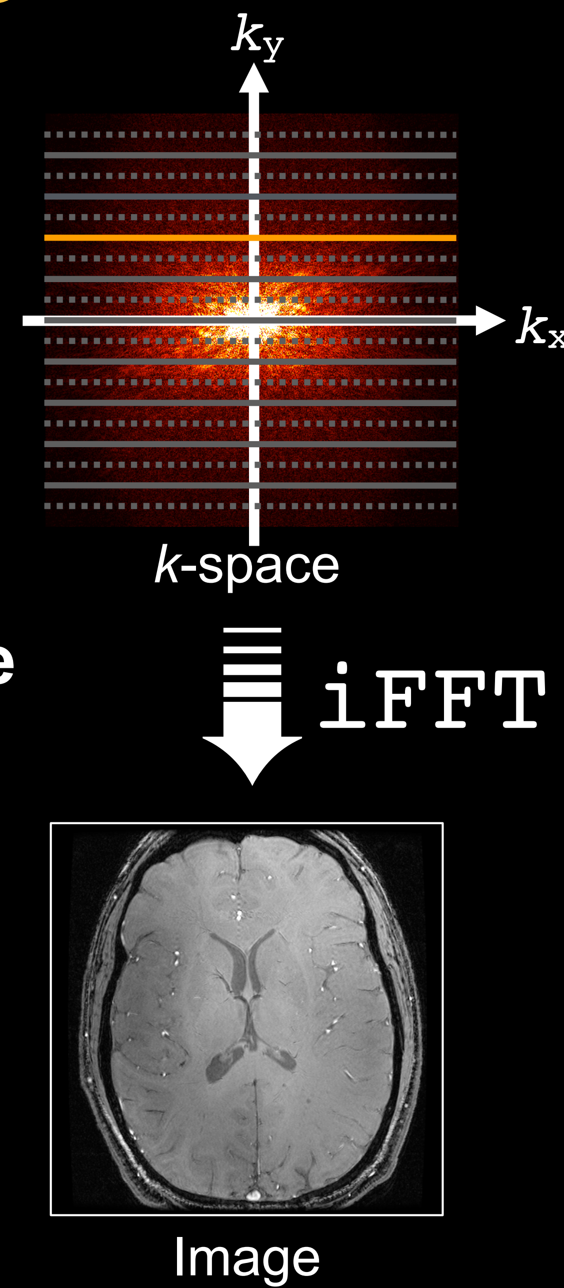
3 Steps for Spatial Localization

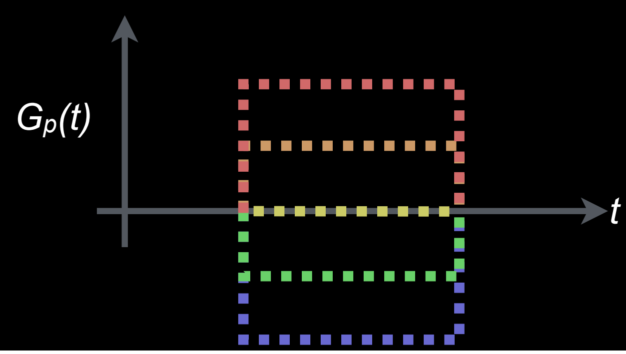


Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

Phase Encoding

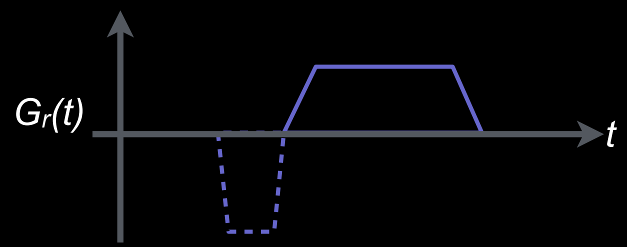
- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**



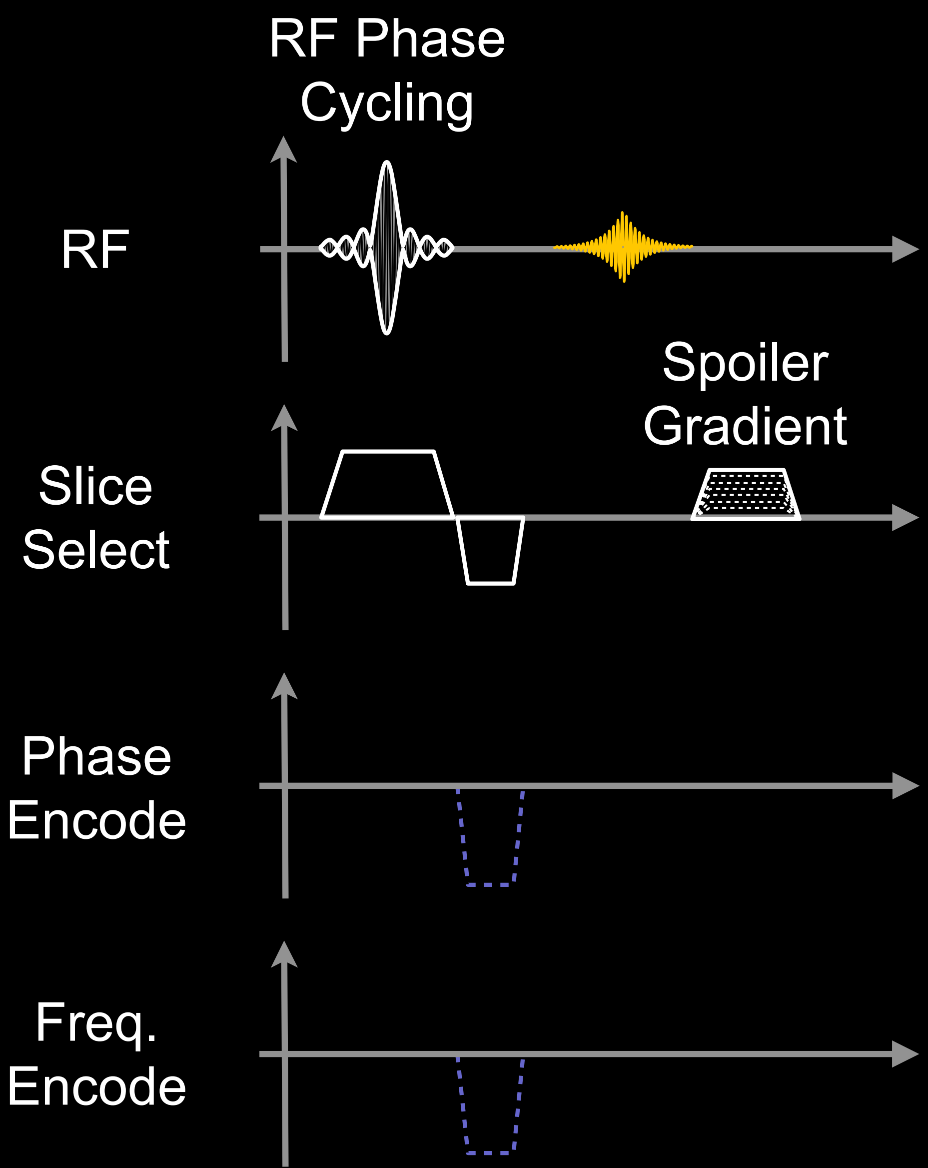


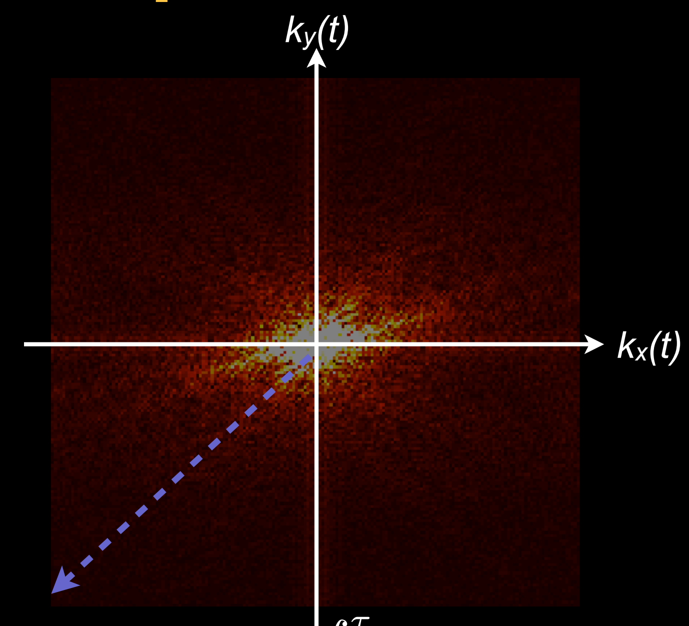
Frequency Encoding

- **Consists of:**
  - **Frequency encoding gradient**
    - **Constant magnitude for Cartesian imaging**
  - **No simultaneous**
    - **RF (B_1)**
    - **Other gradients**
      - **phase encoding, slice encoding, crushers**
  - **Readout pre-phasing gradient**
    - **Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)**
    - **AKA "readout de-phasing gradient"**
- **Adds linear spatial variation of frequency**
- **Helps form an *echo***

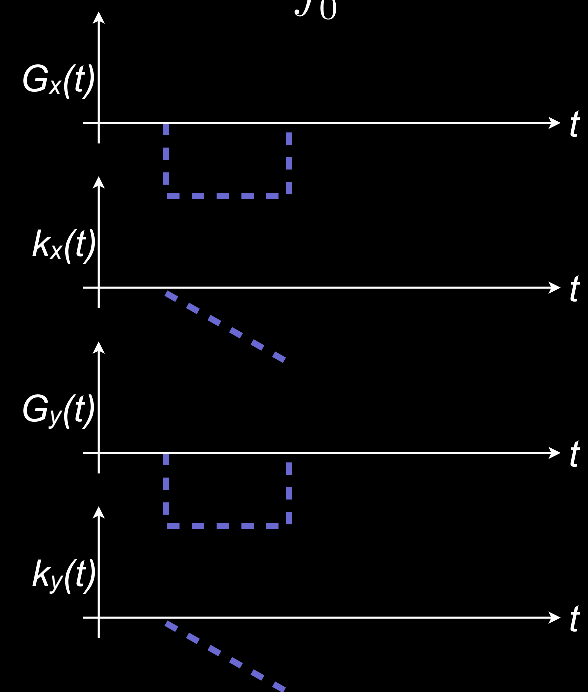


Where am I in *k*-space?



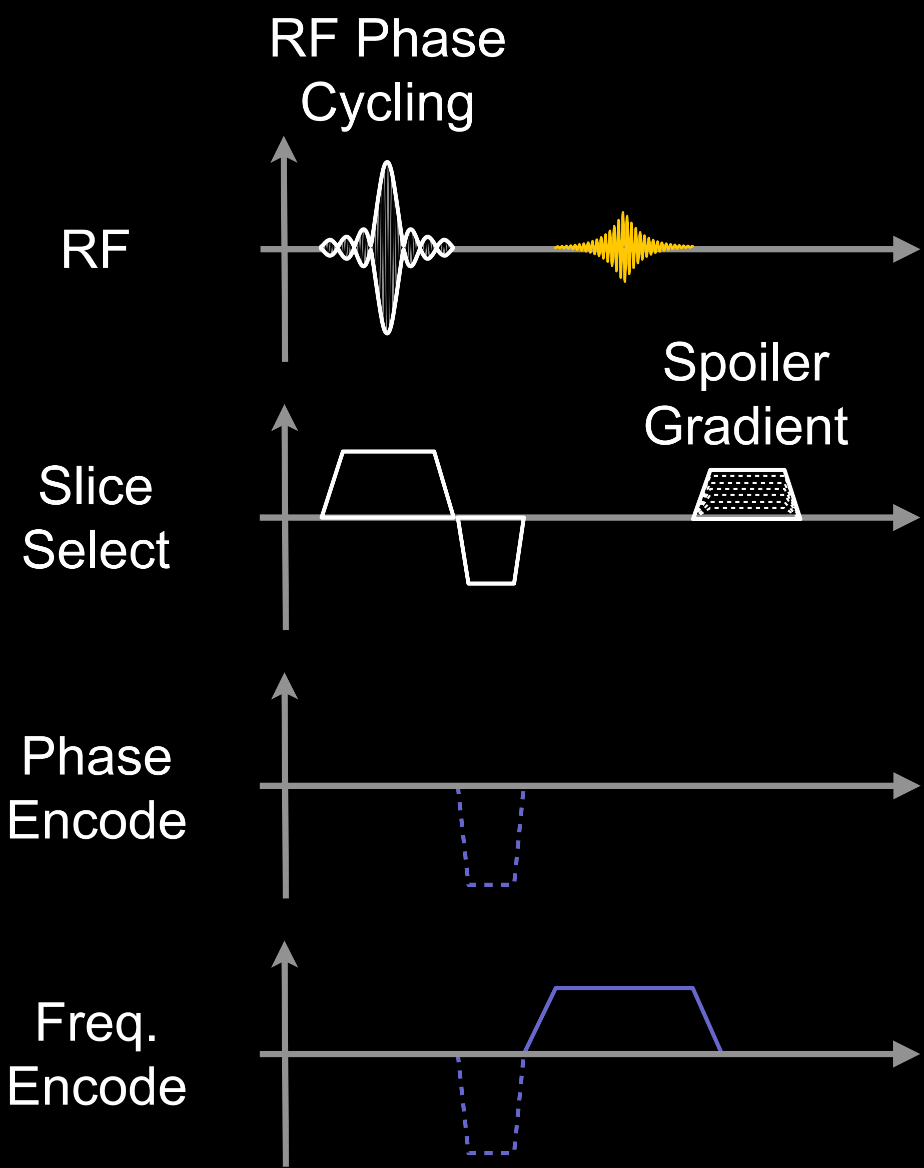


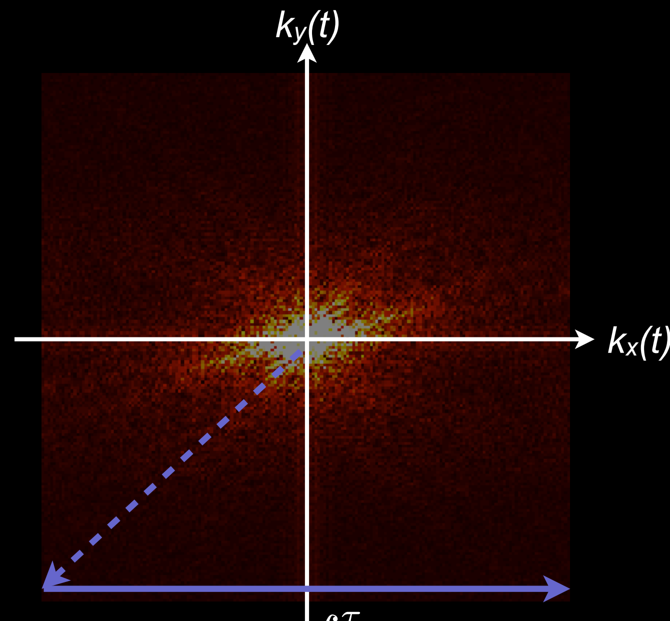
$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



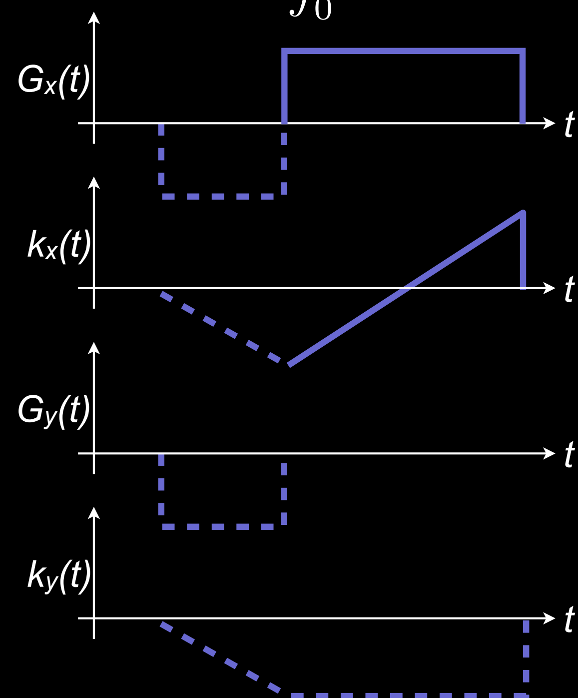
One phase encoded echo is acquired per TR.

Where am I in *k*-space?



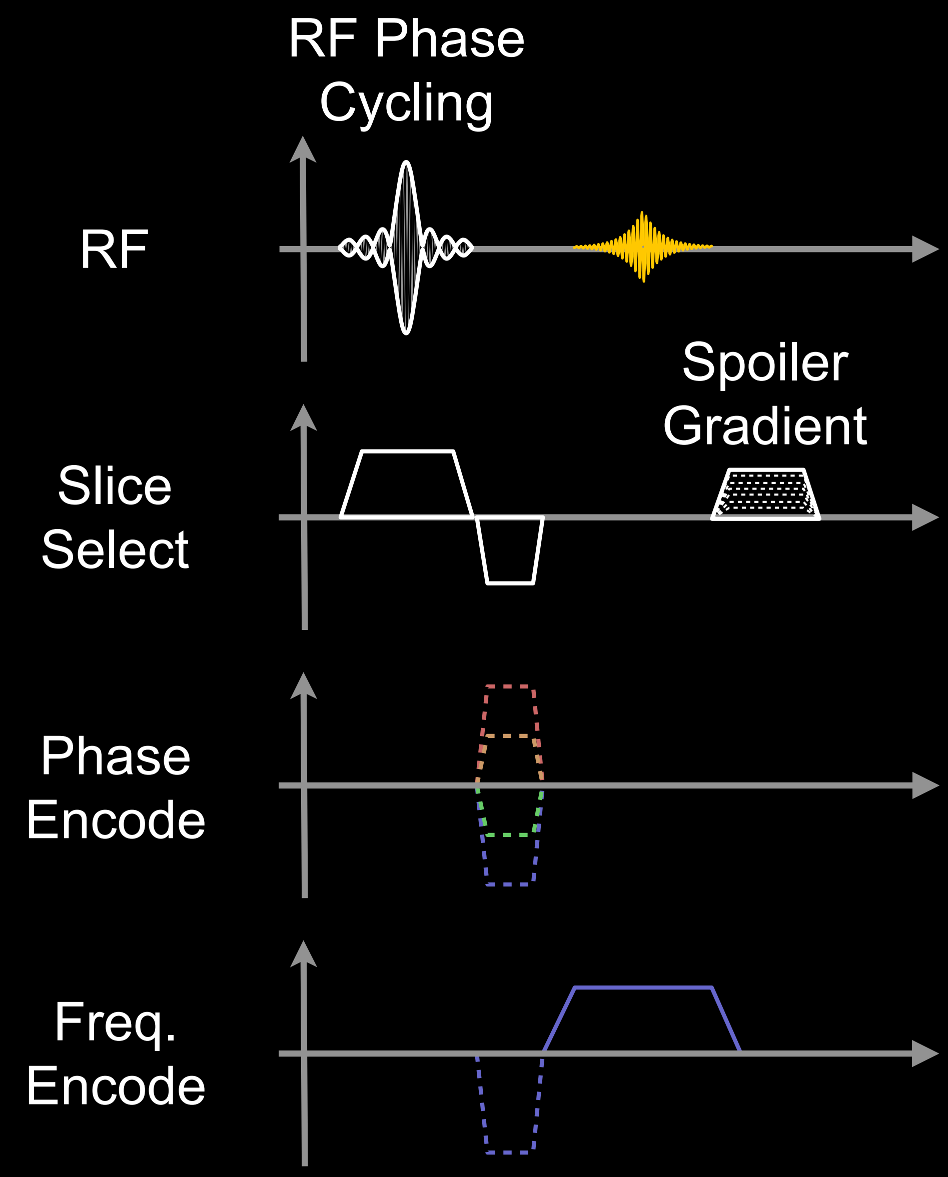


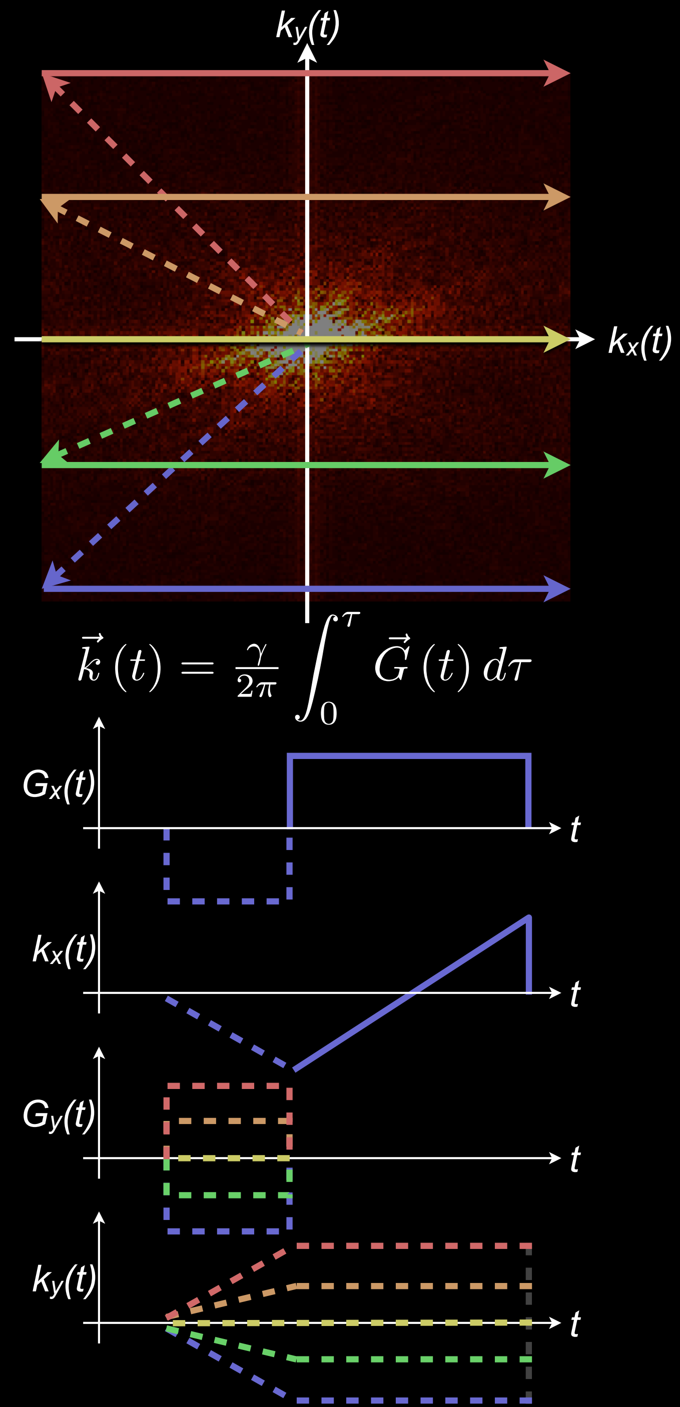
$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



One phase encoded echo is acquired per TR.

Where am I in *k*-space?





$$\vec{k}(t) = \frac{\gamma}{2\pi} \int_0^{\tau} \vec{G}(t)\, d\tau$$

One phase encoded echo is acquired per TR.

To the Board

MRI Sampling Requirements

k-space Sampling

Remember that the collected data in MRI is discrete

Discrete sampling can lead to artifacts if not careful

Sampling considerations
- Field of View
- Spatial Resolution

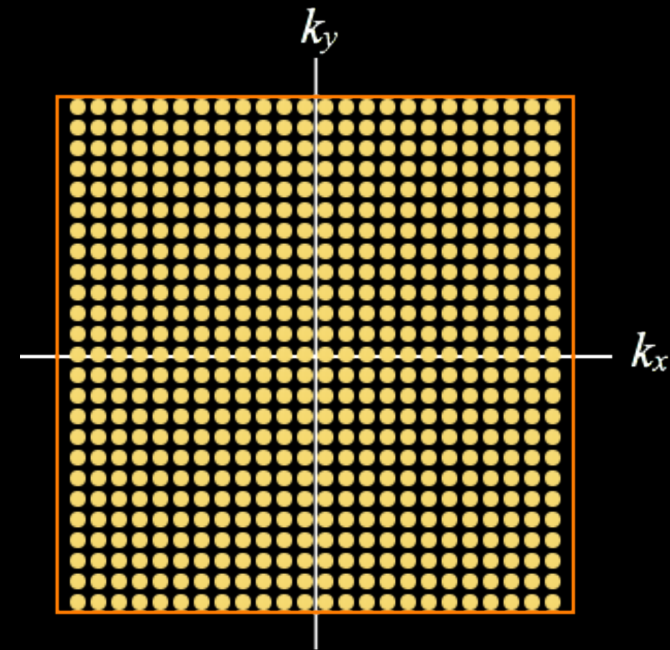
Sampling Considerations

$$s(t) = m(\, k_x(t), k_y(t) \,)$$

$$s(n\Delta t) = M(k_x(n\Delta t), k_y(n\Delta t))$$

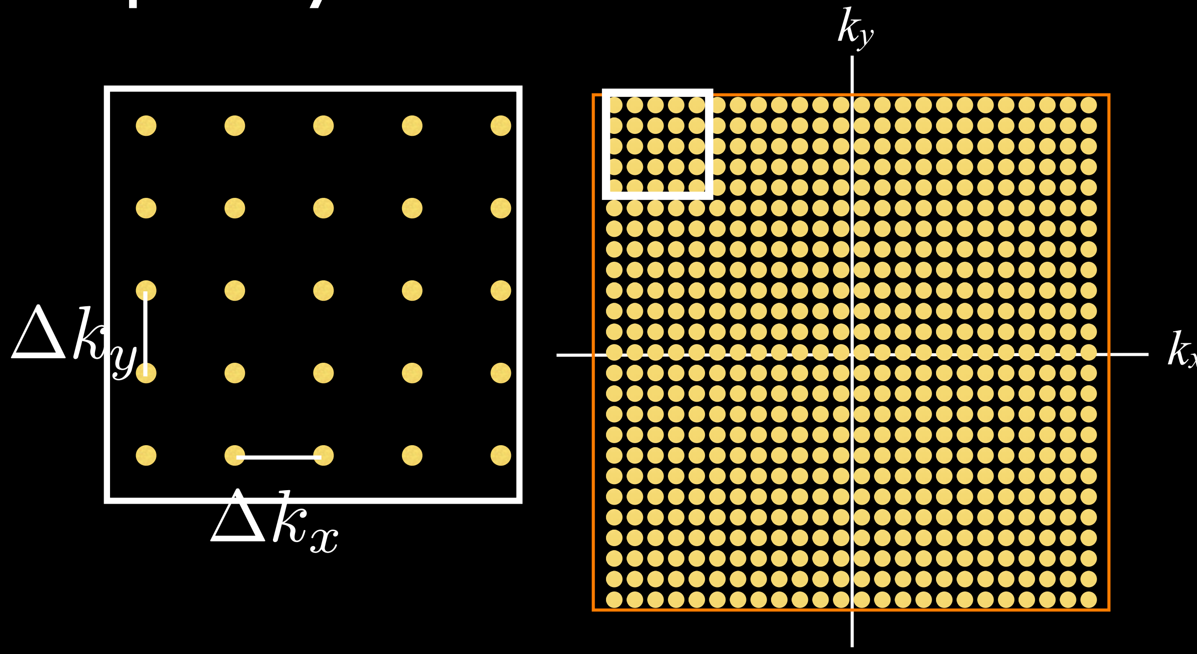
Index   Sampling period

discrete sampling in spatial frequency domain



Sampling Considerations

discrete sampling in spatial frequency domain



$$w_{k_x} = N_{read} \times \Delta k_x$$

$$w_{k_y} = N_{PE} \times \Delta k_y$$

Review: Properties of DFT

Convolution

$$f(x) * h(x) \longleftrightarrow F(k_x)\, H(k_x)$$

Similarity (scaling)

$$f(ax) \longleftrightarrow \frac{1}{|a|} F(\frac{k_x}{a})$$

Shift

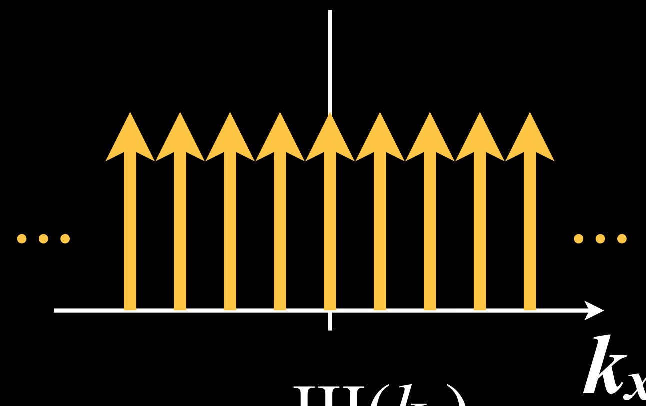
$$f(x-a) \longleftrightarrow \exp(-i2\pi(ak_x)) \cdot F(k_x)$$

Review: Properties of DFT

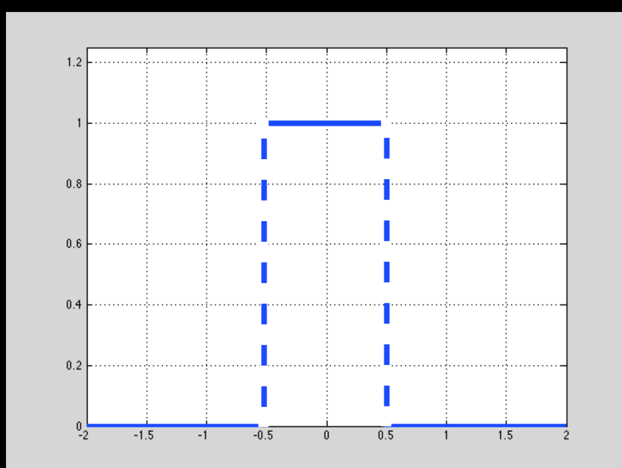
comb or “Shah”

$\mathrm{III}(x)$

FT $\longleftrightarrow$

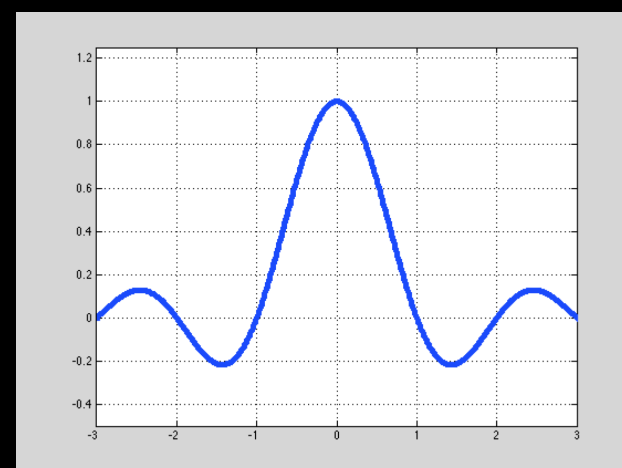


$\mathrm{III}(k_x)$



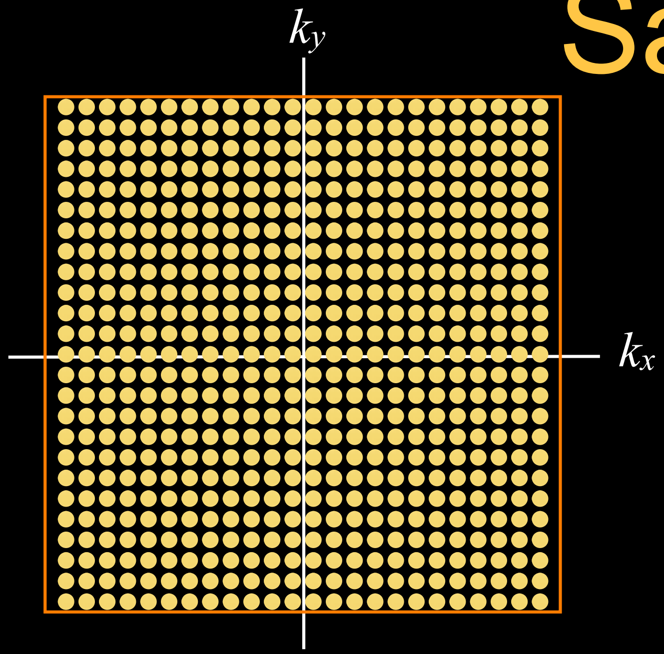
rect

FT $\longleftrightarrow$



$$\mathrm{sinc}(k_x) = \frac{\sin(\pi k_x)}{\pi k_x}$$

Sampling Model



Sampled Version

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \mathrm{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)\frac{1}{\Delta k_x \Delta k_y} \sqcap \left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling — Extent

FT ↕

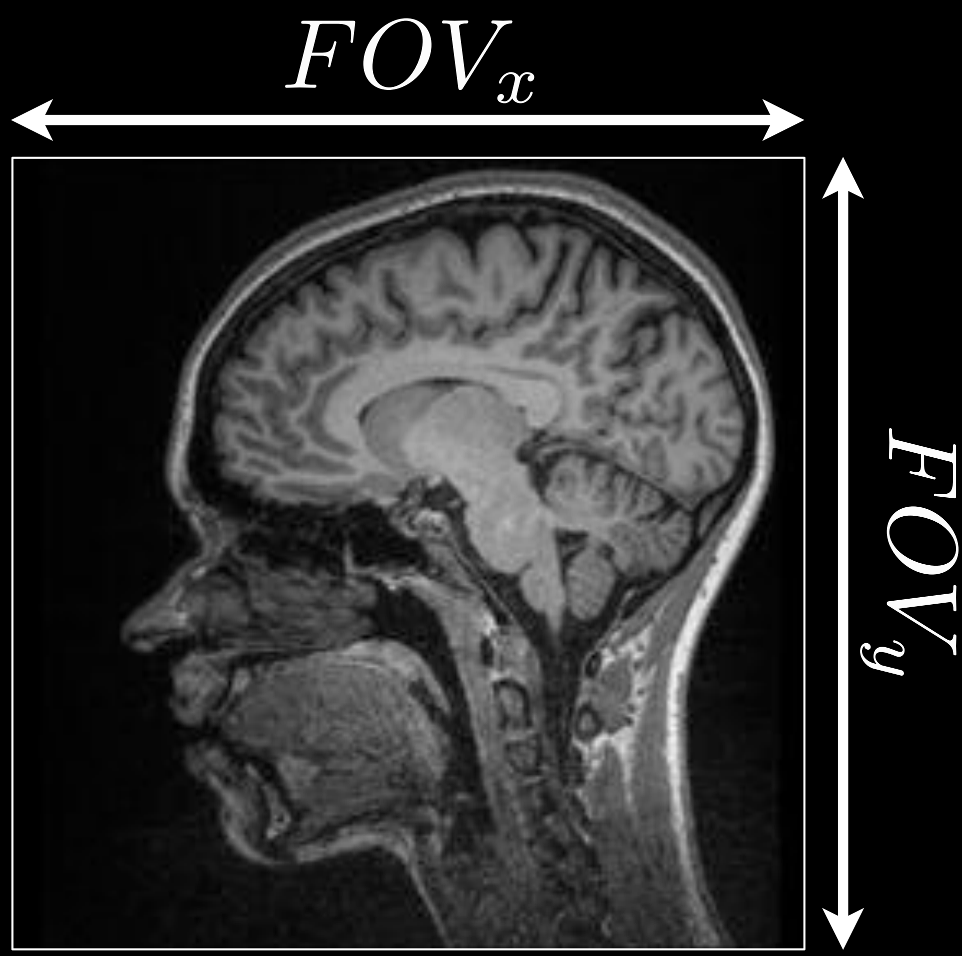
$$\hat{m}(x, y) = m(x, y) * \mathrm{III}(\Delta k_x x, \Delta k_y y) * sinc(w_{k_x} x) sinc(w_{k_y} y)$$

Field of View — Spatial Resolution

Field of View

$$m(x, y) * \mathrm{III}(\Delta k_x x, \Delta k_y y)$$

Field of View



Eq. 5.76

$$\Delta k_x = \frac{1}{FOV_x} = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$\Delta k_y = \frac{1}{FOV_y} = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

To the Board

Field of View

To avoid any aliasing artifacts:

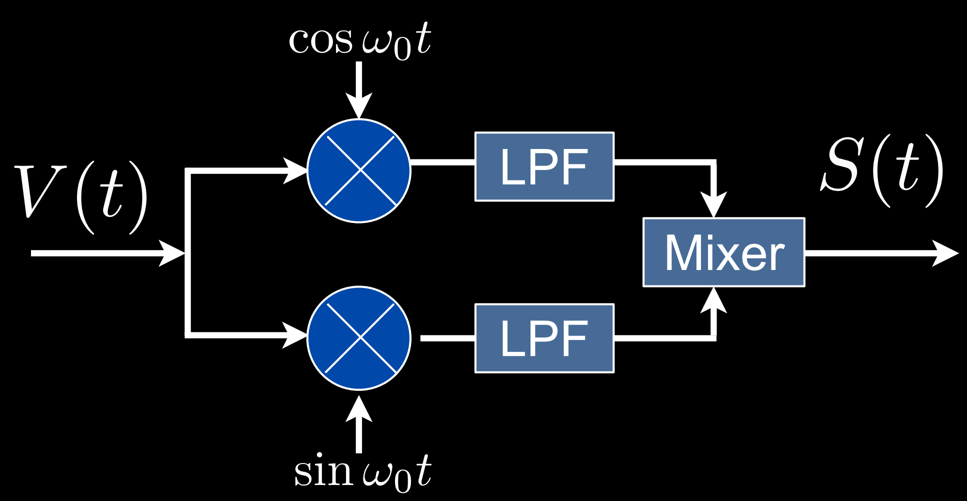
In phase encoding,
- Reduce Δk_y

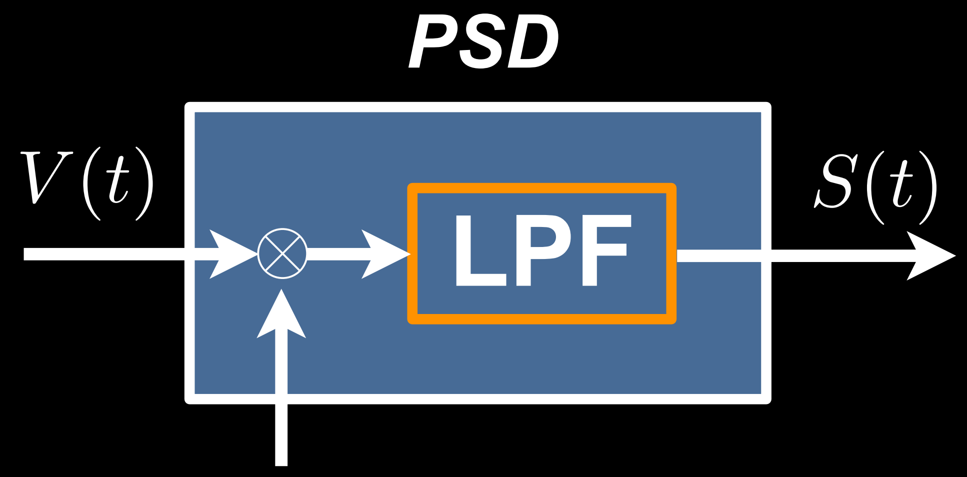
Either lose spatial resolution
or
increase scan time

Field of View

To avoid any aliasing artifacts:

In frequency encoding,
- Reduce Δk_x
- Utilize LPF (low pass filter)

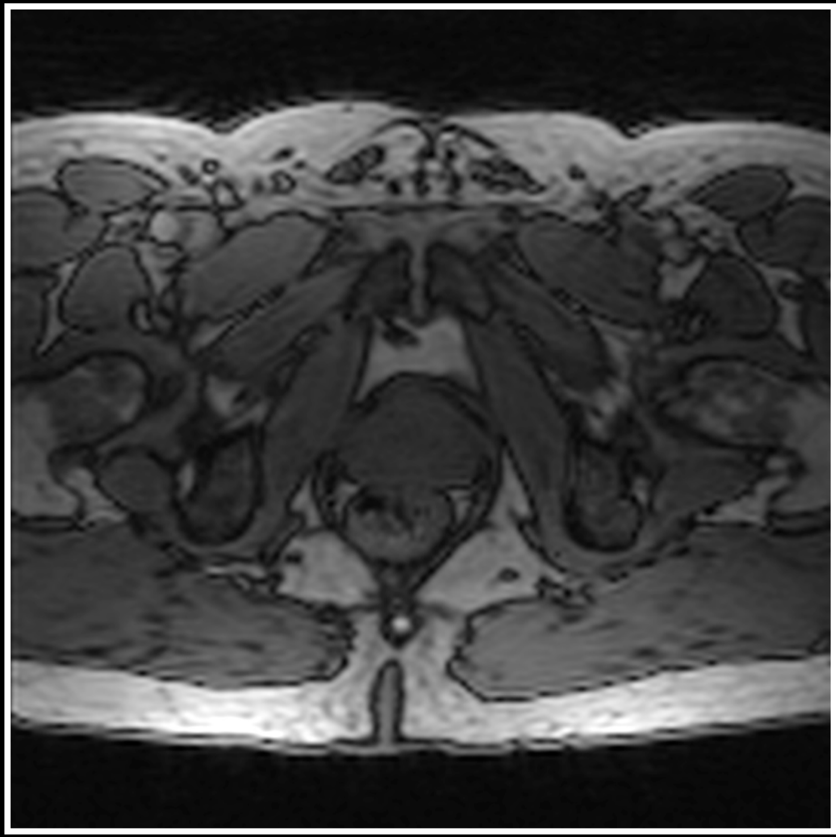




Typically, put long axis of object in readout direction

Field of View

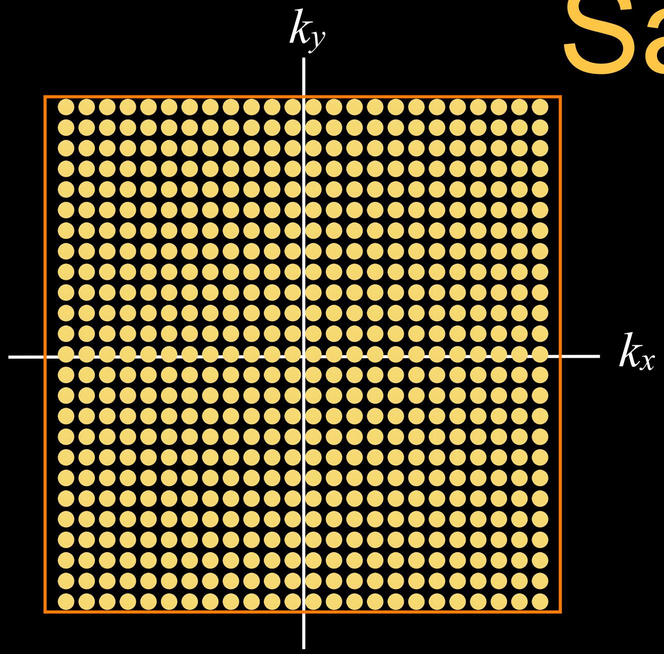
Prostate Imaging Example



Which direction will be readout direction?

Spatial Resolution

Sampling Model



Sampled Version

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)\frac{1}{\Delta k_x \Delta k_y} \sqcap \left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling — Extent

FT ↕

$$\hat{m}(x, y) = m(x, y) * \text{III}(\Delta k_x x, \Delta k_y y) * sinc(w_{k_x} x) sinc(w_{k_y} y)$$

Field of View — Spatial Resolution

Point Spread Function (PSF)

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}) \frac{1}{\Delta k_x \Delta k_y} \underbrace{\sqcap(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}})}_{\text{Extent}}$$

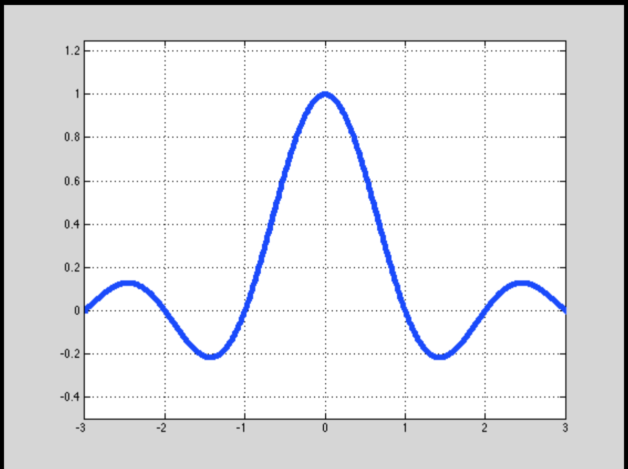
$$\hat{M}'(k_x, k_y) = \hat{M}(k_x, k_y) \cdot \text{window}$$

$$\text{PSF} = \text{FT}(\text{window})$$

Point spread function can show the extent of blurring of the image

Spatial Resolution

$$m(x,y) * sinc(w_{k_x} x) sinc(w_{k_y} y) w_{k_x} w_{k_y}$$



Main lobe causes blurring!
(spatial resolution)

Spatial resolution: δ_x, δ_y

$$\delta_x = \frac{1}{w_{k_x}} \qquad \delta_y = \frac{1}{w_{k_y}}$$

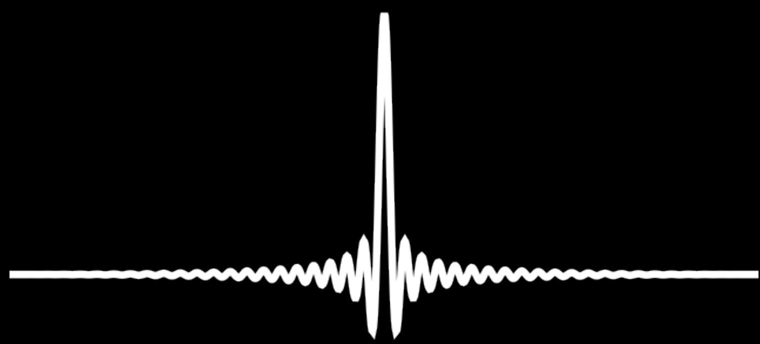
Spatial Resolution

- Spatial resolution of an imaging system is the smallest separation δx of two point sources necessary for them to remain resolvable in the resultant image.

$$\hat{I}\left(x\right) = I\left(x\right) * h\left(x\right)$$

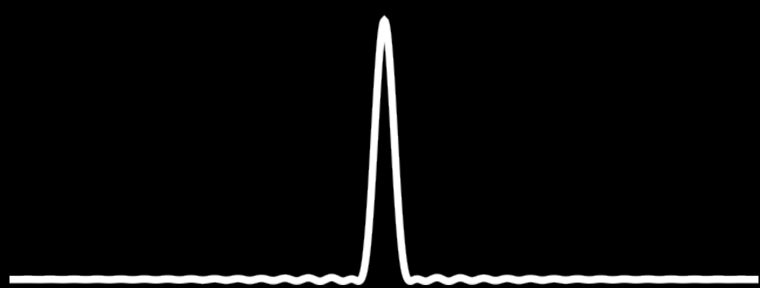
Image — Object — Point Spread Function

PSFs



$\mathcal{F}\{\Pi(t)\}$

Narrower central peak, but lots of ringing



$\mathcal{F}\{H(t)\}$

Reduced ringing, but broader central peak

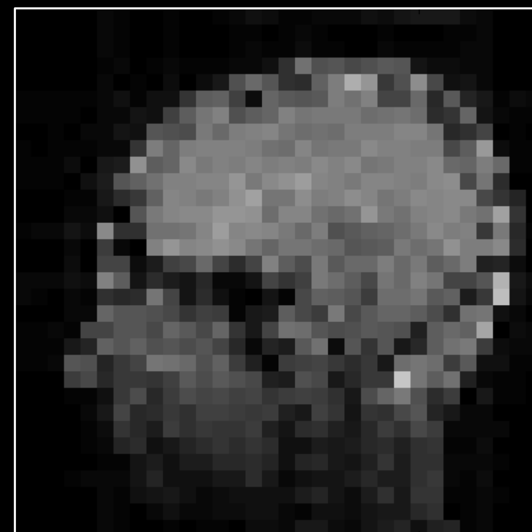
PSFs

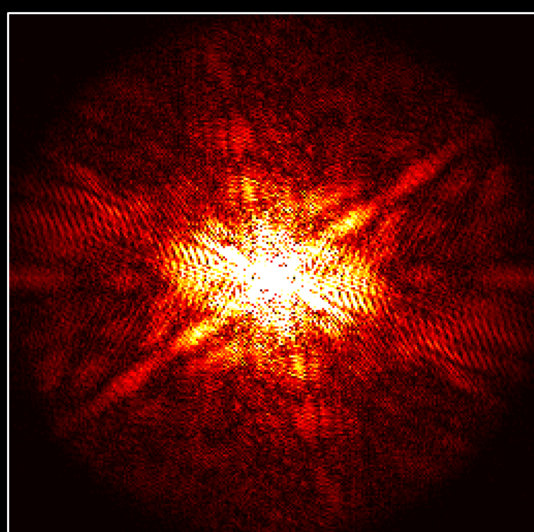
Filters can be used to reduce ringing artifacts but often at the expense of spatial resolution

Hamming window seems to have good balance in reducing ringing

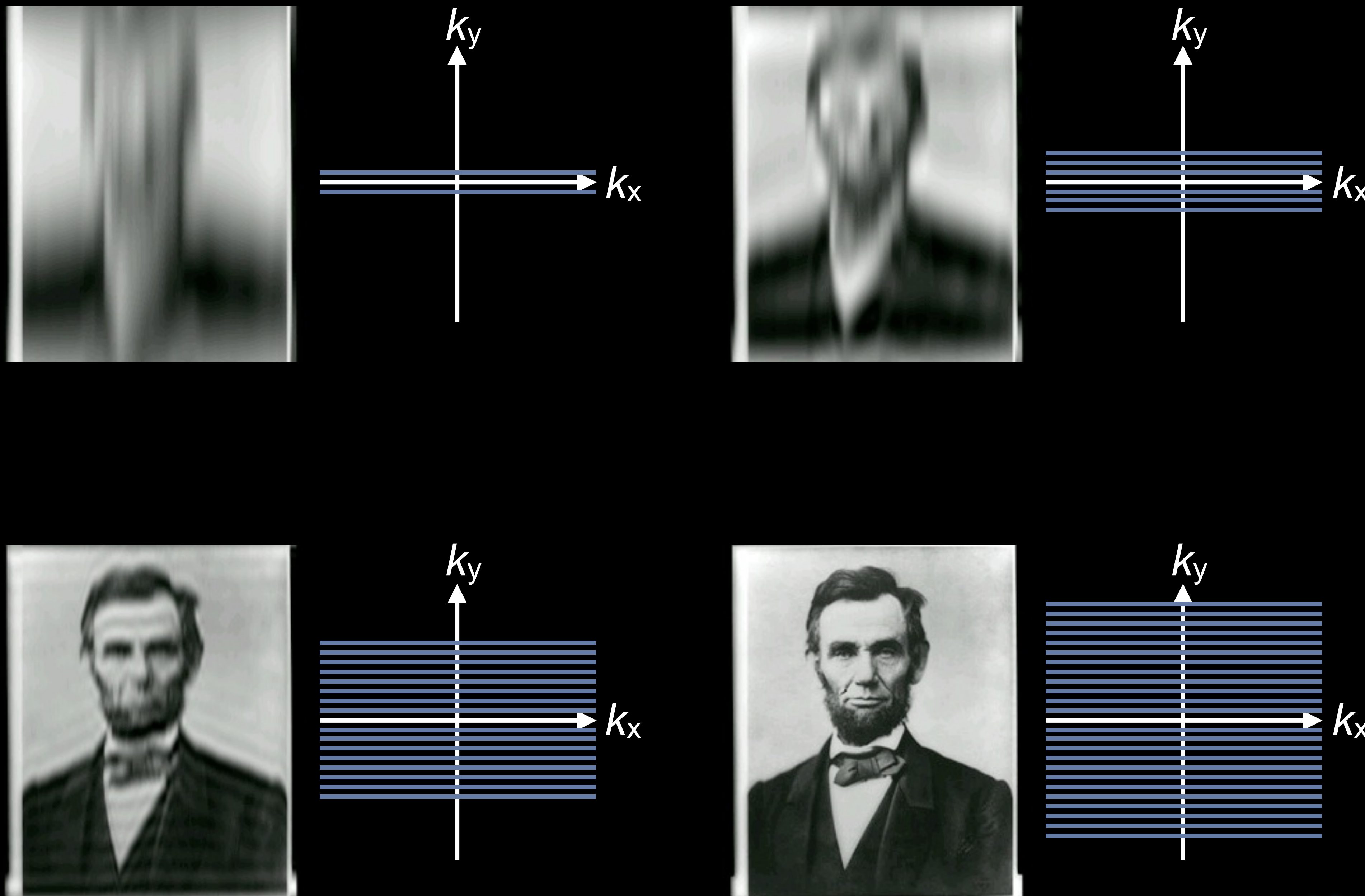
Finite Sampling

$$W_h = \frac{1}{N\Delta k} = \frac{FOV}{N}$$



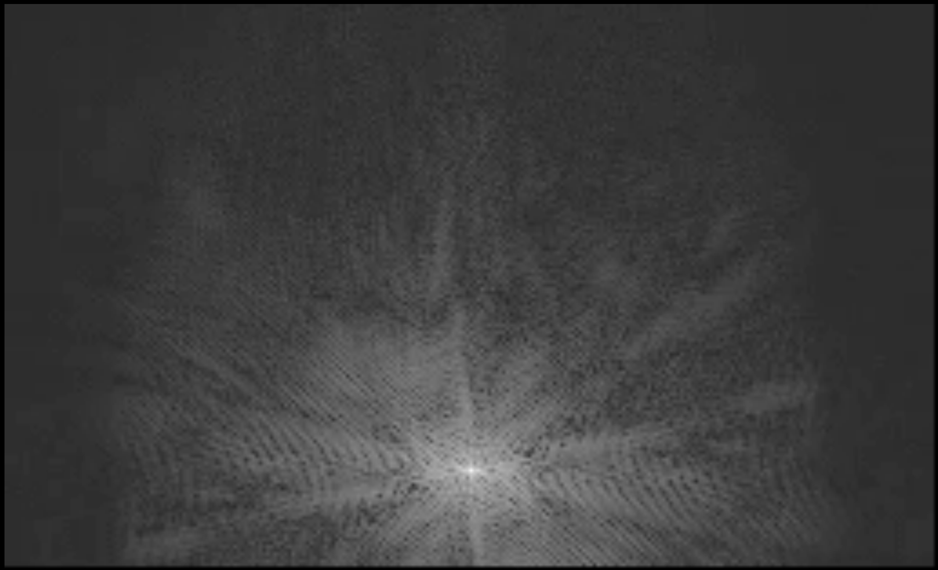


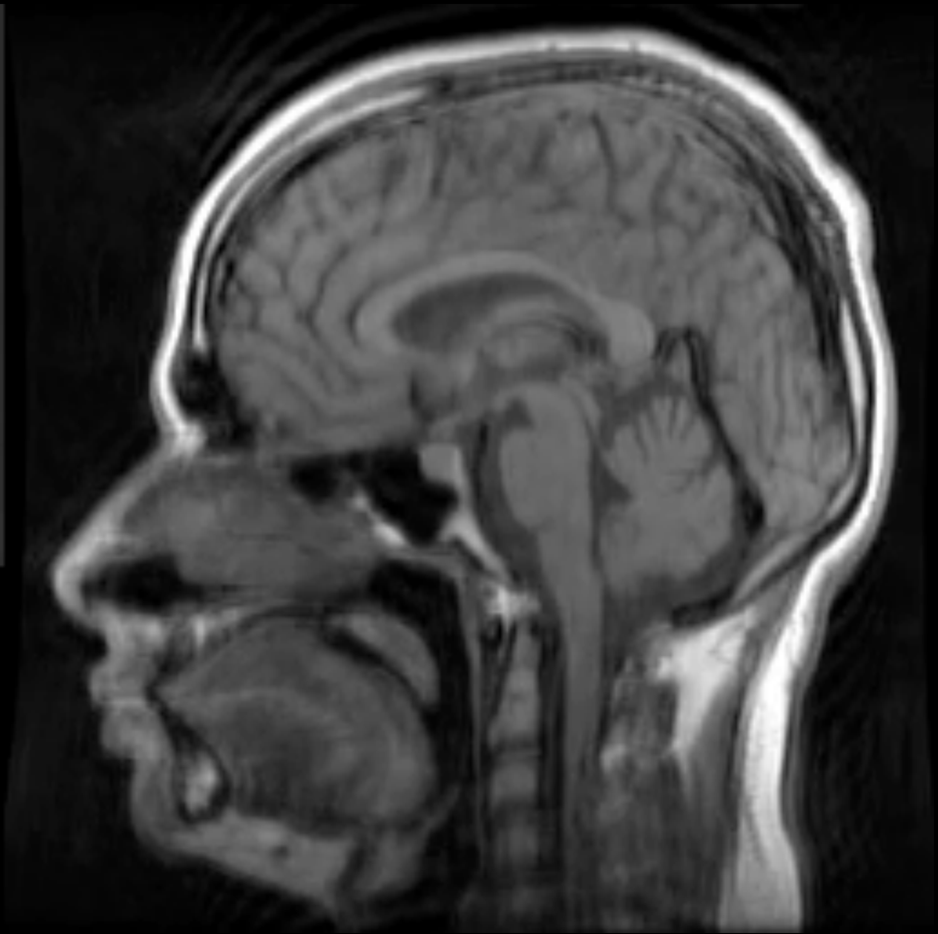
k-space Sampling



example from Dan

k-space Sampling





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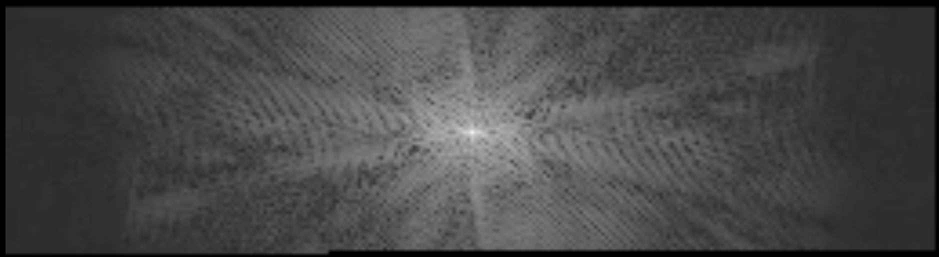
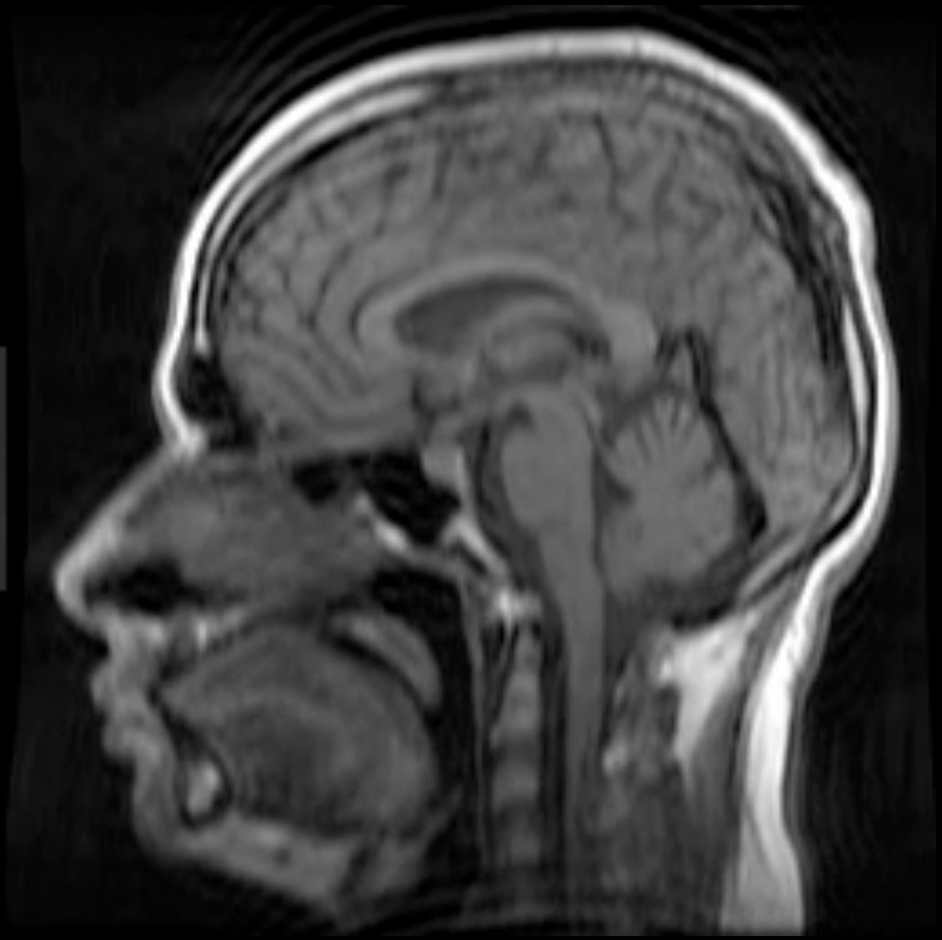
k-space Sampling

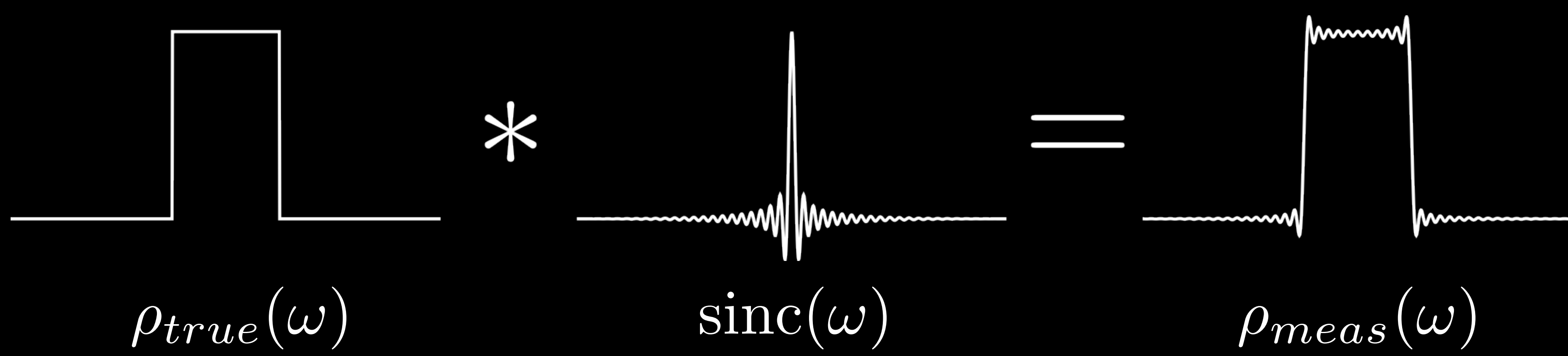




example from Dan

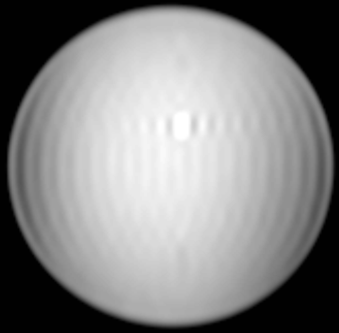
Gibb's Ringing

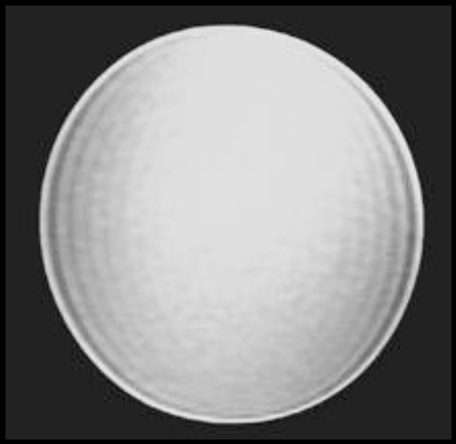
Distortions in the profile arising from the finite sampling of the data

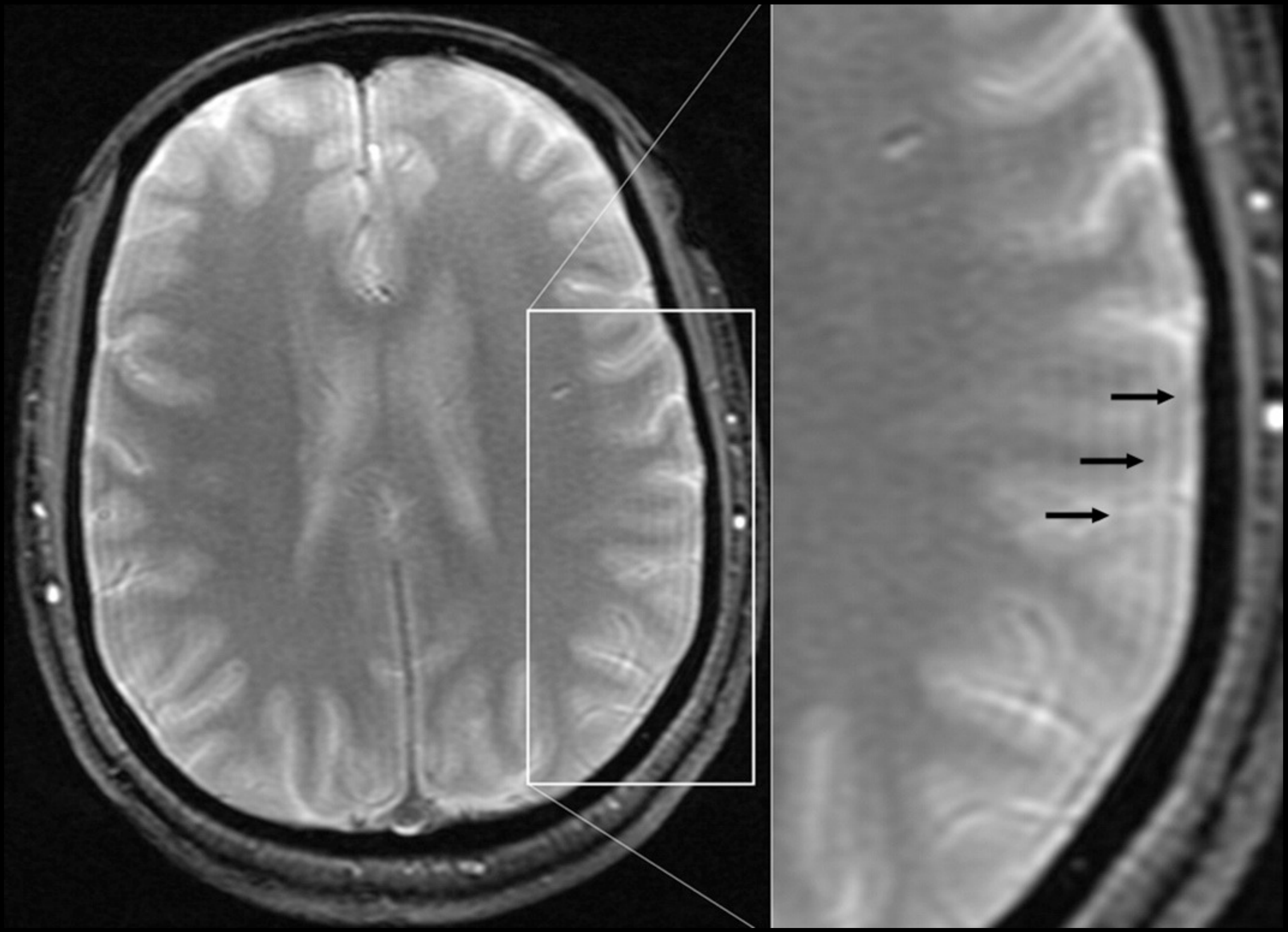


This type of distortion is most commonly referred to as Gibb's ringing

Examples of Gibb's Ringing

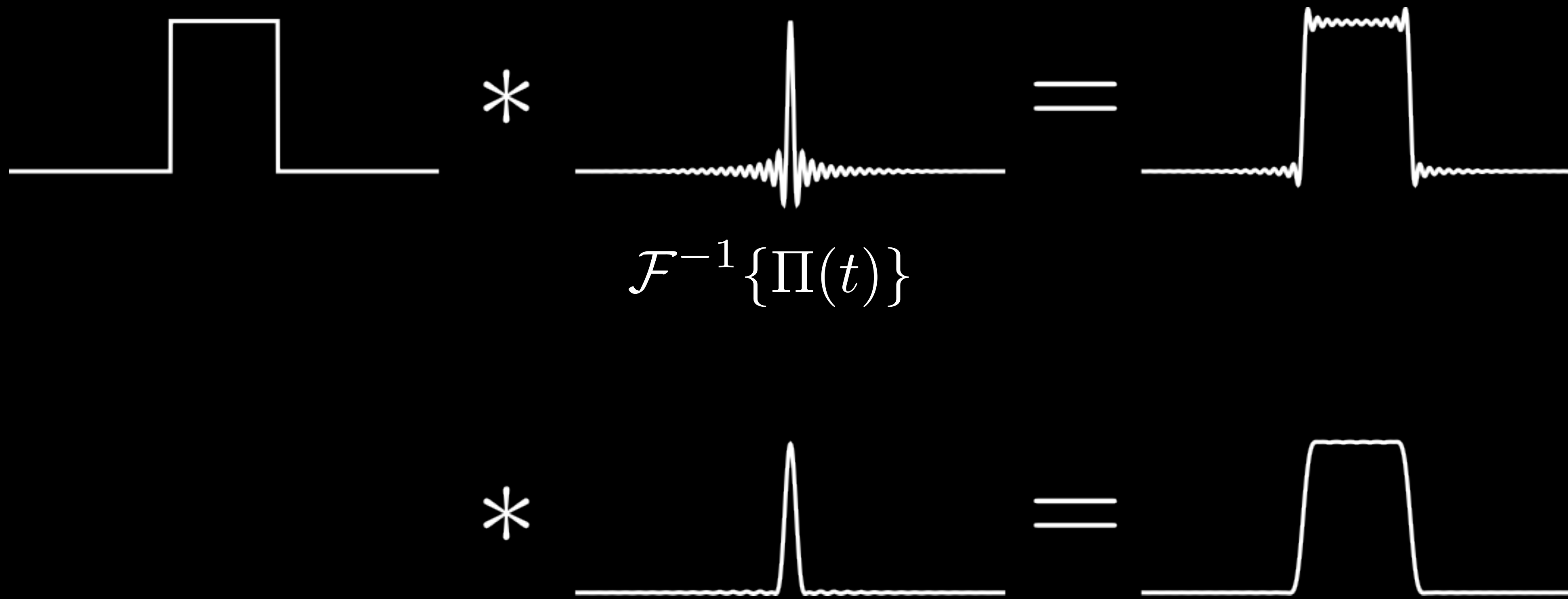






Gibb's Ringing

how to reduce ringing



$\mathcal{F}^{-1}\{\Pi(t)\}$

$\mathcal{F}^{-1}\{H(t)\}$

Hamming window can be used to reduce ringing

Questions?

- Related reading materials
  - Nishimura - Chap 5

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