Spatial Localization II

M219 - Principles and Applications of MRI

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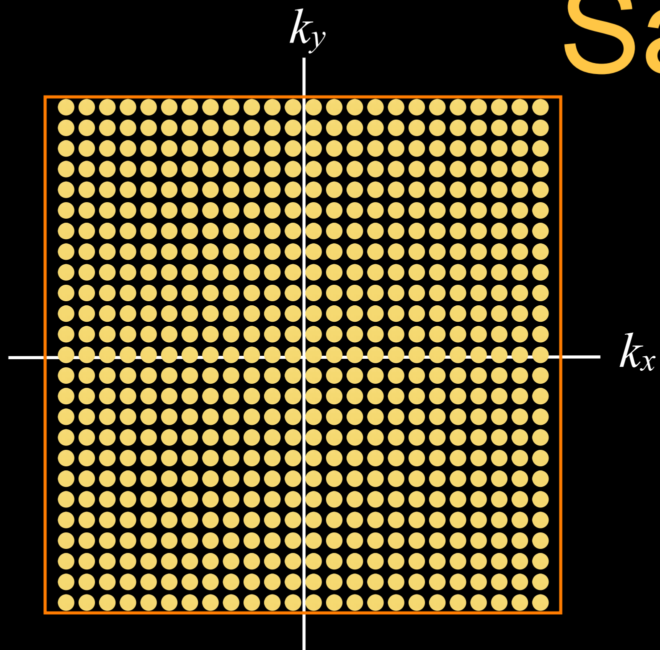
2/5/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #2 due on 2/12

- TA office hours, Mon 4-6pm
- Office hours, Fri 10-11am

Sampling Model



Sampled Version

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \mathrm{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)\frac{1}{\Delta k_x \Delta k_y} \sqcap \left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling — Extent

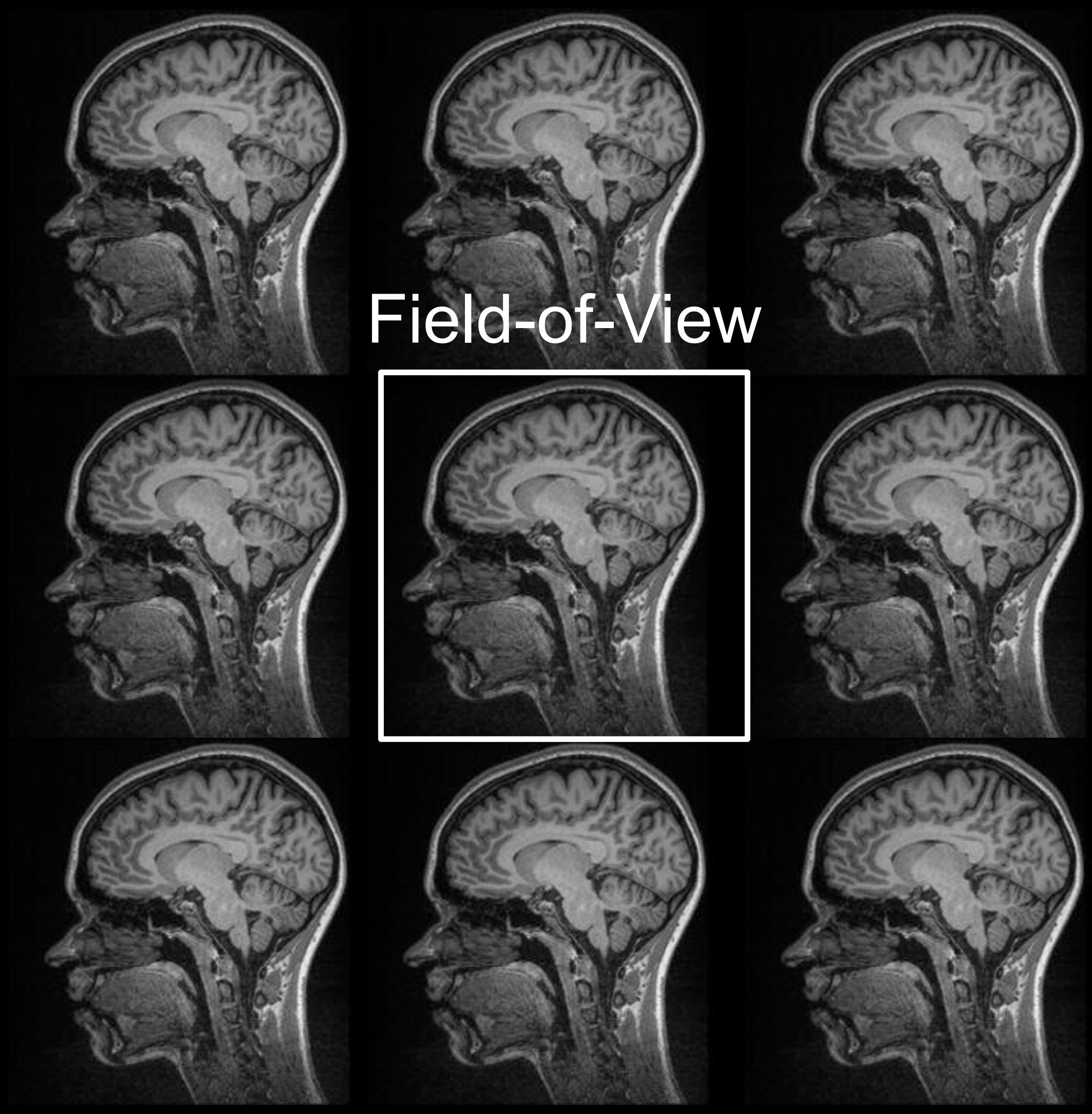
FT ↕

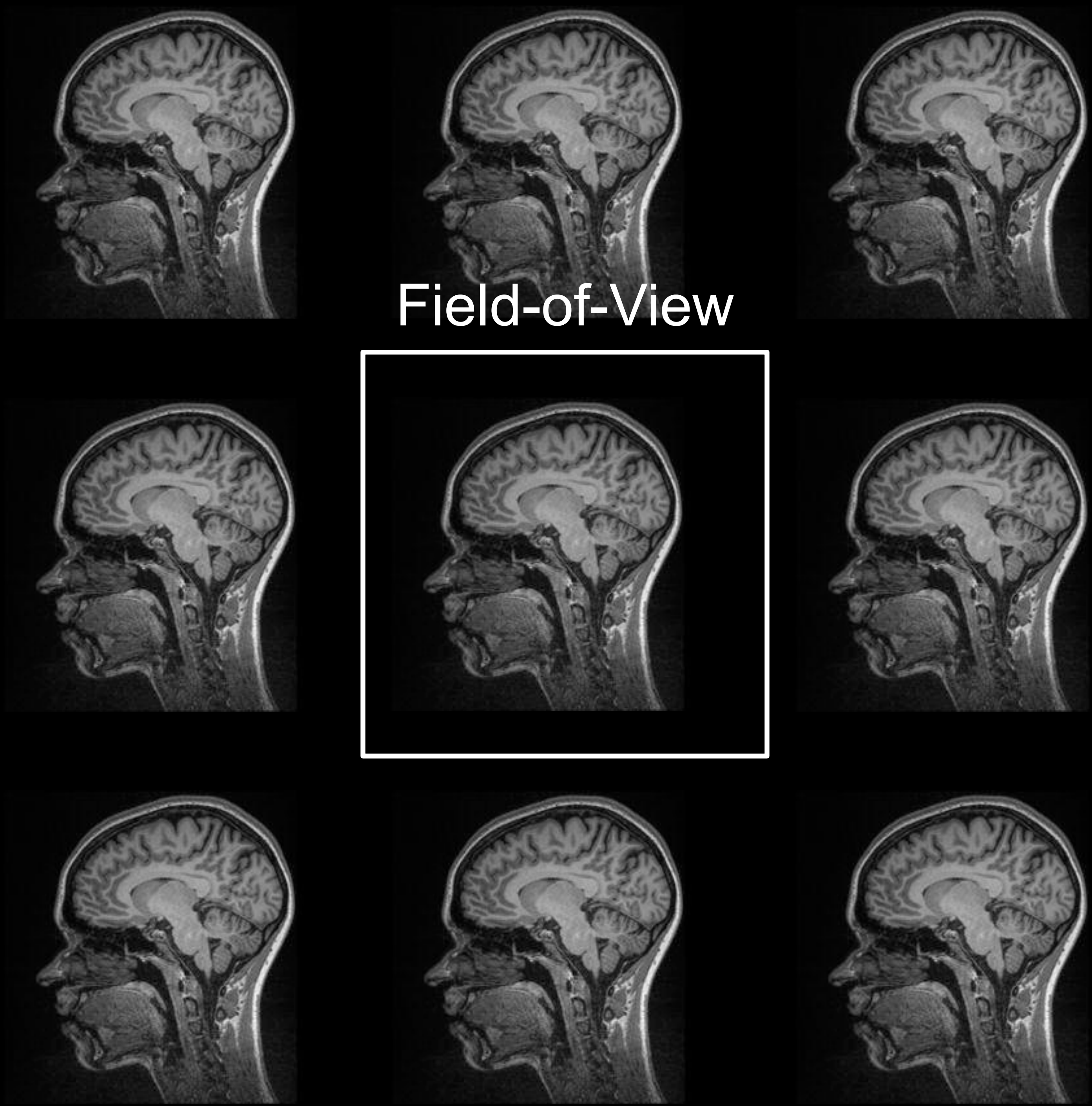
$$\hat{m}(x, y) = m(x, y) * \mathrm{III}(\Delta k_x x, \Delta k_y y) * sinc(w_{k_x} x) sinc(w_{k_y} y)$$

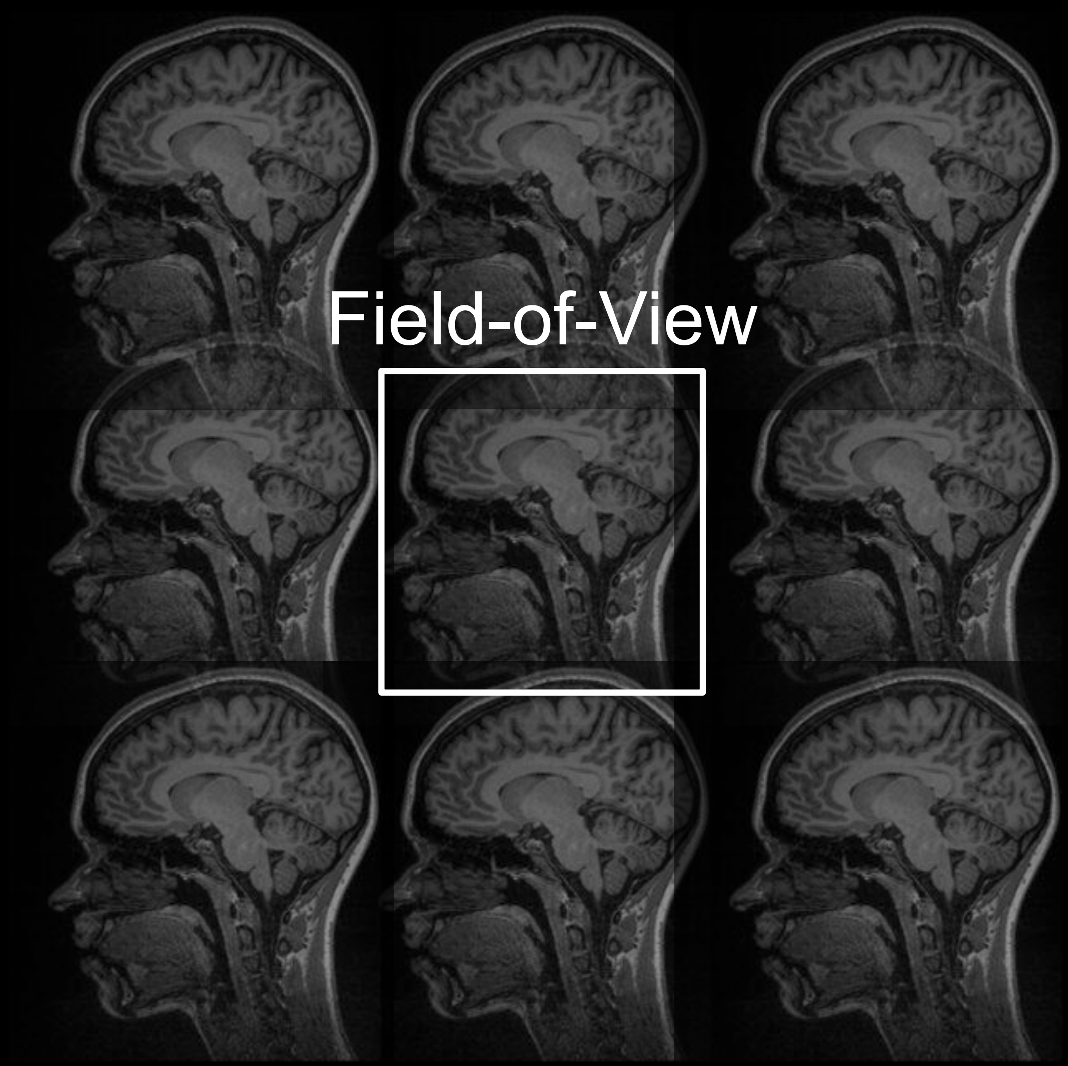
Field of View — Spatial Resolution

Field of View

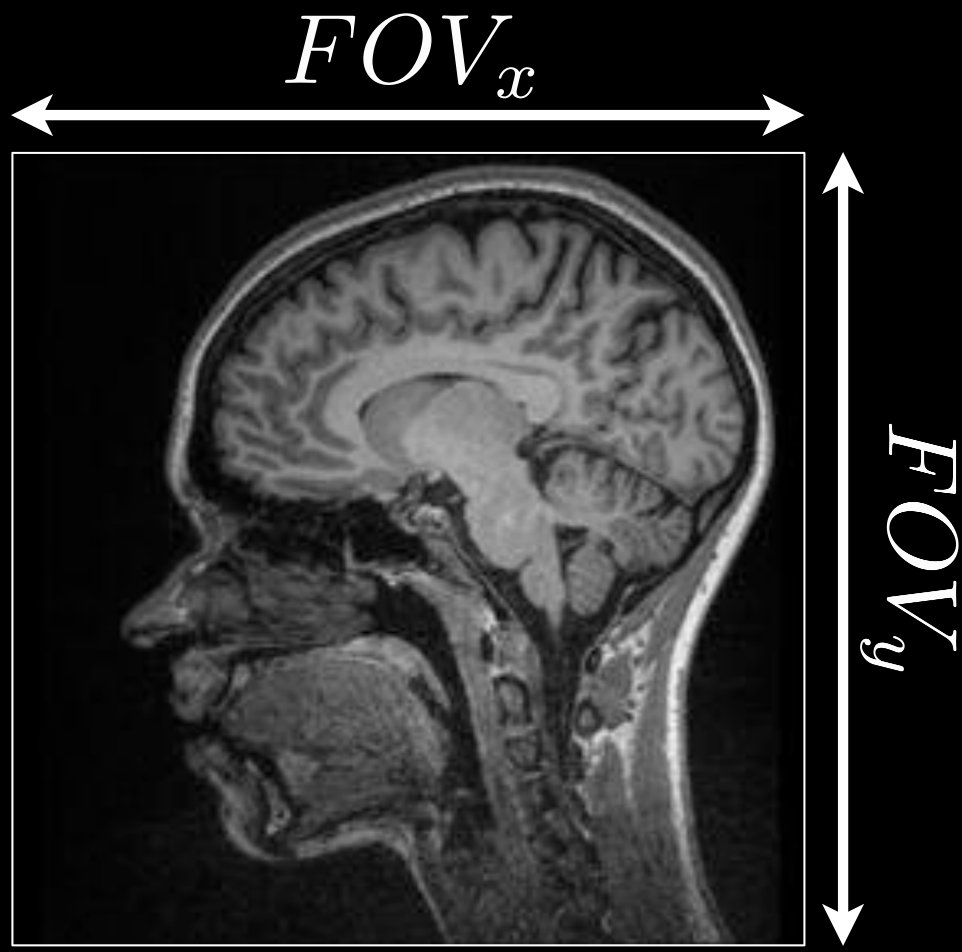
$$m(x, y) * \mathrm{III}(\Delta k_x x, \Delta k_y y)$$


Field-of-View


Field-of-View


Field-of-View

Field of View



$$\Delta k_x = \frac{1}{FOV_x} = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$\Delta k_y = \frac{1}{FOV_y} = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

Eq. 5.76

Point Spread Function (PSF)

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \mathrm{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}) \frac{1}{\Delta k_x \Delta k_y} \underbrace{\sqcap(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}})}_{\text{Extent}}$$

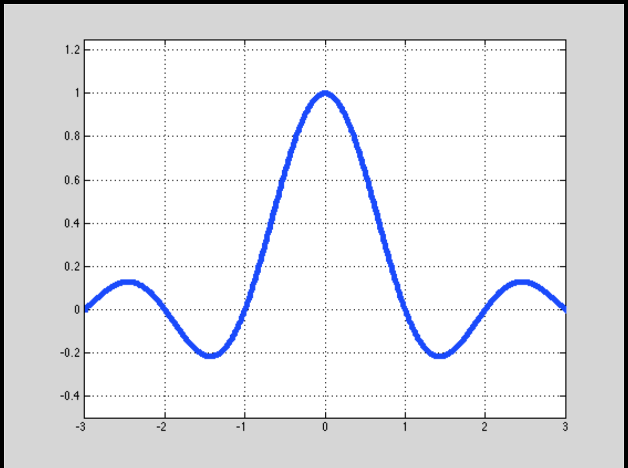
$$\hat{M}'(k_x, k_y) = \hat{M}(k_x, k_y) \cdot \mathrm{window}$$

$$\mathrm{PSF} = \mathrm{FT}(\mathrm{window})$$

Point spread function can show
the extent of blurring of the image

Spatial Resolution

$$m(x,y) * sinc(w_{k_x} x) sinc(w_{k_y} y) w_{k_x} w_{k_y}$$



Main lobe causes blurring!
(spatial resolution)

Spatial resolution: δ_x, δ_y

$$\delta_x = \frac{1}{w_{k_x}} \qquad \delta_y = \frac{1}{w_{k_y}}$$

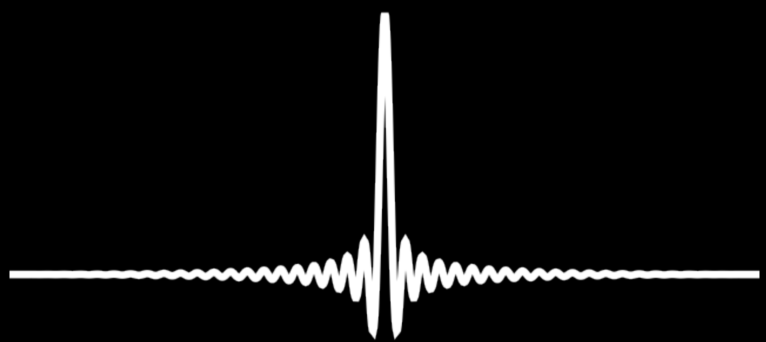
Spatial Resolution

- Spatial resolution of an imaging system is the smallest separation δx of two point sources necessary for them to remain resolvable in the resultant image.

$$\hat{I}(x) = I(x) * h(x)$$

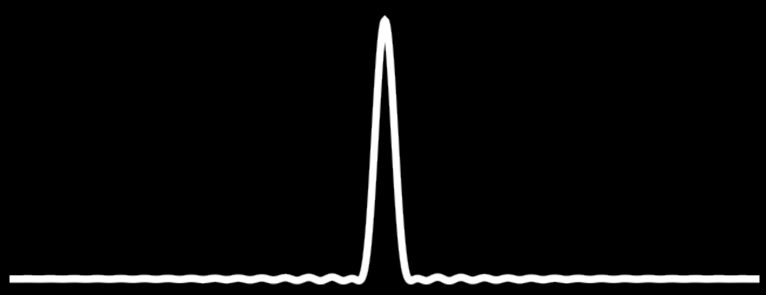
Image — Object — Point Spread Function

PSFs



$\mathcal{F}\{\Pi(t)\}$

Narrower central peak, but lots of ringing



$\mathcal{F}\{H(t)\}$

Reduced ringing, but broader central peak

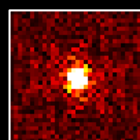
PSFs

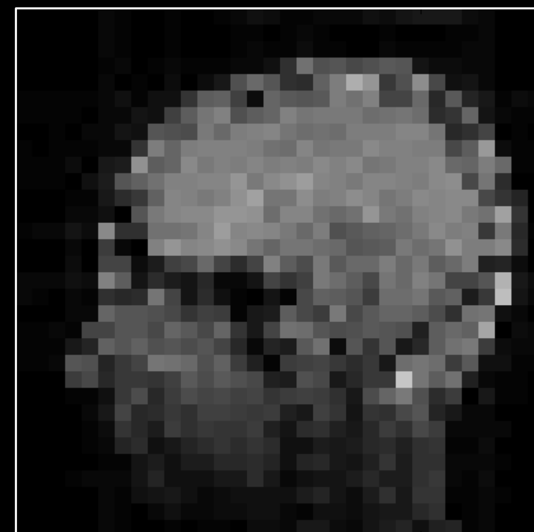
Filters can be used to reduce ringing artifacts but often at the expense of spatial resolution

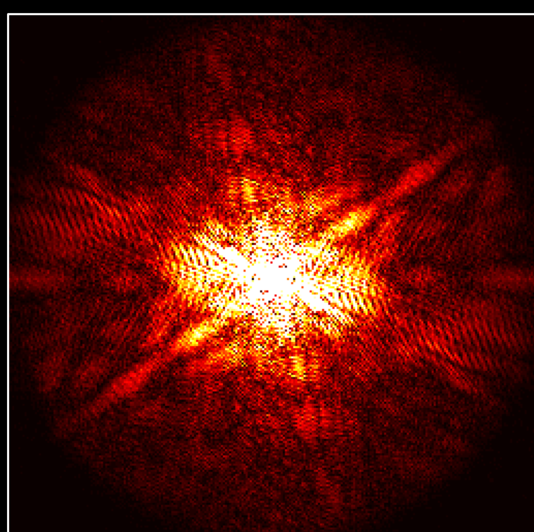
Hamming window seems to have good balance in reducing ringing

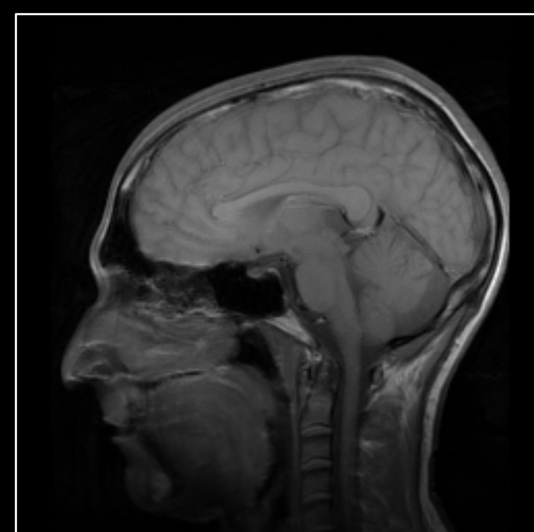
Finite Sampling

$$W_h = \frac{1}{N\Delta k} = \frac{FOV}{N}$$

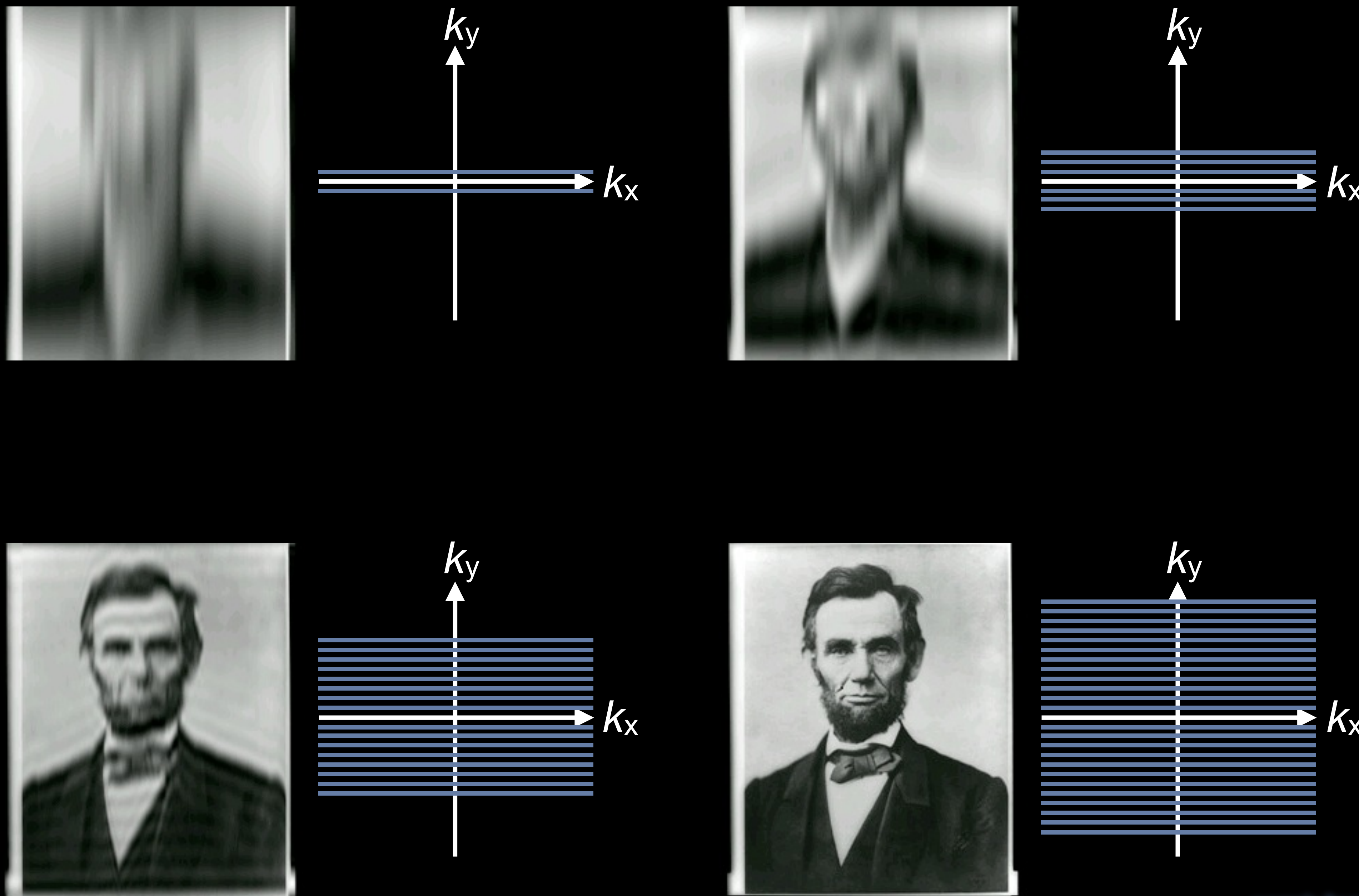






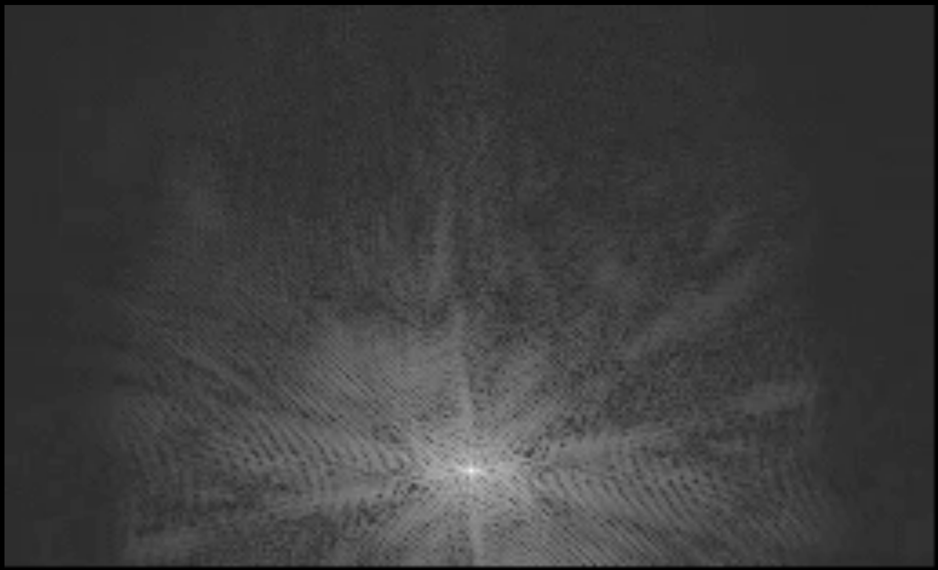


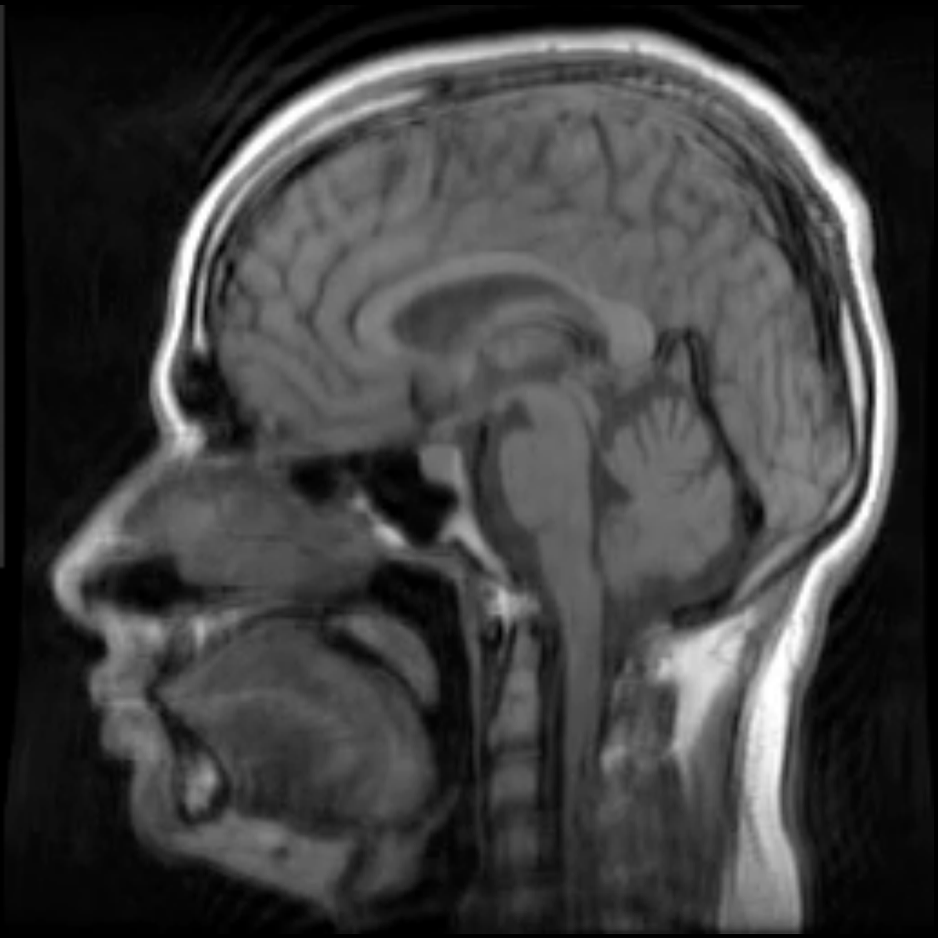
k-space Sampling



example from Dan

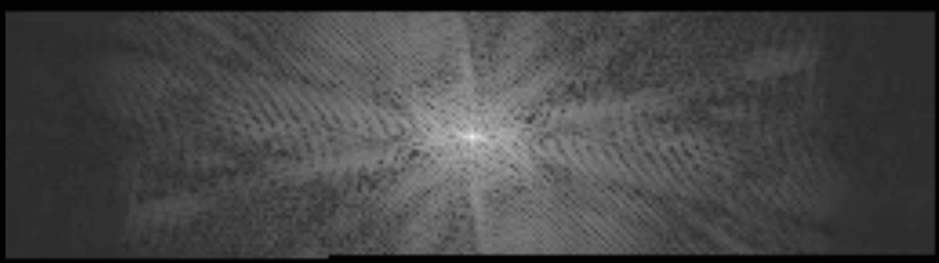
k-space Sampling

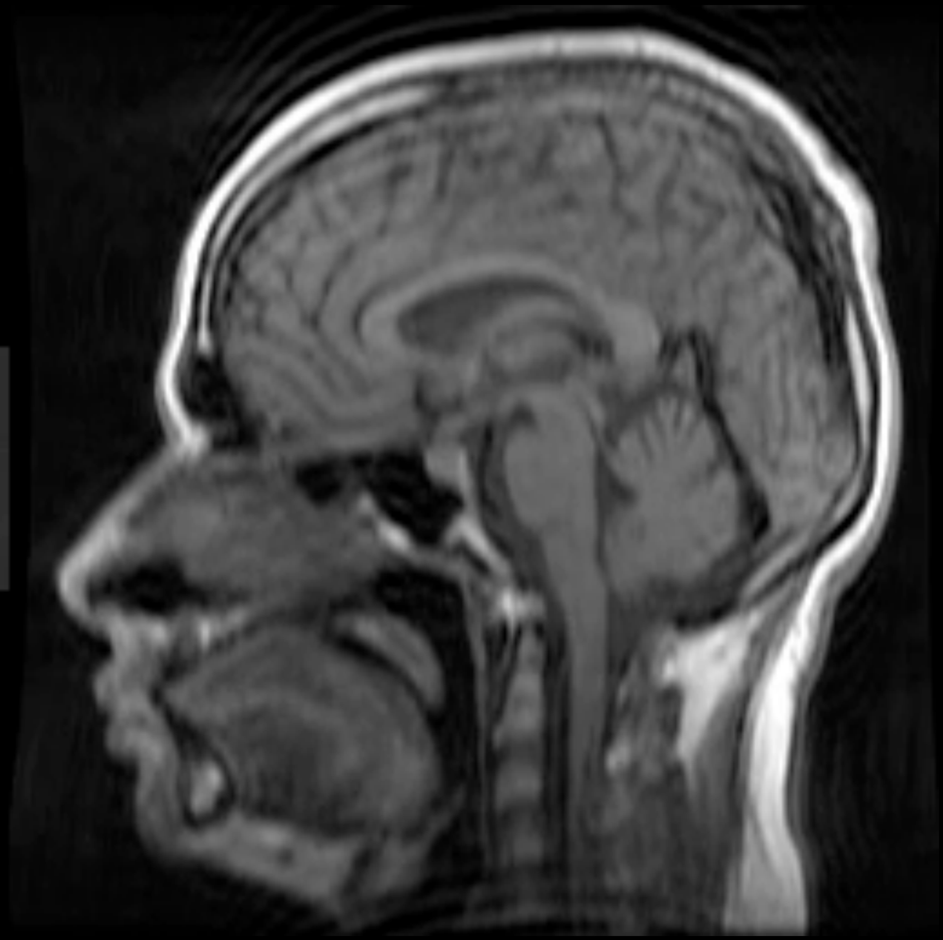




example from Dan

k-space Sampling

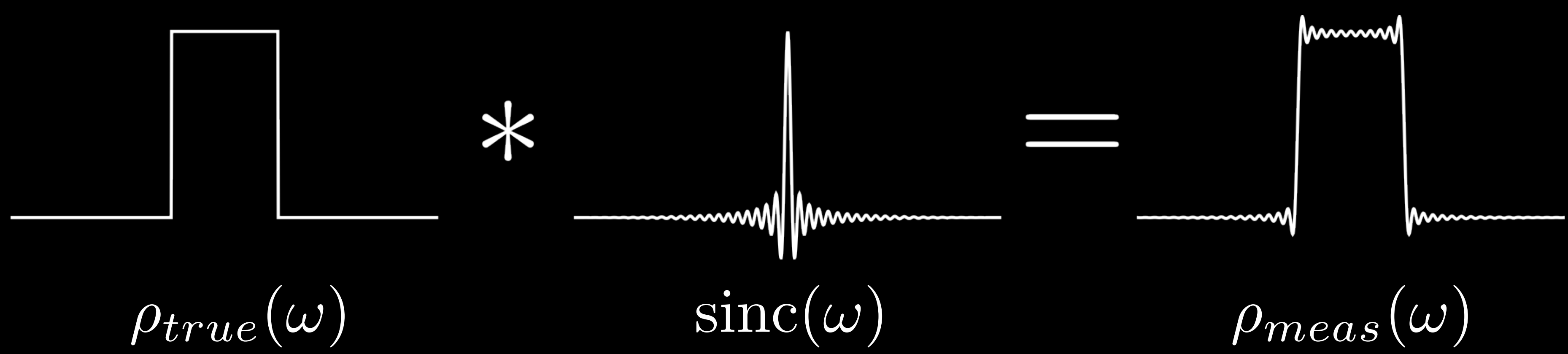




example from Dan

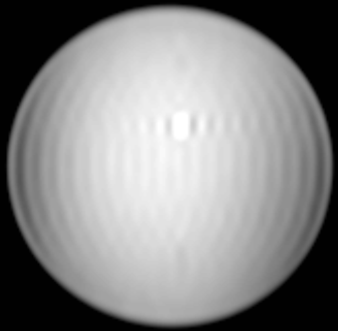
Gibb's Ringing

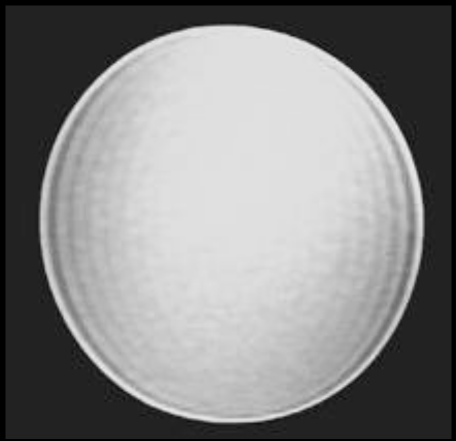
Distortions in the profile arising
from the finite sampling of the data

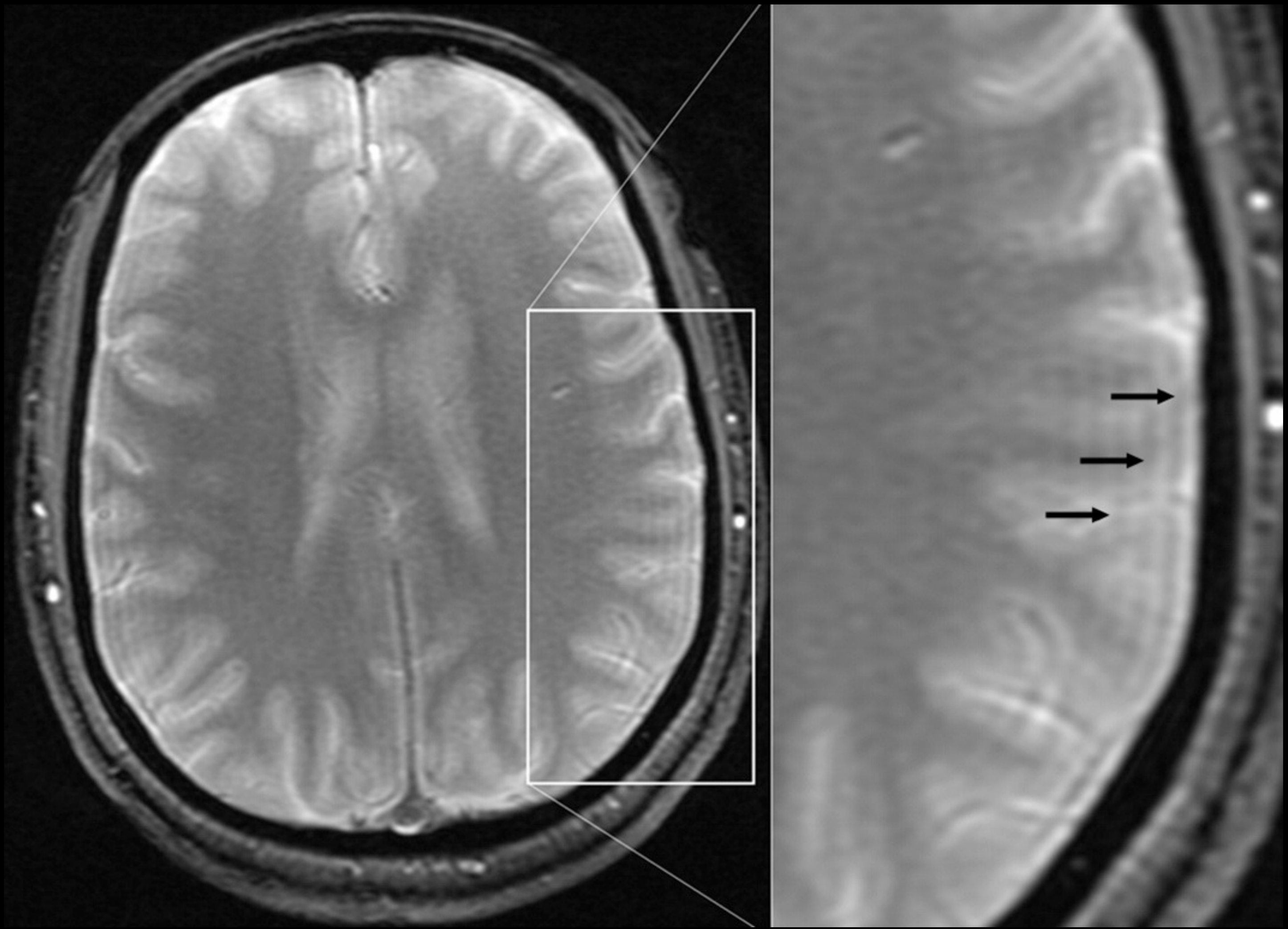


This type of distortion is most commonly
referred to as Gibb's ringing

Examples of Gibb's Ringing

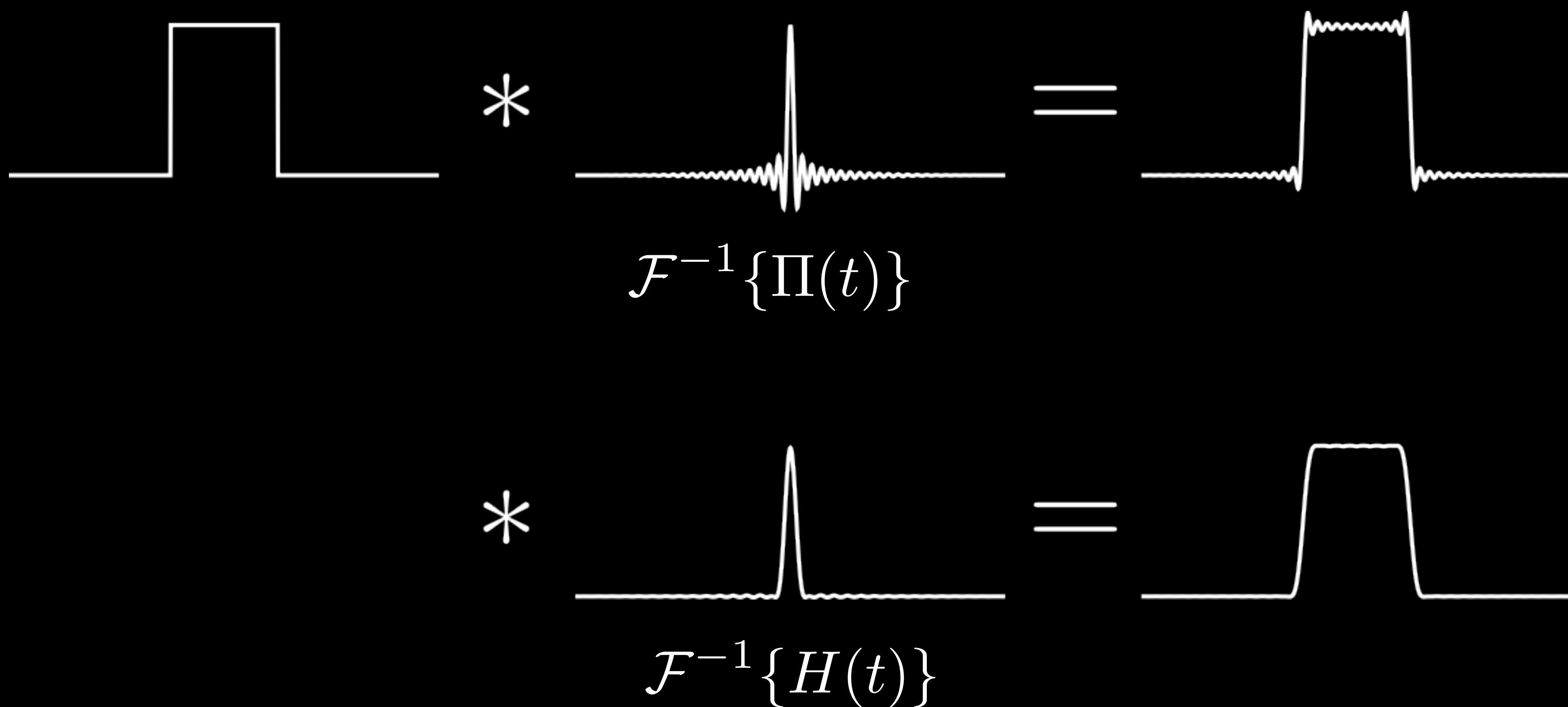






Gibb's Ringing

how to reduce ringing



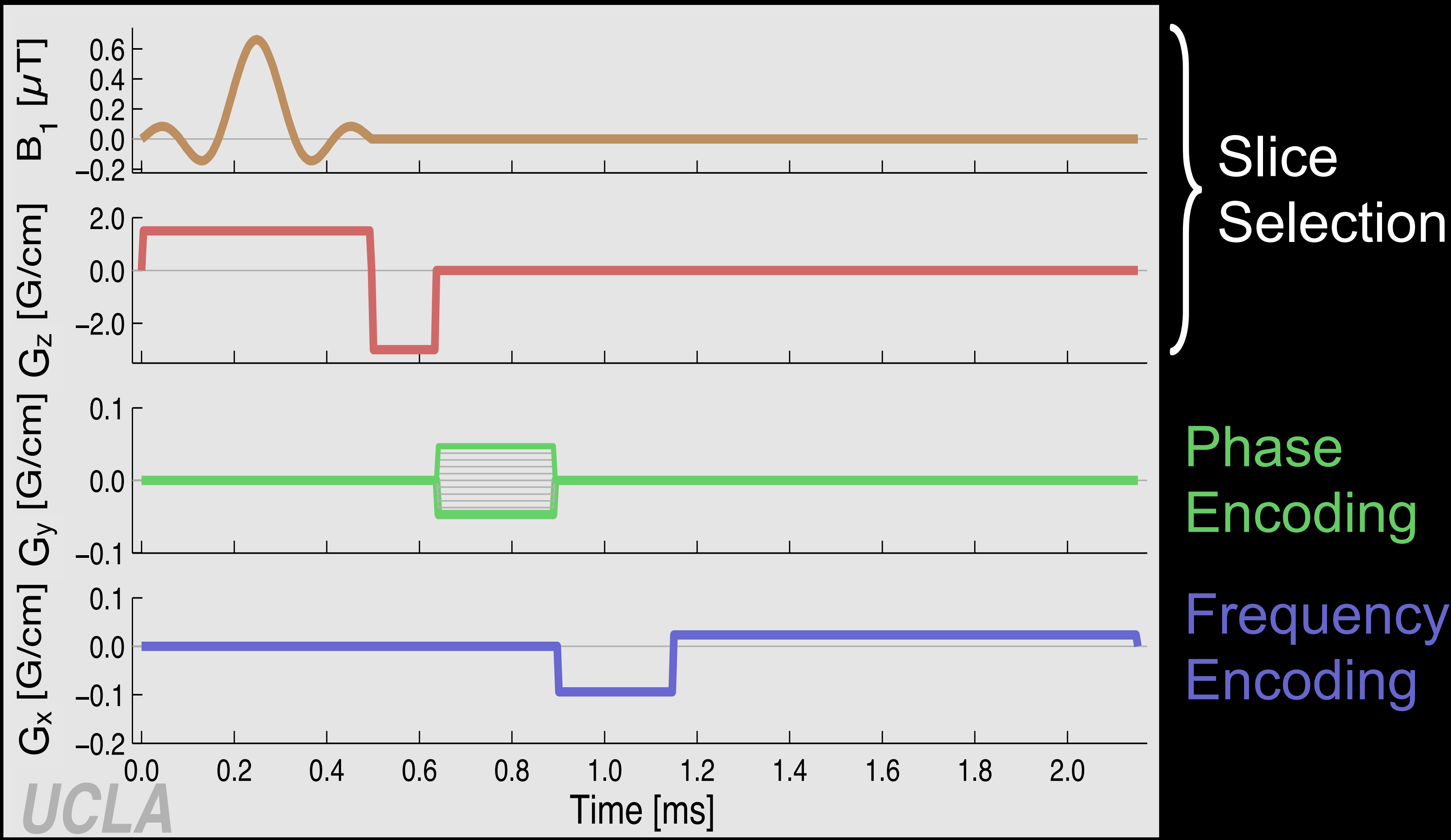
Hamming window can be used to reduce ringing

Spatial Localization

Spatial Encoding

- Three key steps:
  - – Slice selection
    - You have to pick slice!
  - – Phase Encoding
    - You have to encode 1 of 2 dimensions within the slice.
  - – Frequency Encoding (aka *readout*)
    - You have to encode the other dimension within the slice.

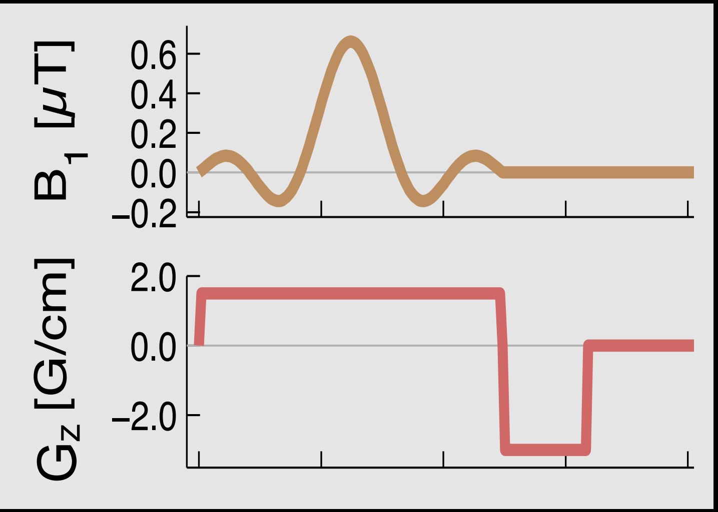
3 Steps for Spatial Localization



Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

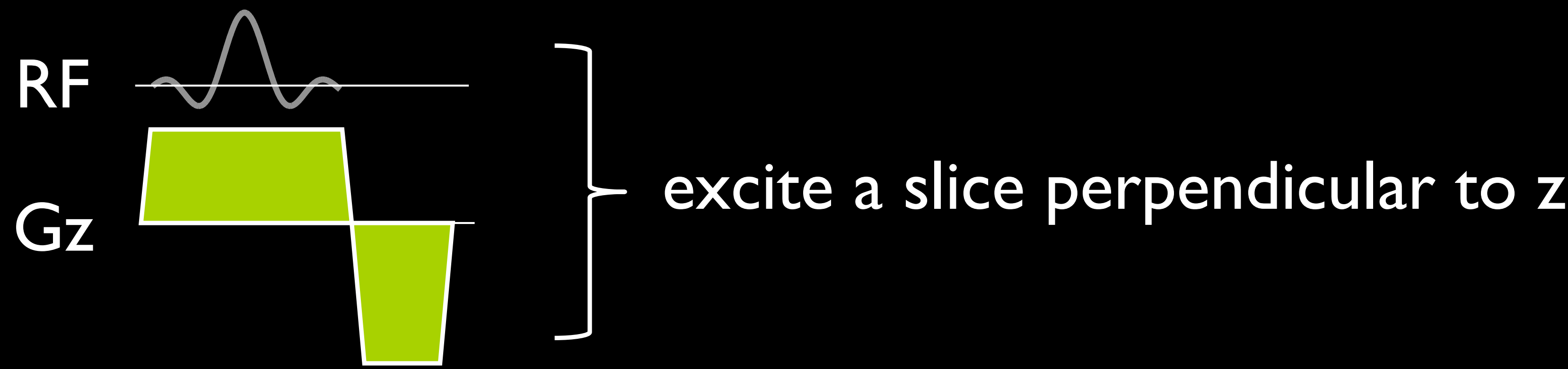
Slice Selection

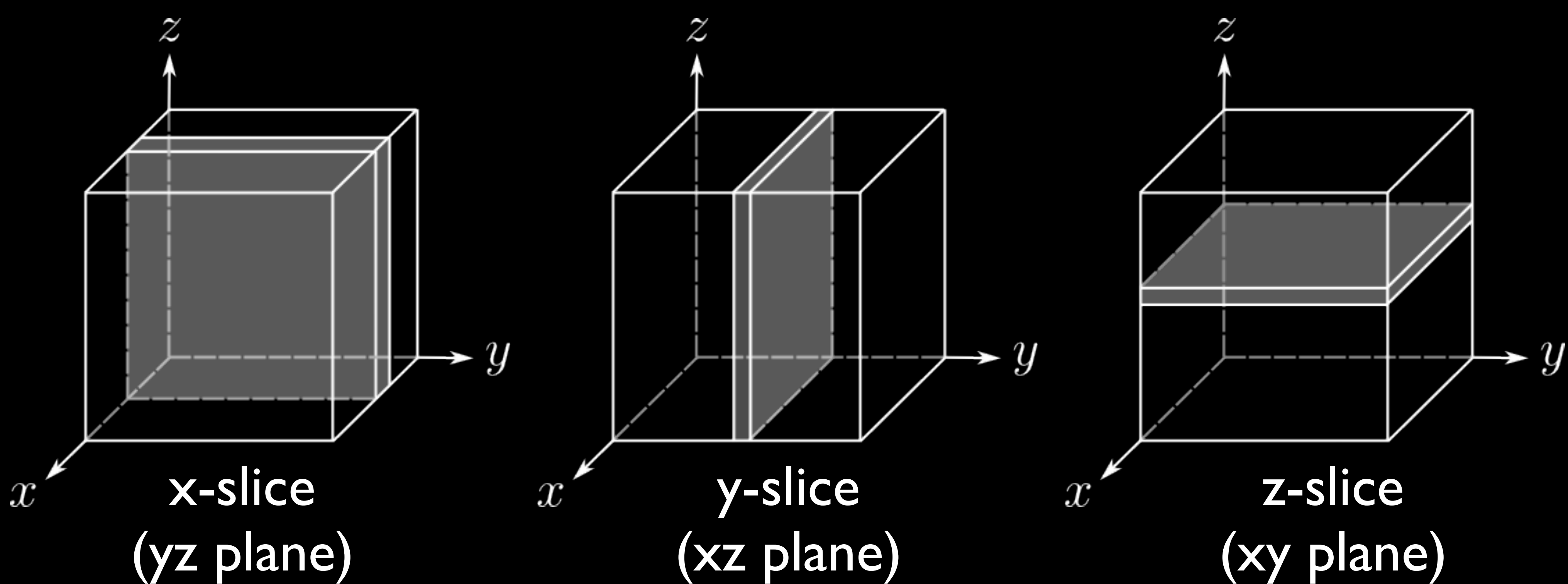
- Consists of:
  - RF (B_1) Pulse
    - Contains frequencies matched to slice of interest
  - Slice selection gradient
    - Constant magnitude
  - Slice re-phasing gradient
    - Increases SNR
    - Re-phases spins within slice
    - AKA “slice refocusing gradient”



- Permits exciting the slice of interest.

Selective Excitation

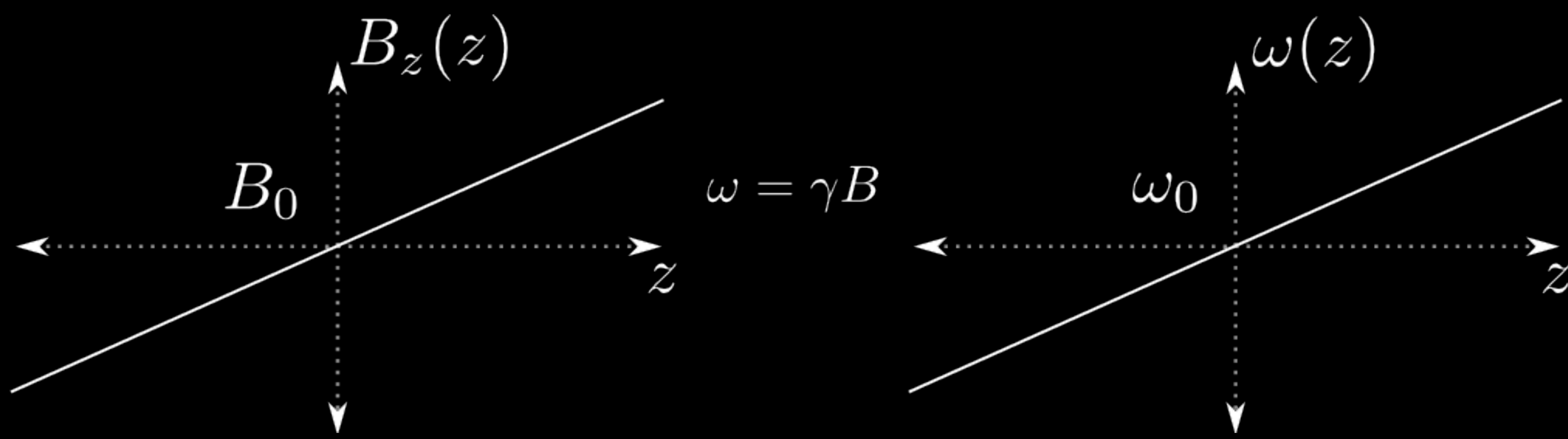




Gradients?

gradients produce a spatial distribution of frequencies

$$B_z(z) = B_0 + G_z \cdot z$$



$$\omega(z) = \omega_0 + \gamma G_z \cdot z$$

there is a direct correspondence between frequency and spatial position

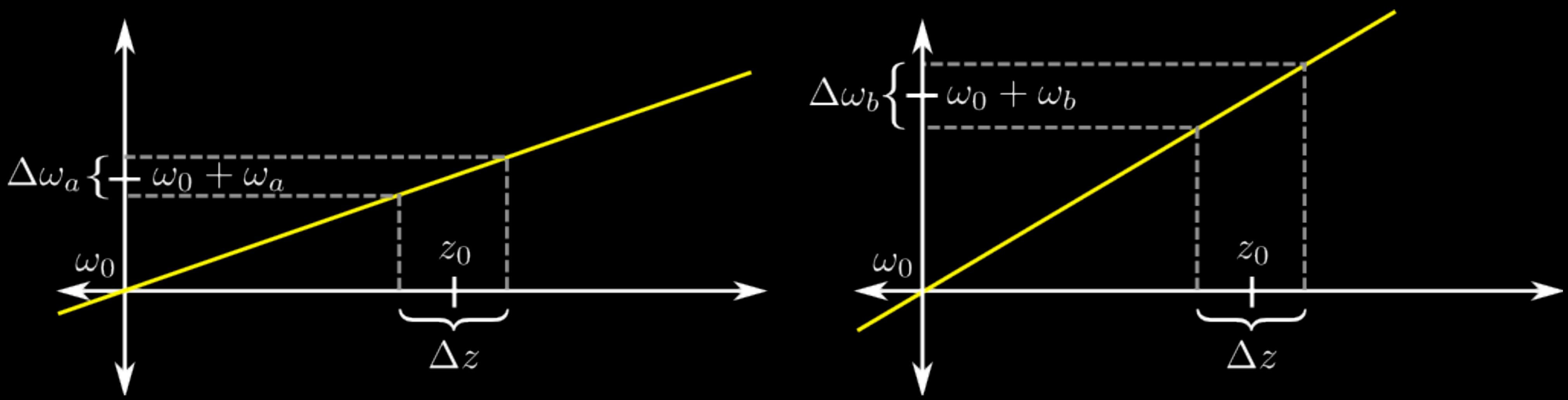
Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \left(\begin{array}{c} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{array} \right) \Rightarrow \vec{B}_{eff} = \left(\begin{array}{c} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} + G_z z \end{array} \right)$$

Slice Selection

how do we "excite" a certain slice?



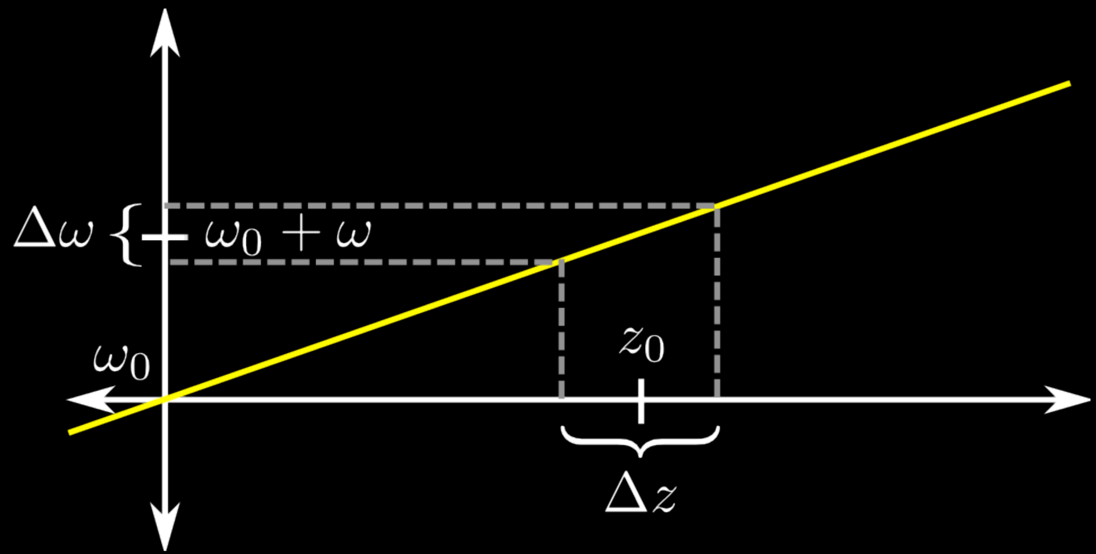
the strength of the gradient affects all parameters for the same spatial location

$$\Delta\omega_a < \Delta\omega_b$$

$$\omega_a < \omega_b$$

Slice Selection

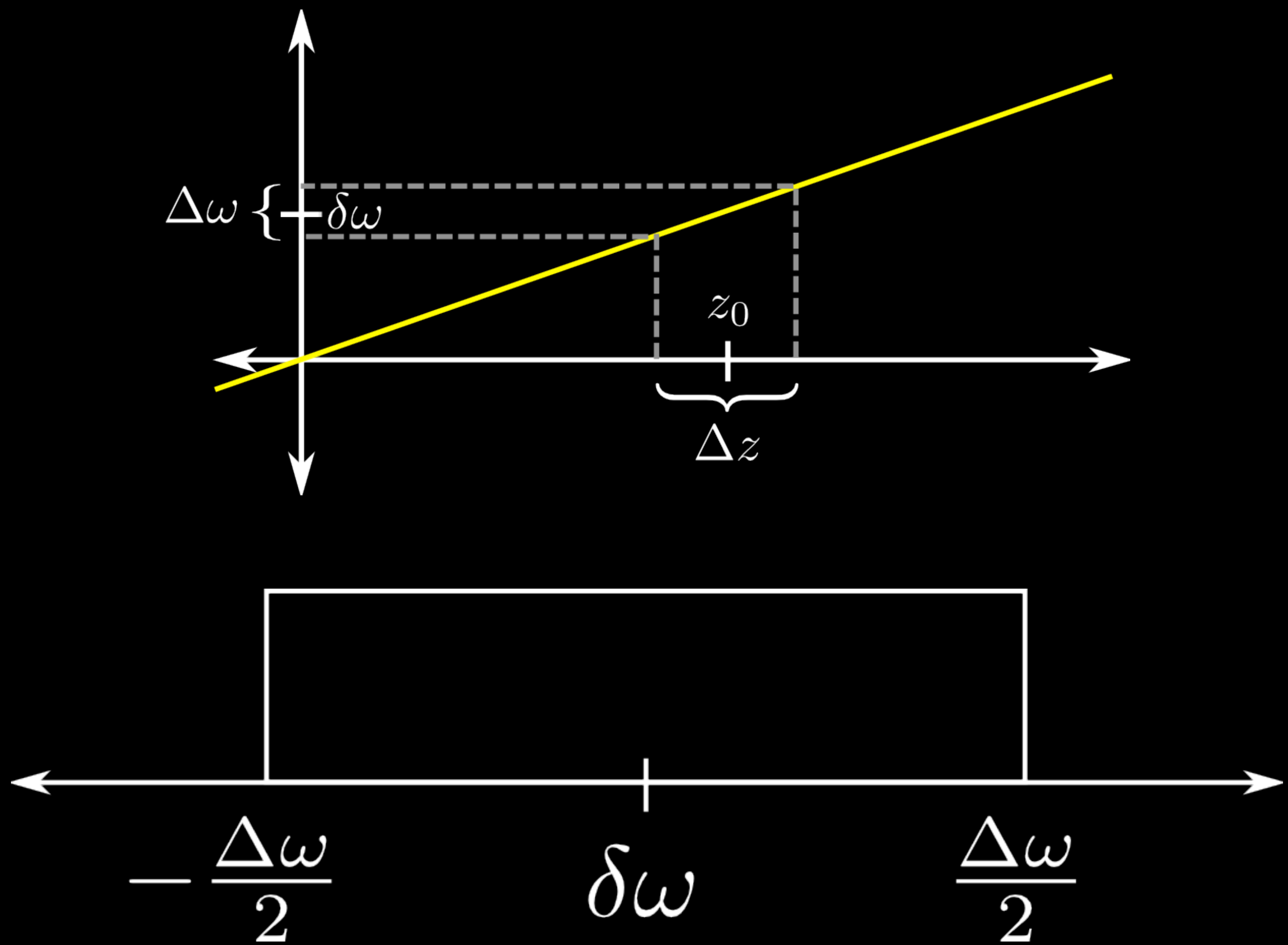
how do we physically set the parameters?



ω - the carrier frequency of the RF pulse

$\Delta\omega$ - frequency bandwidth of the RF pulse

Slice Selection



we want a pulse with as rectangular of an slice profile as possible

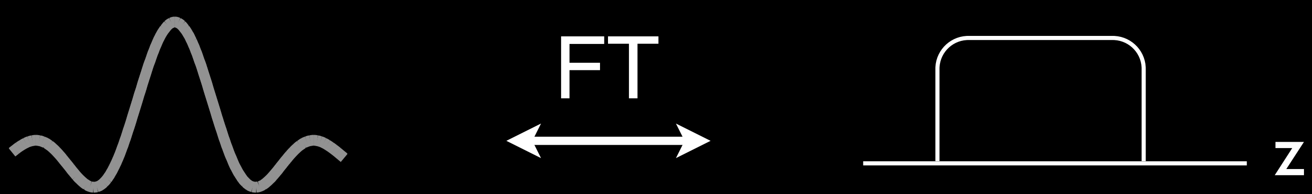
Selective Excitation

changing the shape of the pulse affects
the bandwidth of excitation

how we do know which shape to use?

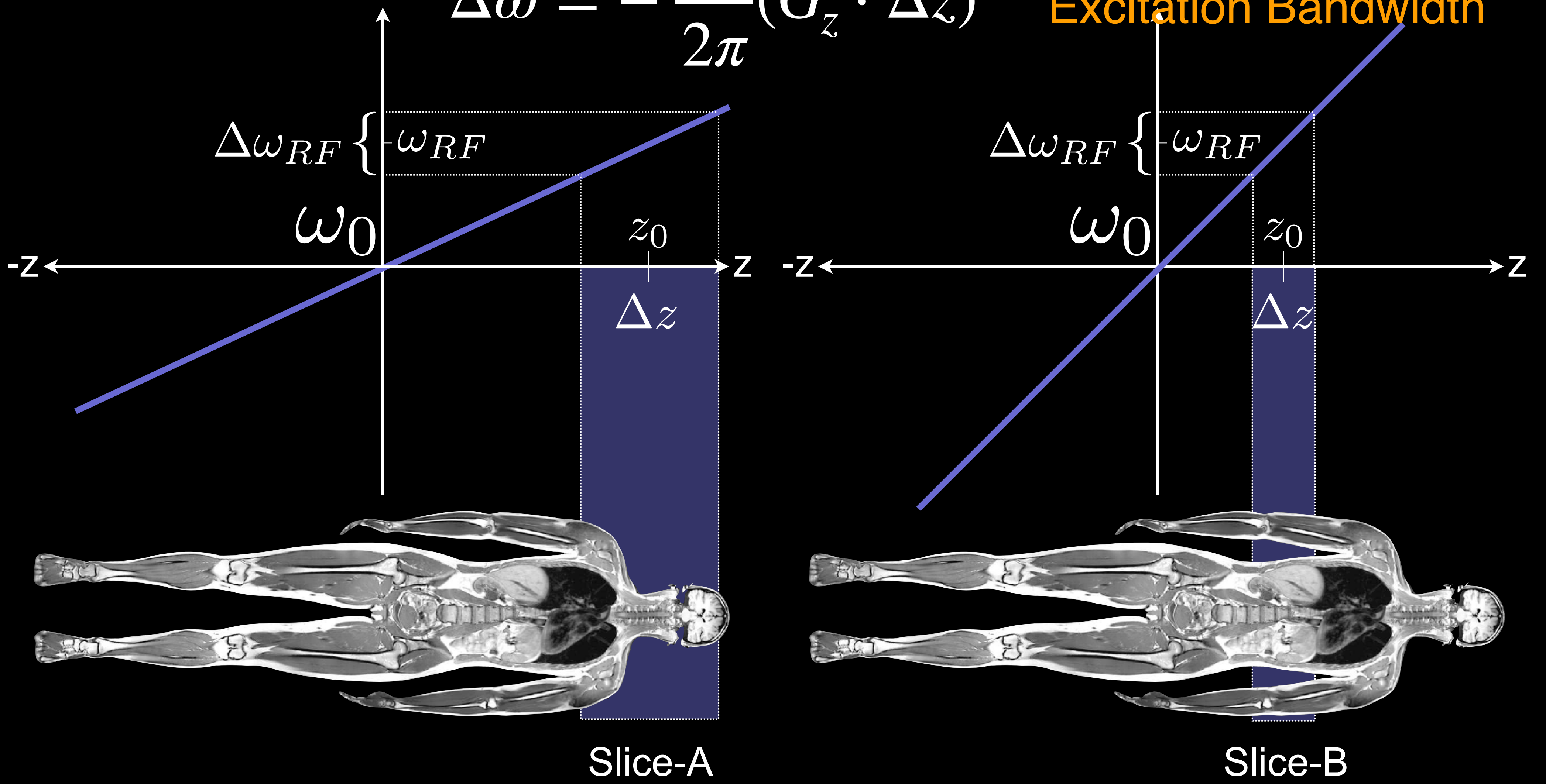
small angle approximation

We will show the slice profile depends on



Slice Selective Excitation

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z)$$



How do you move the slice along ±z?

Compare Δω and ω_{RF} for Slice-A and Slice-B.

Do we usually acquire $\omega_{RF}>\omega_0$?

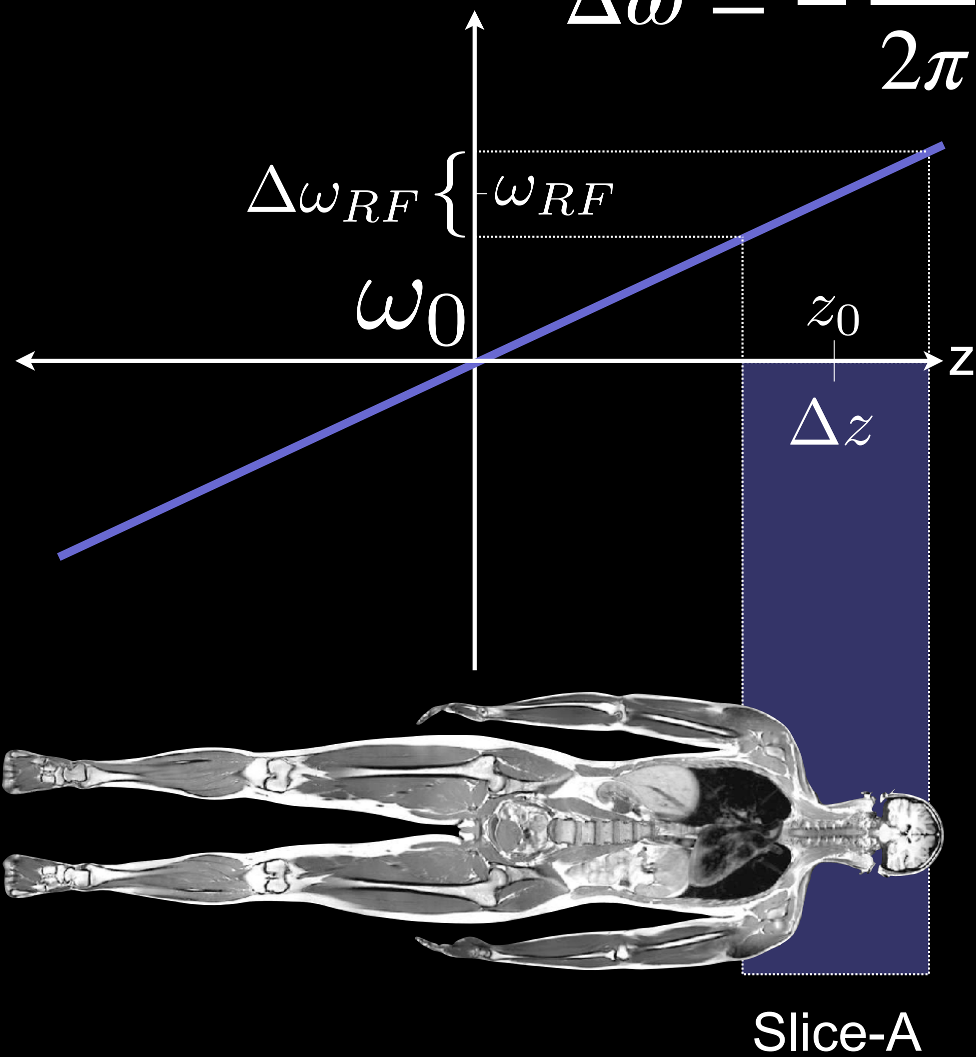
Time Bandwidth Product (TBW)

- **Time bandwidth (TBW) product:**
  - **Pulse Duration [s] x Pulse Bandwidth [Hz]**
  - **Unitless**
  - **# of zero crossings**
  - **High TBW**
    - Large # of zero crossings ∴ fewer truncation artifacts
    - Longer duration pulse
- **Examples:**
  - **TBW = 4, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?
  - **TBW = 16, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?

Slice Selective Excitation - Example

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z)$$

Excitation Bandwidth



$$\mathrm{TBW} = \tau_{RF} \cdot \Delta\omega_{RF}$$

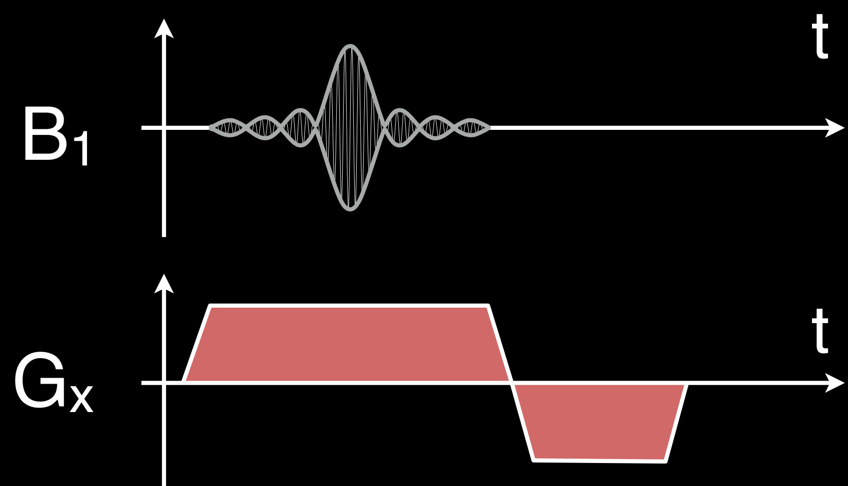
$$\Delta\omega_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}} = \frac{4}{1\mathrm{ms}} = 4\mathrm{kHz}$$

$$G_z = \frac{\Delta\omega_{RF}}{\gamma\Delta z} = \frac{4000\mathrm{Hz}}{42.57\mathrm{e}6\,\frac{\mathrm{Hz}}{\mathrm{T}}\,\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}} = 0.94\,\frac{\mathrm{G}}{\mathrm{cm}}$$

Selective Excitation

- **What factors control slice selection?**

$B_1^e(t)$	**Pulse envelope function** (e.g. $B_{1,max}$ and $\Delta\omega$)
ω_{RF}	**Excitation carrier frequency**
BW	**RF pulse bandwidth**
$\vec{G}$	**Gradient amplitude**



To the Board

RF Pulse Bandwidth and Slice Profile:
Small Tip Angle Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

sin θ ≈ θ
cos θ ≈ 1
$M_z \approx M_0 \rightarrow$ constant

$$\left.\right\} \frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

$$M_{xy}(t, z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z\cdot(t-s)} ds$$

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

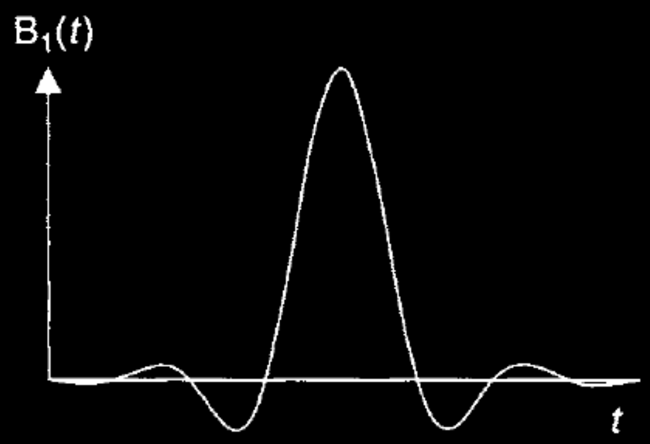
(See the note for complete derivation)

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

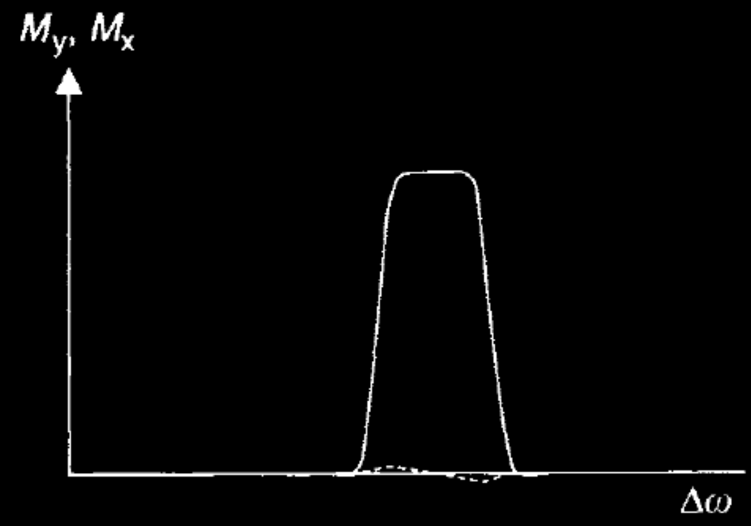
To the Board

Small Tip Approximation

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \mid_{f=-(\gamma/2\pi)G_z z}$$



Fourier transform



- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of B1(t)”
- The approximation works surprisingly well even for flip angles up to 90°

Small Tip Approximation

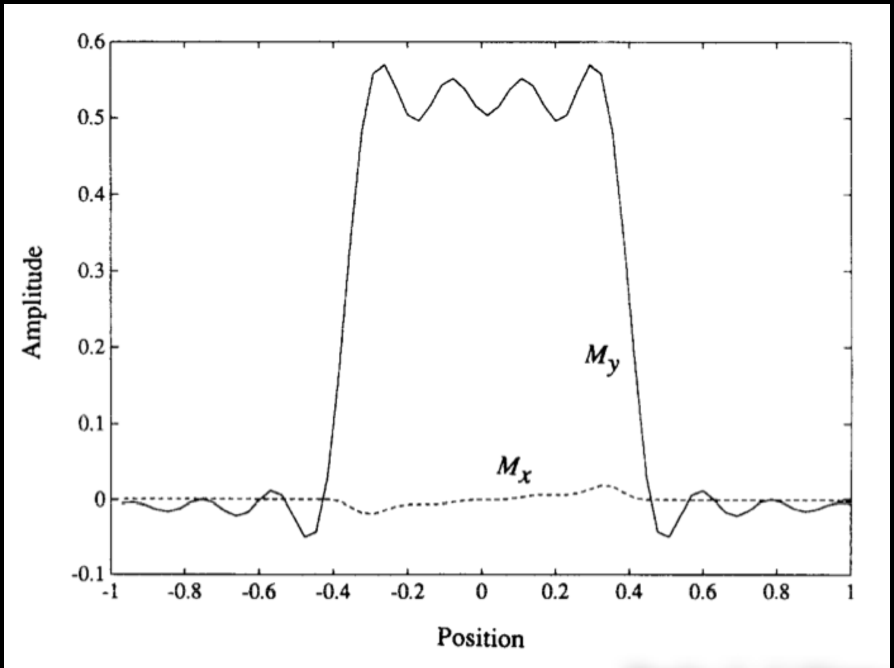
the excitation profile, within the small angle approximation, is just the Fourier transform of the pulse

remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly

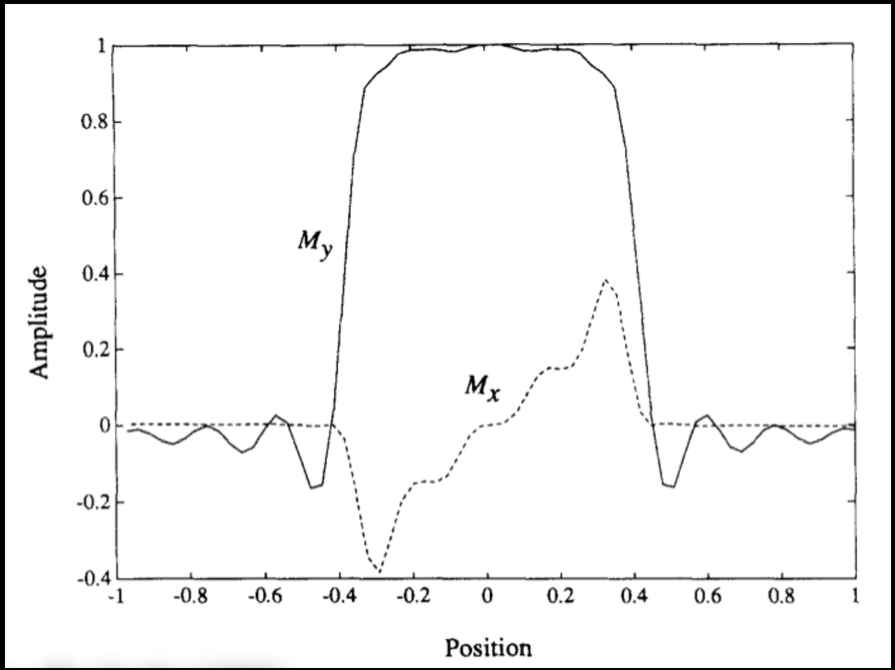
the approximation works surprisingly well even for flip angles up to 90^0

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

small-angle approximation still works reasonably well for flip angles that aren't necessarily "small"

Truncation Artifacts

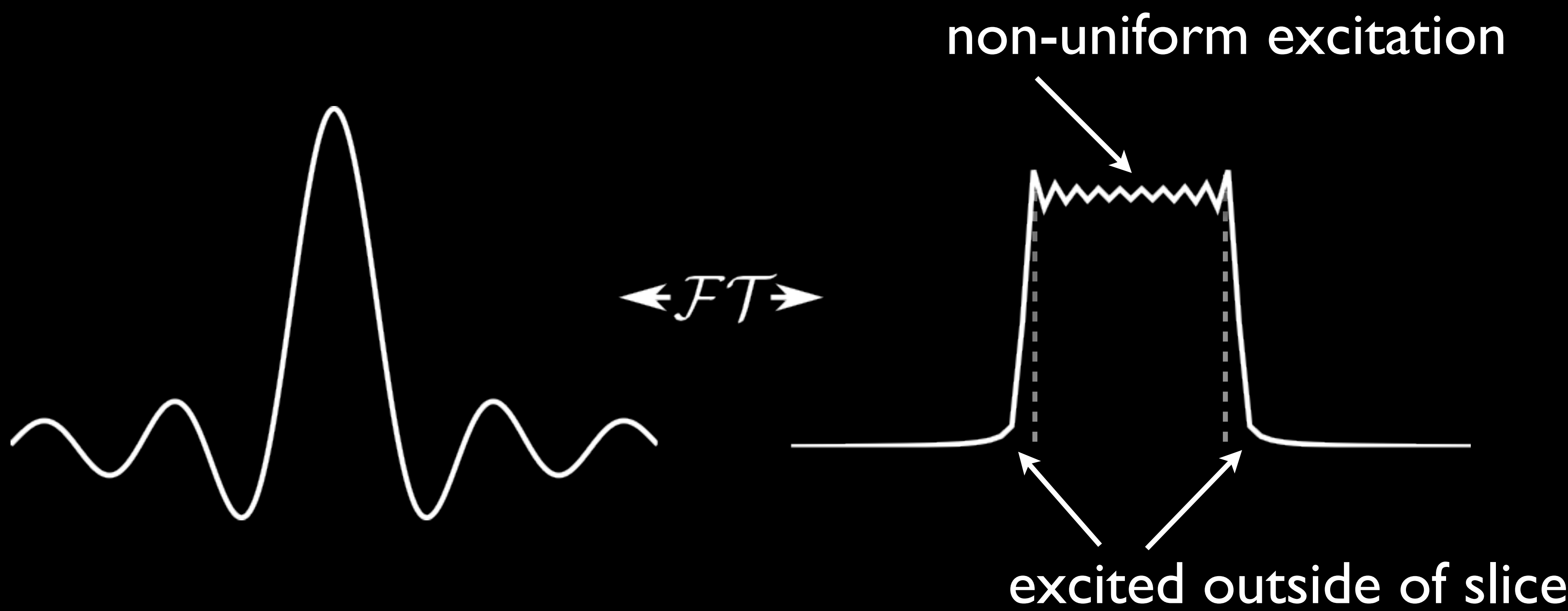
in MRI we want pulses to be as short as possible to avoid relaxation effects

the sinc function is defined over all time which is impractical in any experiment

the sinc pulse needs to be truncated to be appropriate for clinical scans

Truncation Artifacts

what happens when we truncate our pulses?



these deviations from the ideal are known as truncation artifacts

Truncation Artifacts

alternative Pulse Shapes

gaussian

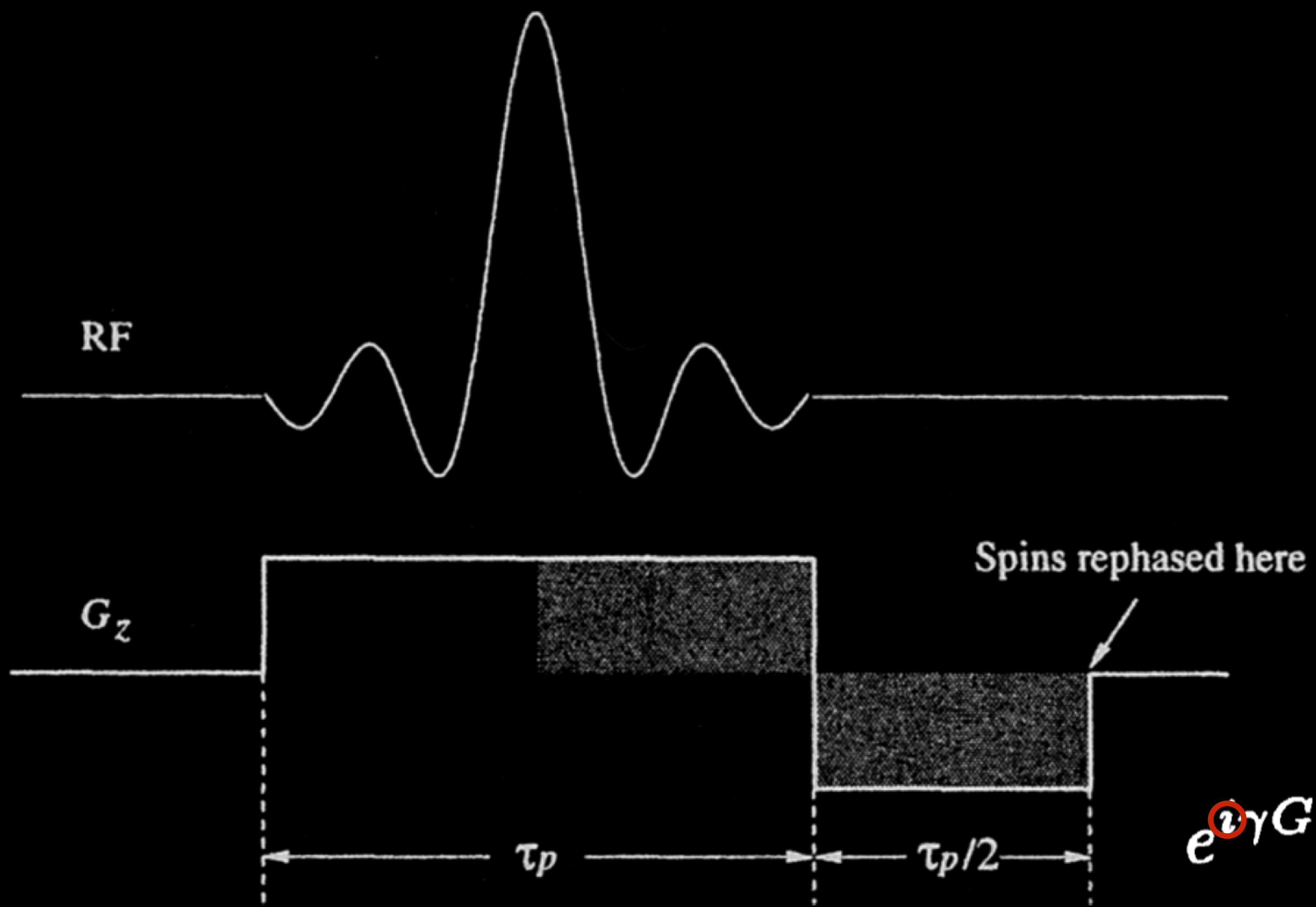
$$B_x(t) = A\exp\left[-a(t-\tau/2)^2\right]$$

reduced side-lobes, but not as flat of a profile

Window Functions

Hamming, Hanning, …

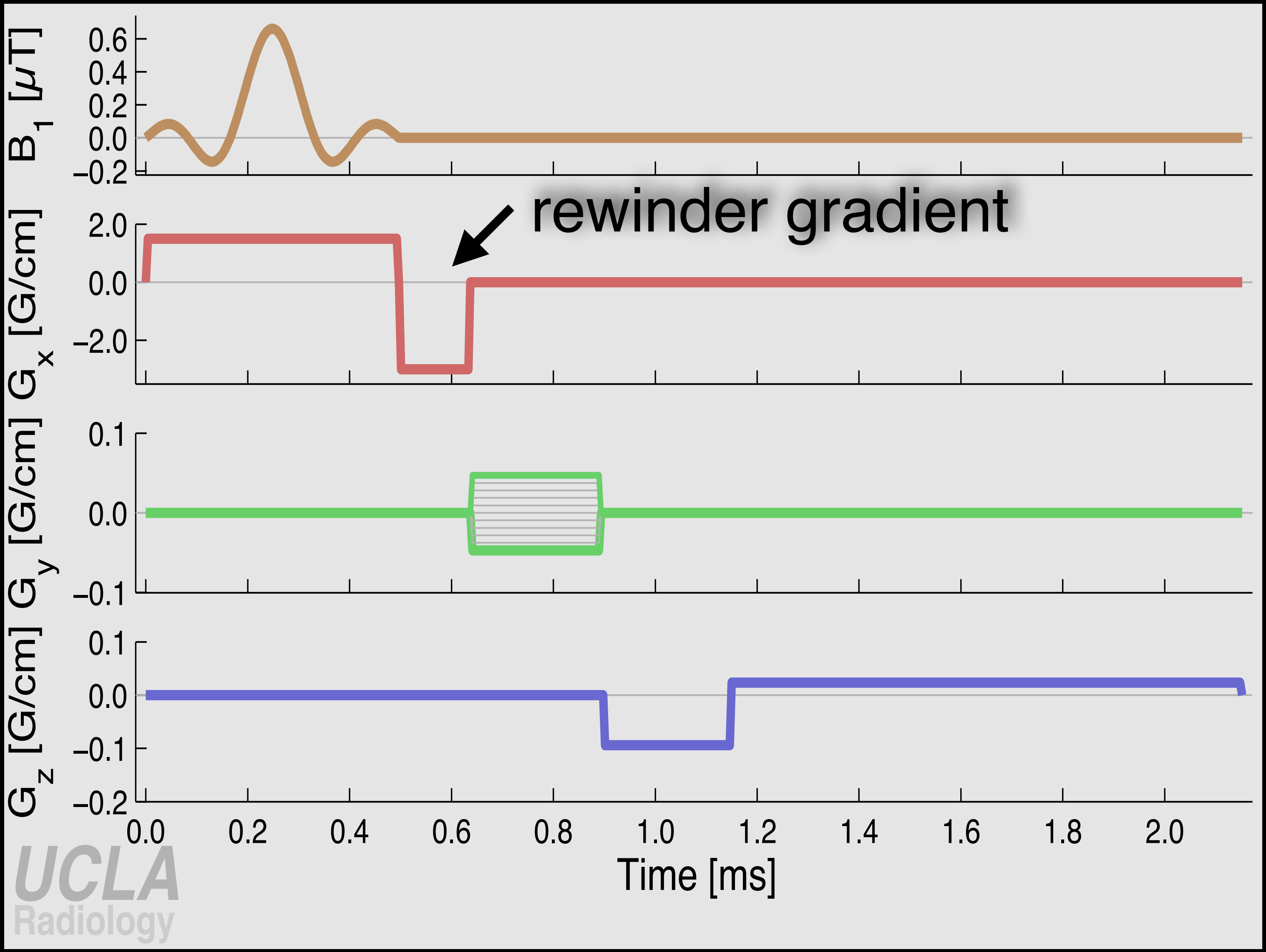
Slice Rewinder



$$e^{i\gamma G_z(z-z_0)\tau_p/2}$$

Opposite Polarity

Slice Selective Excitation Example



slice select gradient rewinder eliminates the linear phase ramp

Selective Excitation: Conclusion

B1 amplitude
-> flip angle

B1 amplitude profile
-> bandwidth, slice profile

B1 carrier frequency
-> slice location

B1 phase profile
-> slice location, etc.

Small Tip Approximation
-> slice profile = FT of B1 envelope function

MATLAB Demo

```
%% Design of Windowed Sinc RF Pulses
tbw = 4;
samples = 512;
rf = wsinc(tbw, samples);
```

```
%% Plot RF Amplitude
flip_angle = pi/2;
rf = flip_angle*rf;

pulseduration = 1;        % in msec
dt  = pulseduration/samples;
rfs = rf/(gamma*dt);      % Scaled to Gauss

bw = tbw/pulseduration; % in kHz
gmax = bw/gamma_2pi;

b1    = [rfs zeros(1,samples/2)];                 % in Gauss
g     = [ones(1,samples) -ones(1,samples/2)]*gmax;  % in G/cm
t_all = (1:length(g))*dt;  % in msec
```

MATLAB Demo

```
%% Simulate Slice Profile using Bloch Simulation
x  = (-2:.01:2);                % in cm
f  = 0;                         % in Hz
dt = pulseduration/samples/1e3;
t  = (1:length(b1))*dt;  % in usec

% Bloch Simulation
[mx,my,mz] = bloch(b1(:),g(:),t(:),1,.2,f(:),x(:),0);

% Transverse Magnetization
mxy_bloch = mx+1i*my;
```

```
%% Simulate Slice Profile using Small Tip Approximation
samples_st = 4096;
f_st = linspace(-0.5/dt,0.5/dt,samples_st)/1e3;
x_st = -f_st/(gamma_2pi*gmax);

rfs_zp = zeros(1,samples_st);
rfs_zp(1:samples) = rfs;

mxy_st = fftshift(fftn(fftshift(rfs_zp)))/30;
```

Questions?

- Related reading materials
  - Nishimura - Chap 6.1, 6.2, 6.4

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