RF Pulse Design

RF Pulses / Adiabatic Pulses

M229 Advanced Topics in MRI
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Class Business

• Homework 1 is due on 4/23 (Friday)
Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion
Review of RF Pulses
Notation and Conventions

\[ \vec{B} = B_0 \hat{k} + B_1(t)[\cos \omega t \hat{i} - \sin \omega t \hat{j}] \]

- \(\omega\) = carrier frequency
- \(\omega_0\) = resonant frequency
- \(B_1(t)\) = complex valued envelop function
RF Pulse - Excitation

\[ \vec{B} = B_0 \hat{k} + B_1(t)[\cos \omega t \hat{i} - \sin \omega t \hat{j}] \]

\[ B_1(t) \cdot [\cos(\omega t) \hat{i} - \sin(\omega t) \hat{j}] \]

carrier frequency \( \omega \)

envelope \( B_1(t) \)
Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.
Rotating Frame Definitions

\[ \vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \quad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \quad B_{z'} \equiv B_z \quad M_{z'} \equiv M_z \]

\[ \vec{M}_{lab}(t) = R_Z(w_0 t) \cdot \vec{M}_{rot}(t) \]

\[ \vec{B}_{lab}(t) = R_Z(w_0 t) \cdot \vec{B}_{rot}(t) \]

\[ \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff} \]
Bloch Equation (Rotating Frame)

\[ \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff} \]

where

\[ \vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma} \]

\[ \vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \]
Bloch Equation (Rotating Frame)

\[
\vec{B}_{\text{eff}} = \vec{B}_{\text{rot}} + \frac{\vec{w}_{\text{rot}}}{\gamma}
\]

\[
\vec{B}_{\text{lab}} = \begin{pmatrix}
B_1(t) \cos \omega_0 t \\
B_1(t) \sin \omega_0 t \\
B_0
\end{pmatrix} \quad \vec{B}_{\text{rot}} = \begin{pmatrix}
B_1(t) \\
B_1(t) \\
B_0
\end{pmatrix}
\]

Assume real-valued \( B_1(t) \)

\[
\vec{B}_{\text{rot}} = \begin{pmatrix}
B_1(t) \\
0 \\
B_0
\end{pmatrix} \quad \vec{B}_{\text{eff}} = \begin{pmatrix}
B_1(t) \\
0 \\
B_0 \frac{\omega}{\gamma}
\end{pmatrix}
\]
To the board ...
Bloch Equation with Gradient

\[
\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}
\]

\[
\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \quad \rightarrow \quad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}
\]
Bloch Equation (at on-resonance)

\[ \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff} \]

where \( \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 + \frac{\omega}{\gamma} + G_z z \end{pmatrix} \)

\[ \frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M} \]

\[ \omega(z) = \gamma G_z z \quad \omega_1(t) = \gamma B_1(t) \]
To the board ...
B1 Variations

- In MRI, B1 field is not always uniform across the imaging volume
- B1 inhomogeneity can cause:
  - Image shading
  - Incomplete saturation (e.g. in fat suppression)
  - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
  - Inaccurate/imprecise quantification in T1 mapping
B1 Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

→ Adiabatic Pulses!

“Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform B1 field.”
Adiabatic Passage Principle
Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B1 field
  \[ \theta \neq \int_{0}^{T} B_1(\tau) d\tau \]
- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing
## Adiabatic vs. Non-Adiabatic Pulses

### Adiabatic Pulses:

\[ \theta \neq \int_{0}^{T} B_1(\tau) d\tau \]

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher B1 amplitude (>12 µT)
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

### Non-Adiabatic Pulses:

\[ \theta = \int_{0}^{T} B_1(\tau) d\tau \]

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower B1 amplitude
- Generally multi-purpose
Adiabatic Pulses

- Frequency modulated pulses:
  \[ B_1(t) = A(t) \exp\left(-i \int \omega_1(t') \, dt' \right) \]

- Or phase modulation:
  \[ B_1(t) = A(t) \exp(-i \phi(t)) \]
Bloch Equation (at on-resonance)

\[ B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'} \]

\[ \frac{d \vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff} \]

where \( \vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ -B_0 \omega + \frac{\omega_1(t)}{\gamma} \end{pmatrix} \)

\[ \frac{d \vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M} \]
Magnetization Plot

To the board ...
Adiabatic Excitation

- At the end of the pulse, all the magnetization is in the transverse plane \( \Rightarrow \) so we have adiabatic excitation!
- This is also called an adiabatic half passage (AHP)
- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:
Adiabatic Inversion
Adiabatic Inversion
Design of Adiabatic Inversion

- General equation for an adiabatic pulse:
  \[ B_1(t) = A(t) \exp(-i \int \omega_1(t') \, dt') \]

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions

- The most famous one is…

  The Hyperbolic Secant Inversion Pulse!
Hyperbolic Secant Pulse Equations

\[ B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'} \]

where

\[ A(t) = A_0 \text{sech}(\beta t) \]
\[ \omega_1(t) = -\mu \beta \text{tanh}(\beta t) \]

\( A_0 \): peak amplitude (\( \mu T \))
\( \beta \): frequency modulation parameter (rad/s)
\( \mu \): phase modulation parameter (dimensionless)

_Handbook of MRI Pulse Sequences, Ch 6.2, pp 194-195_
Hyperbolic Secant Pulse Example

Pulse Parameters:
$A_0 = 12 \, \mu T$
$\mu = 5$
$\beta = 672 \, \text{rad/s}$
Duration = 10.24 ms
Comparing Hyperbolic Secant with an AFP Example

Amplitude Modulation, $A(t)$

Frequency Modulation, $\omega_1(t)$

Hyperbolic Secant Pulse

General Adiabatic Full Passage pulse
### Some Examples of Other Adiabatic Inversion Pulses

<table>
<thead>
<tr>
<th>Pulse Name</th>
<th>$A(t)$</th>
<th>$\omega_1(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lorentz</td>
<td>$\frac{1}{1 + \beta \tau^2}$</td>
<td>$\frac{\tau}{1 + \beta \tau^2} + \frac{1}{\sqrt{\beta}} \tan^{-1}(\sqrt{\beta} \tau)$</td>
</tr>
<tr>
<td>HS</td>
<td>$\text{sech}(\beta \tau)$</td>
<td>$\frac{\tanh(\beta \tau)}{\tanh(\beta)}$</td>
</tr>
<tr>
<td>Gauss(^c)</td>
<td>$\exp \left( -\frac{\beta^2 \tau^2}{2} \right)$</td>
<td>$\frac{\text{erf}(\beta \tau)}{\text{erf}(\beta)}$</td>
</tr>
<tr>
<td>Hanning</td>
<td>$\frac{1 + \cos(\pi \tau)}{2}$</td>
<td>$\tau + \frac{4}{3 \pi} \sin(\pi \tau) \left[ 1 + \frac{1}{4} \cos(\pi \tau) \right]$</td>
</tr>
<tr>
<td>HSn(^o) $(n=8)$</td>
<td>$\text{sech}(\beta \tau^n)$</td>
<td>$\int \text{sech}^2(\beta \tau^n) , d\tau$</td>
</tr>
<tr>
<td>Sin40(^d) $(n=40)$</td>
<td>$1 - \left</td>
<td>\sin^n \left( \frac{\pi \tau}{2} \right) \right</td>
</tr>
</tbody>
</table>

*Tannus et al., “Adiabatic Pulses”, NMR in Biomedicine, vol. 10, p423*
Some Examples of Other Adiabatic Inversion Pulses

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The shape of the inversion profile depends on the choice \(A(t)\) and \(\omega_1(t)\)!

HSn° (n=8) sech(\(\beta \tau^n\)) \[ \int \text{sech}^2(\beta \tau^n) \, d\tau \]

Sin40° (n=40) \(1 - \left| \sin^n \left( \frac{\pi \tau}{2} \right) \right| \tau - \int \sin^n \left( \frac{\pi \tau}{2} \right) \left( 1 + \cos^2 \left( \frac{\pi \tau}{2} \right) \right) \, d\tau \)

What Will Inversion Profile Look Like?

Hyperbolic Secant Pulse

Amplitude Modulation, $A(t)$

Frequency Modulation, $\omega_1(t)$

Inversion Profile
Inversion Profiles

- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth

- The inversion efficiency depends strongly on the B1 amplitude (as well as pulse duration, T1, T2 and pulse shape)

- For the hyperbolic secant pulse,

\[
\text{RF spectral bandwidth} = \mu \beta \\
B_{1\text{max}} \gg (\beta \sqrt{\mu})/\gamma \quad \text{(B}_1\text{ threshold for adiabaticity)}
\]
Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
\( B_{1_{\text{max}}} = 12 \, \mu T \)

75% Attenuated Pulse
\( B_{1_{\text{max}}} = 9 \, \mu T \)
Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
$B_{1\text{max}} = 12 \, \mu T$

60% Attenuated Pulse
$B_{1\text{max}} = 7.2 \, \mu T$

$B_1$ Threshold $\approx 6 \, \mu T$
Original Pulse (100%)
$B_1 = 12 \ \mu T$

125% Amplified Pulse
$B_1 = 15 \ \mu T$

150% Amplified Pulse
$B_1 = 18 \ \mu T$
Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic Sech needs to be truncated, which can affect the overall performance
Thank You!

- Further reading
  - Read "Adiabatic Refocusing Pulses" p.200-212

- Acknowledgments
  - John Pauly’s EE469b (RF Pulse Design for MRI)

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