Fast Imaging Trajectories: Non-Cartesian Sampling (2)

M229 Advanced Topics in MRI

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Class Business

- Homework 1 solutions

- Homework 2 due 5/5 Fri

- Final project
  - Proposal due 5/8 Mon
    can send us a draft to get feedback
  - Abstract due 6/8 Thu
  - Presentations on 6/13 Tue

- Office hours

Outline

- Spiral Trajectory

- Non-Cartesian 3D Trajectories
  - 3D stack of radial
  - 3D radial
  - 3D cones

- Non-Cartesian Image Reconstruction
  - Gridding reconstruction
  - Gradient measurement
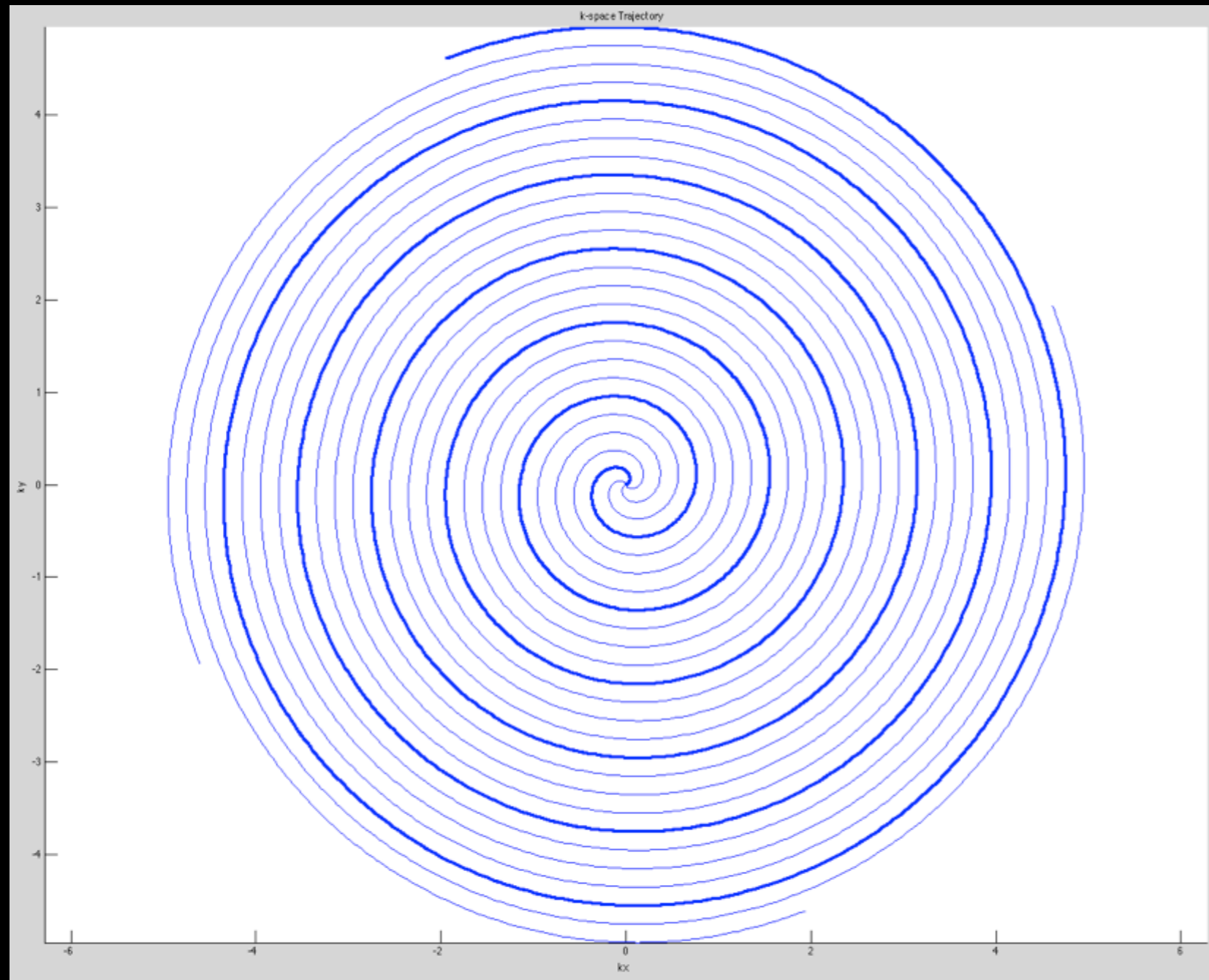 - Off-resonance correction

Spirals
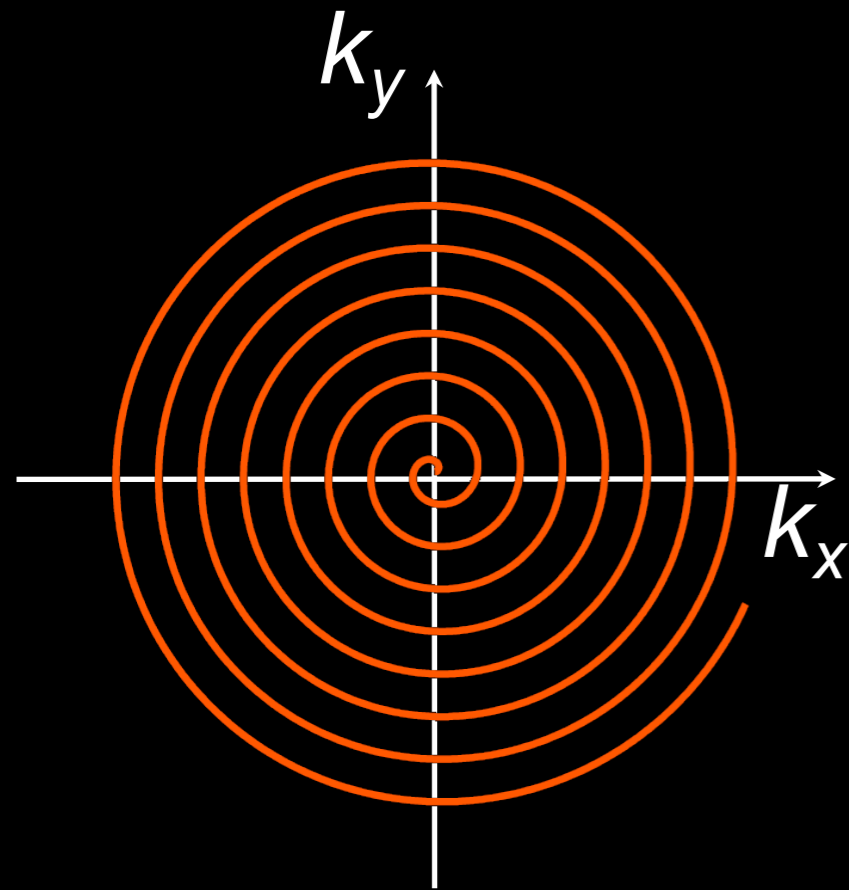
"THE" non-Cartesian trajectory

Highly robust to motion/flow effects

Very fast!

   - optimal use of gradients in 2D

   - can acquire one image in ~100 ms

Spirals: Sampling Requirements
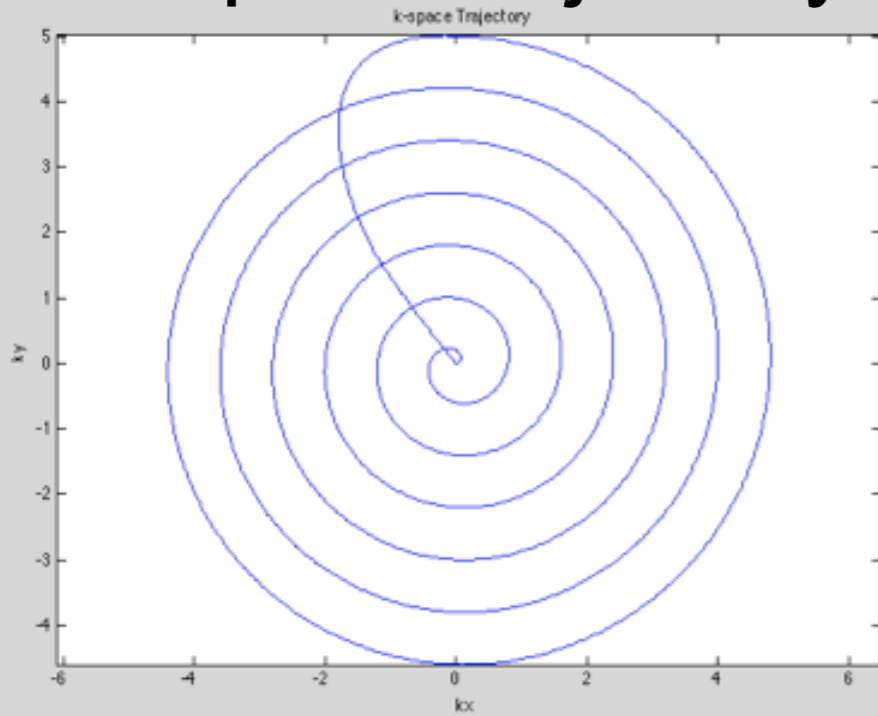

N interleaves

$2 \, k_{r,max} = 1 \, / \, \mathrm{d}x$

$\mathrm{d}k = 1 \, / \, \mathrm{FOV}$

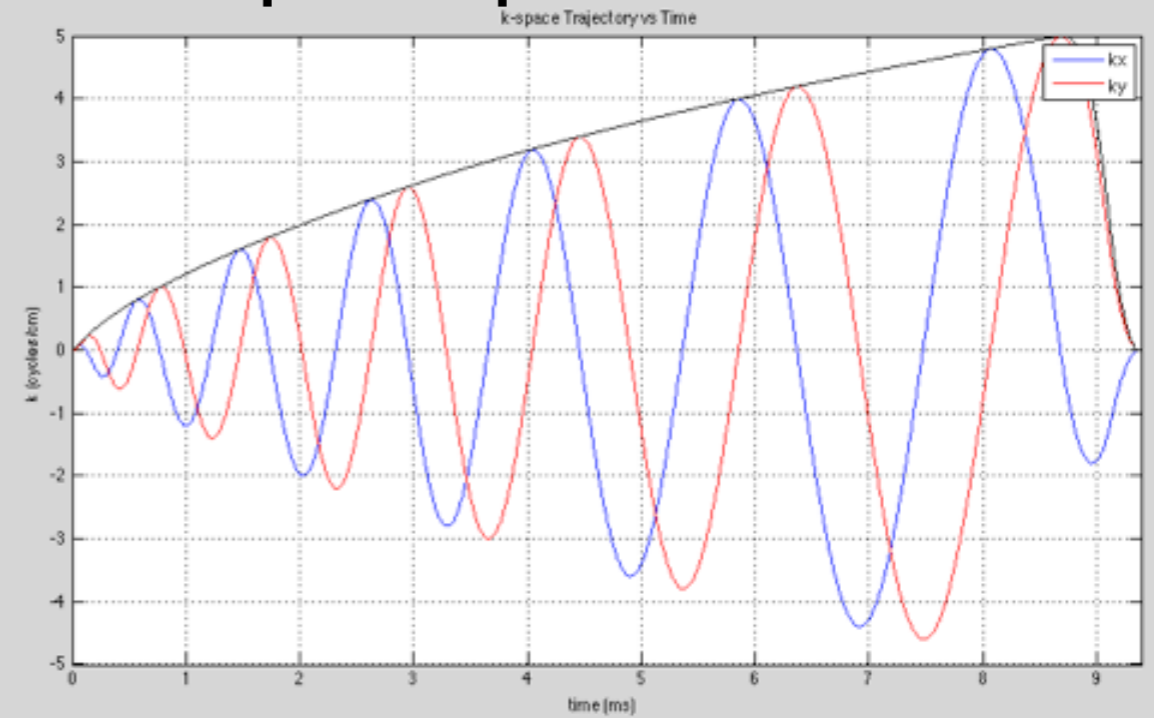Design 1 interleaf
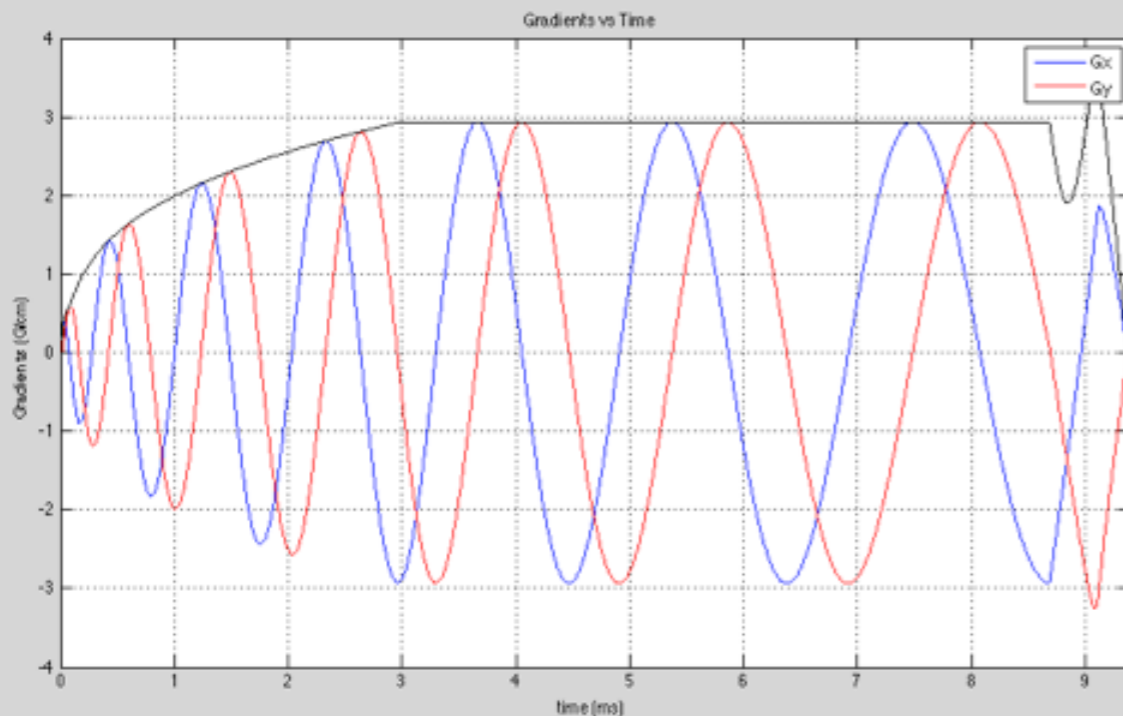
and rotate

Subject to HW limits

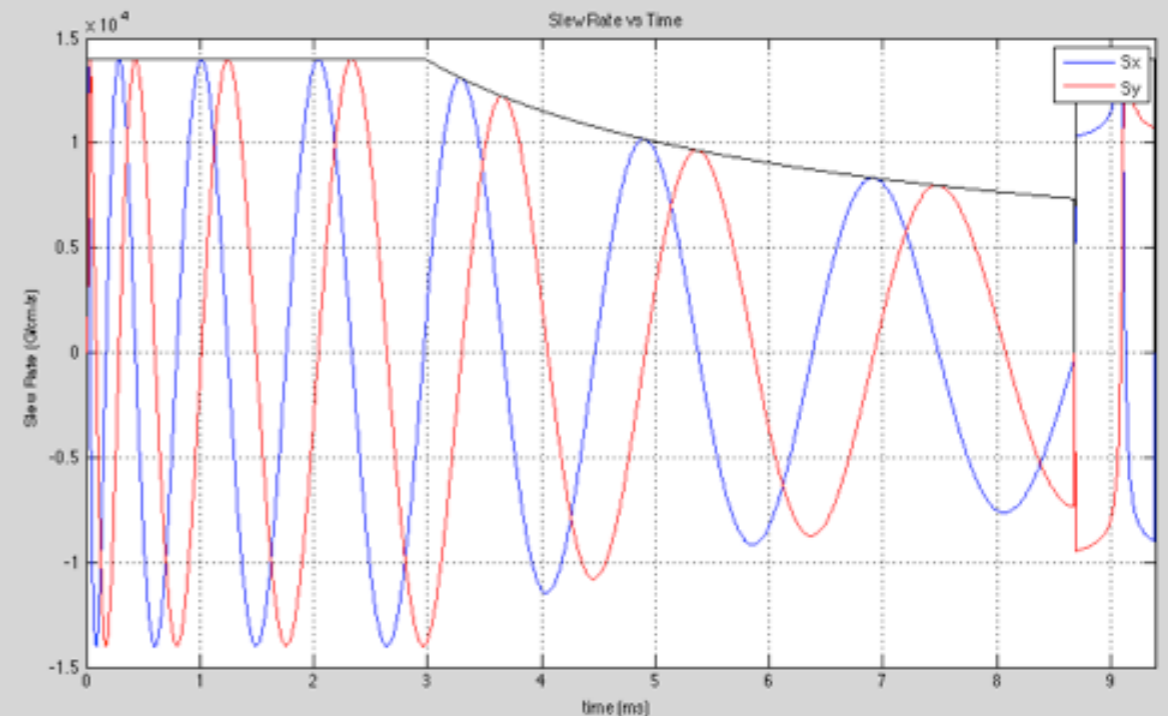# Spirals: Gradient Design
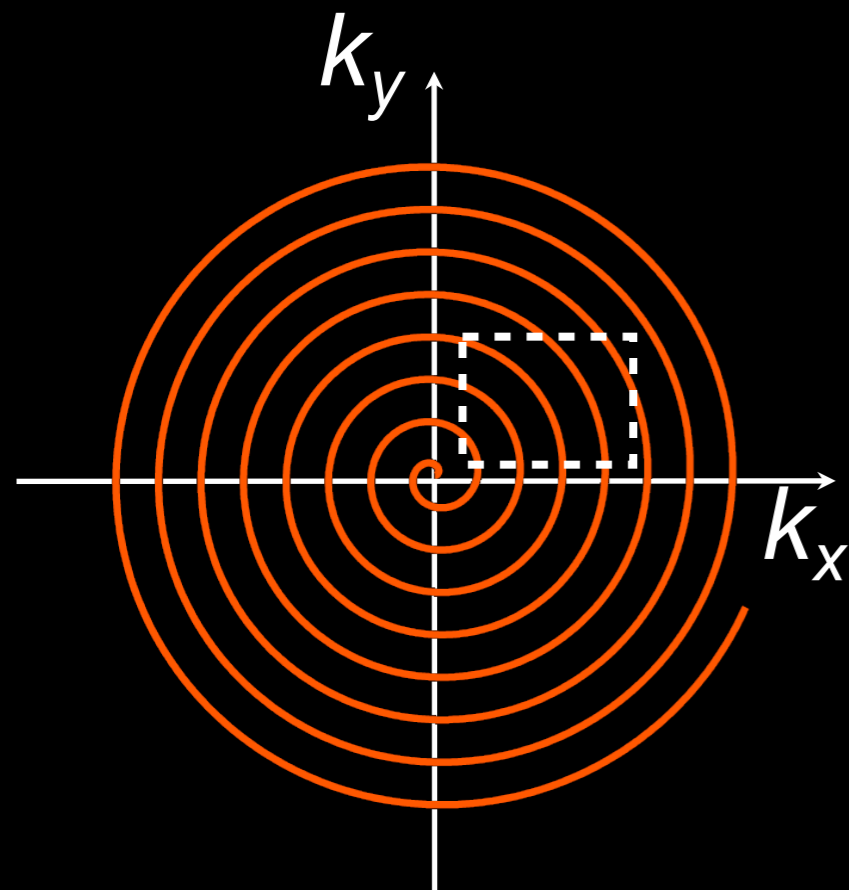
k-space trajectory

k-space pos vs. time
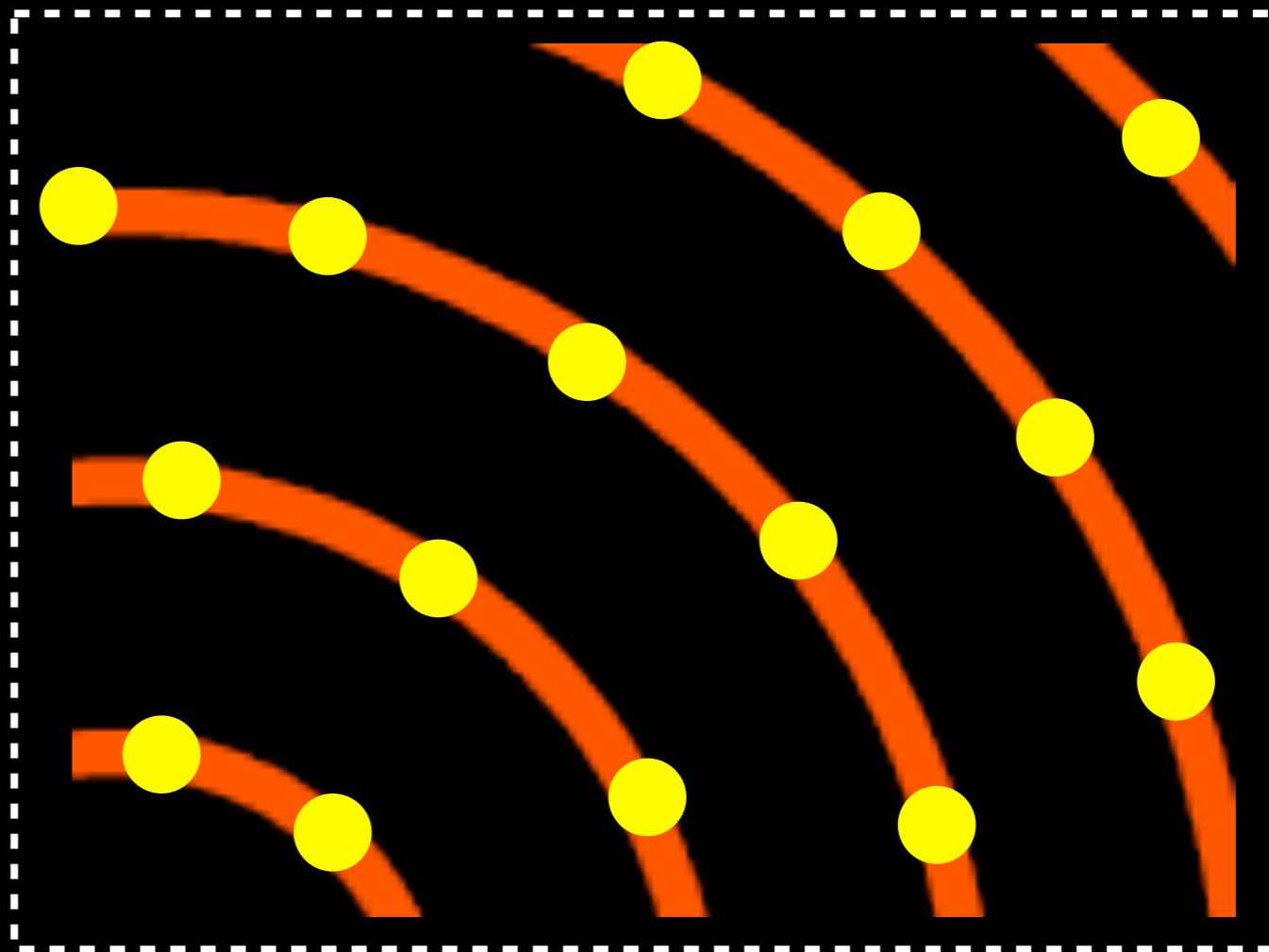

Gradients vs. time

Slew rate vs. time

Spirals: Image Reconstruction
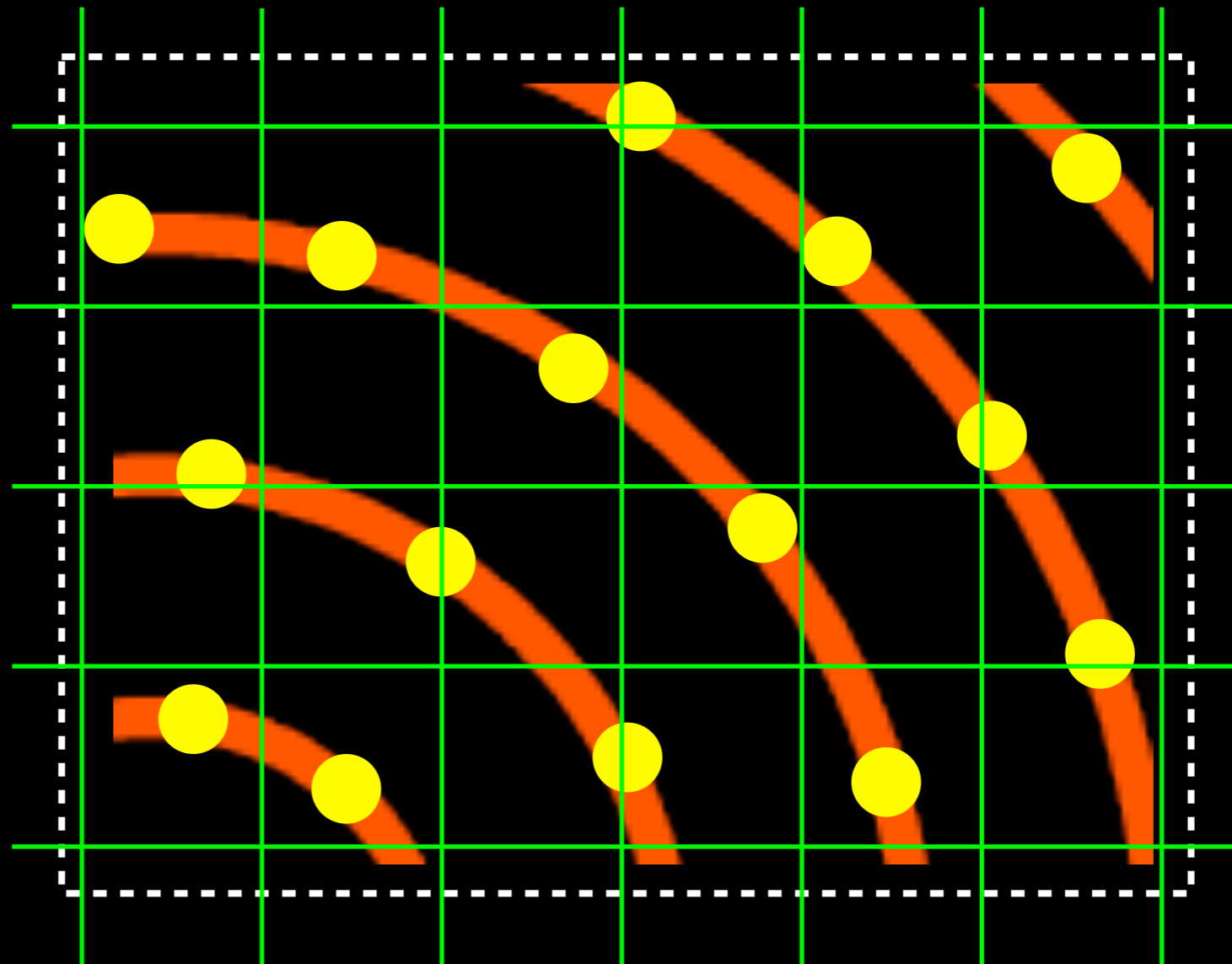
Spirals: Image Reconstruction

k_y

k_x

Gridding Algorithm

Spirals: Image Reconstruction
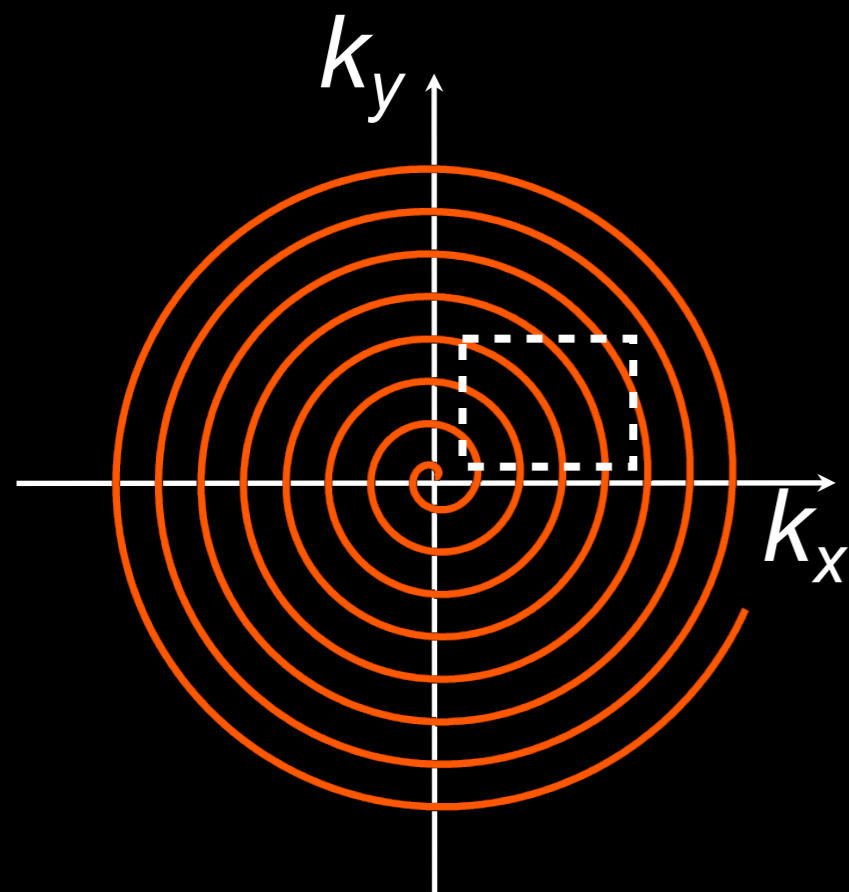
Gridding Algorithm
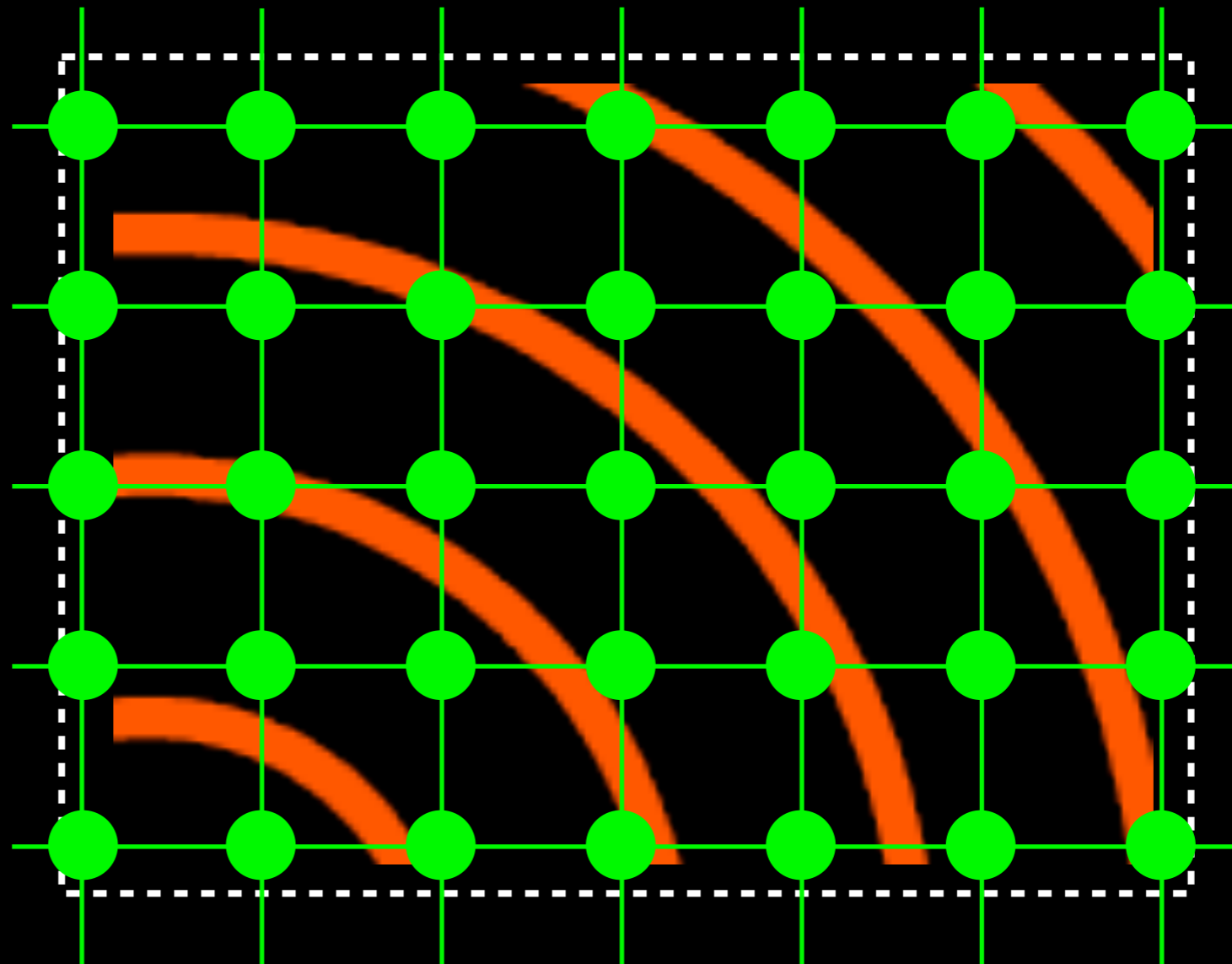
Follow with 2D Fourier Transform ...

Spirals: Gradient Delays

2 sample delay   1 sample delay   calibrated

Spirals: Off-Resonance Effects

$N_{intlv} = 8$

$T_{rd} = 26.67$ ms

$N_{intlv} = 16$

$T_{rd} = 13.41$ ms

$N_{intlv} = 48$

$T_{rd} = 4.61$ ms

Spirals: Practical Considerations
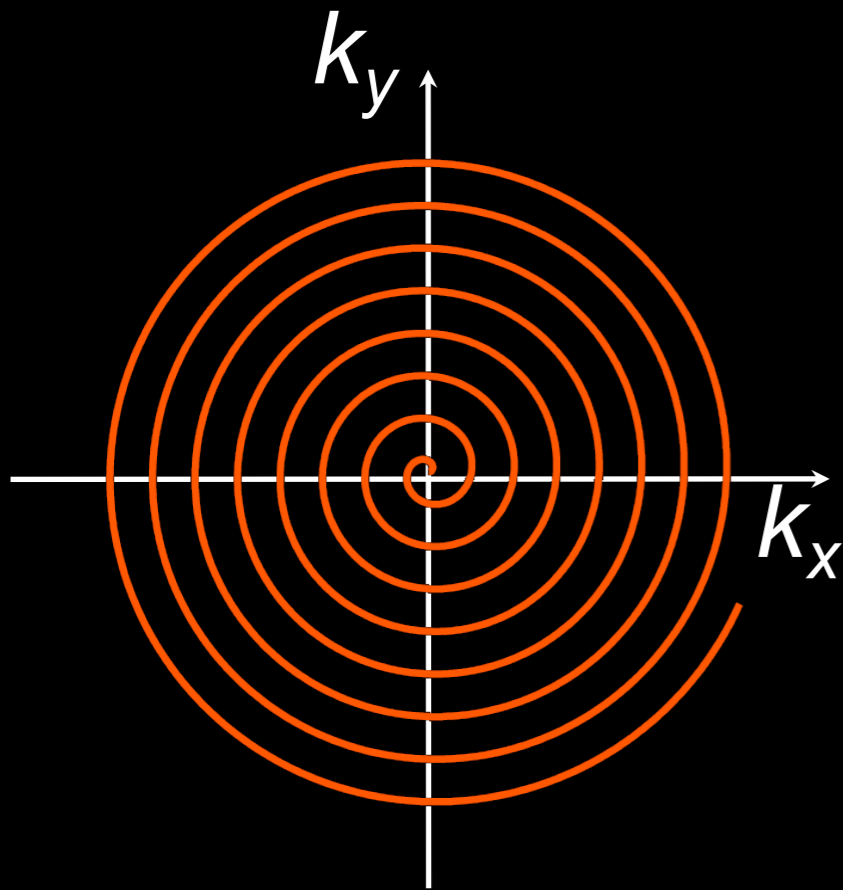
Trajectory design

Gradient waveform calibration

k-Space density compensation

Off-resonance correction

Fat suppression

Gridding reconstruction

applies to non-Cartesian MRI in general

Spirals: Pros and Cons

k_y

k_x
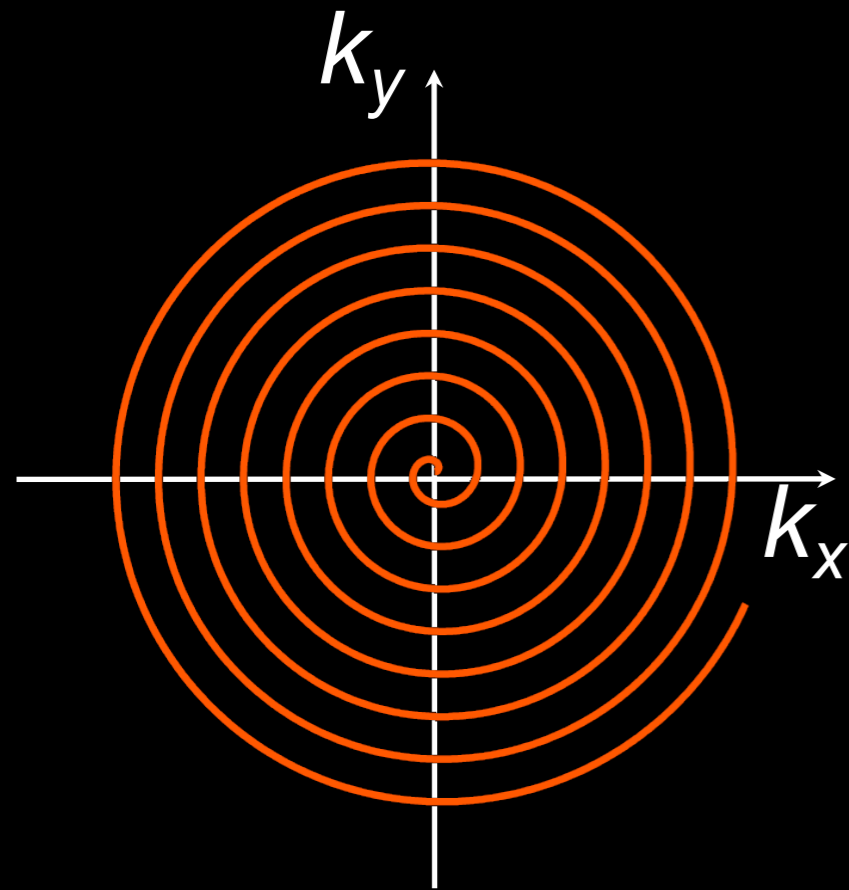
Pros

- Very fast (up to single shot)

- Very short TE

- Robust to motion/flow effects

Cons

- May have mixed contrast

- Sensitive to gradient delays

- Sensitive to off-resonance effects

Spirals: Real-Time Cardiac MRI

- Healthy subject; 1.5 T; 8-ch array
- Golden-angle ordering
- Spiral 2D GRE; 8-mm slice
- Spatial resolution = 1.6 mm
- SPIRiT recon with R = 2
- 40 cm, 1.6 mm
- 250x250 matrix @ 6 fps
- 12-fold reduction in #TRs (vs. 2DFT)
- 8-TR sliding window display (16 fps)

Wu HH et al., ISMRM 2013, p3828
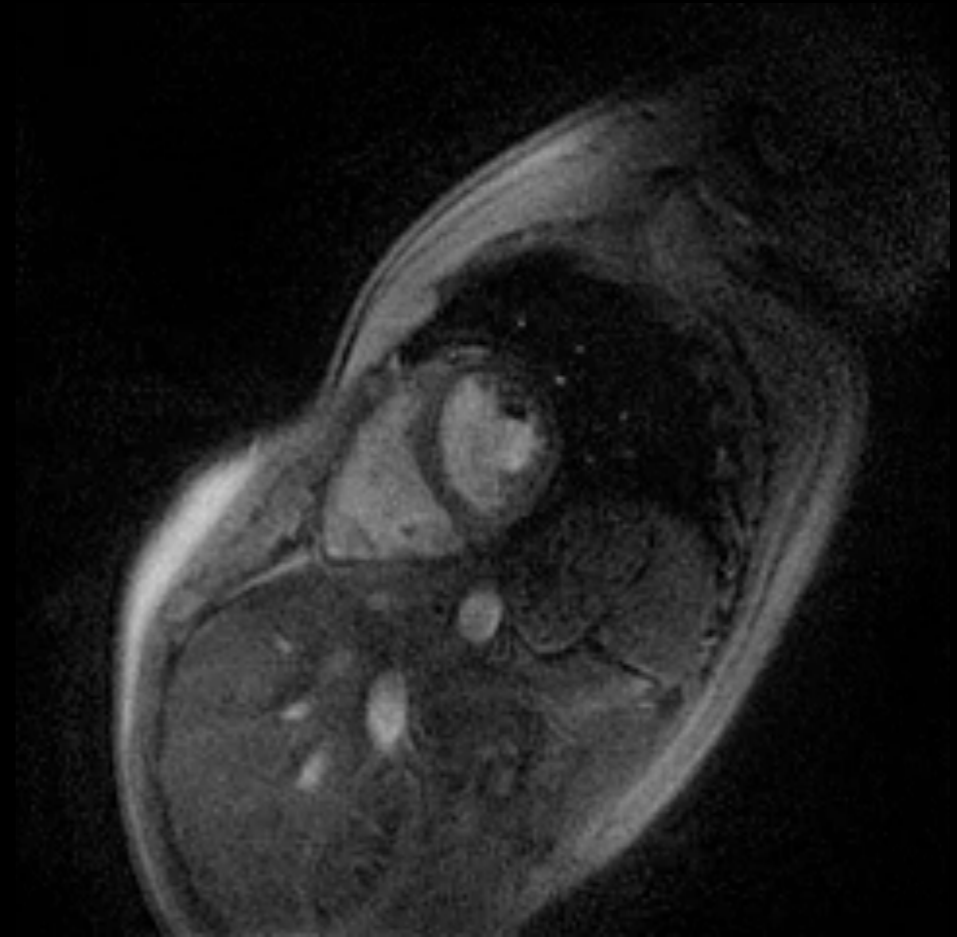
Spirals: 3D LGE MRI

3D Spiral IR-GRE
- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

Reconstruction
- SPIRiT (R = 2)
- ~5-sec recon

1.5 T

courtesy of Joelle Barral & Juan Santos (HeartVista)
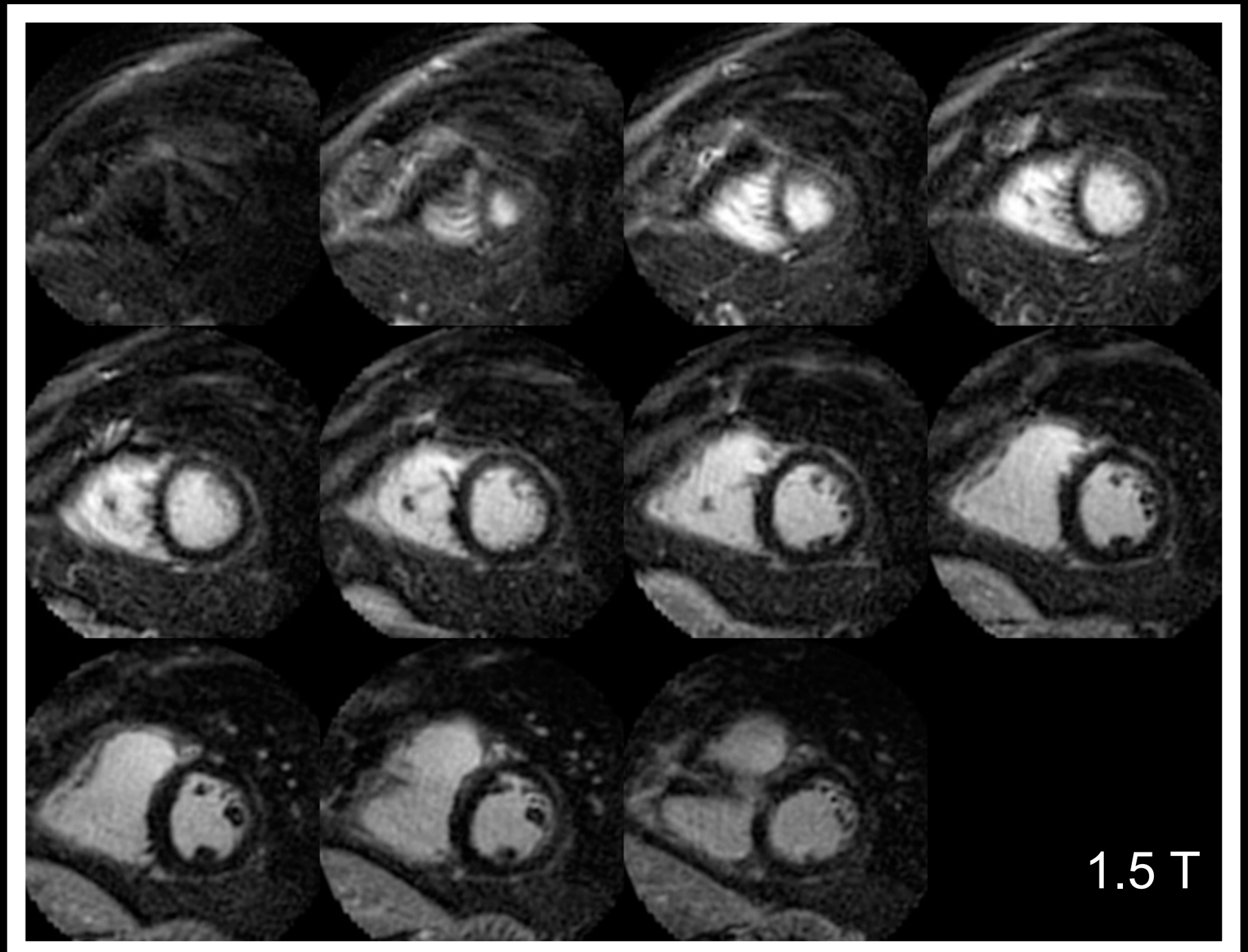
3D Non-Cartesian Sampling
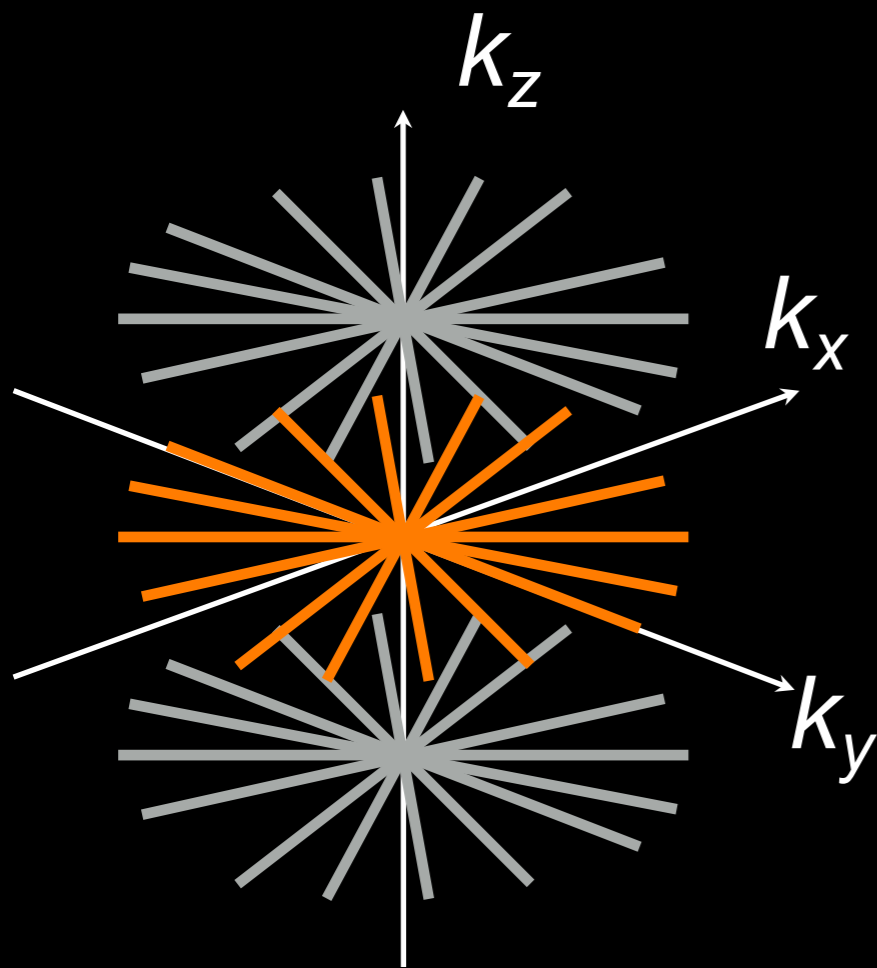
3D Stack of Stars
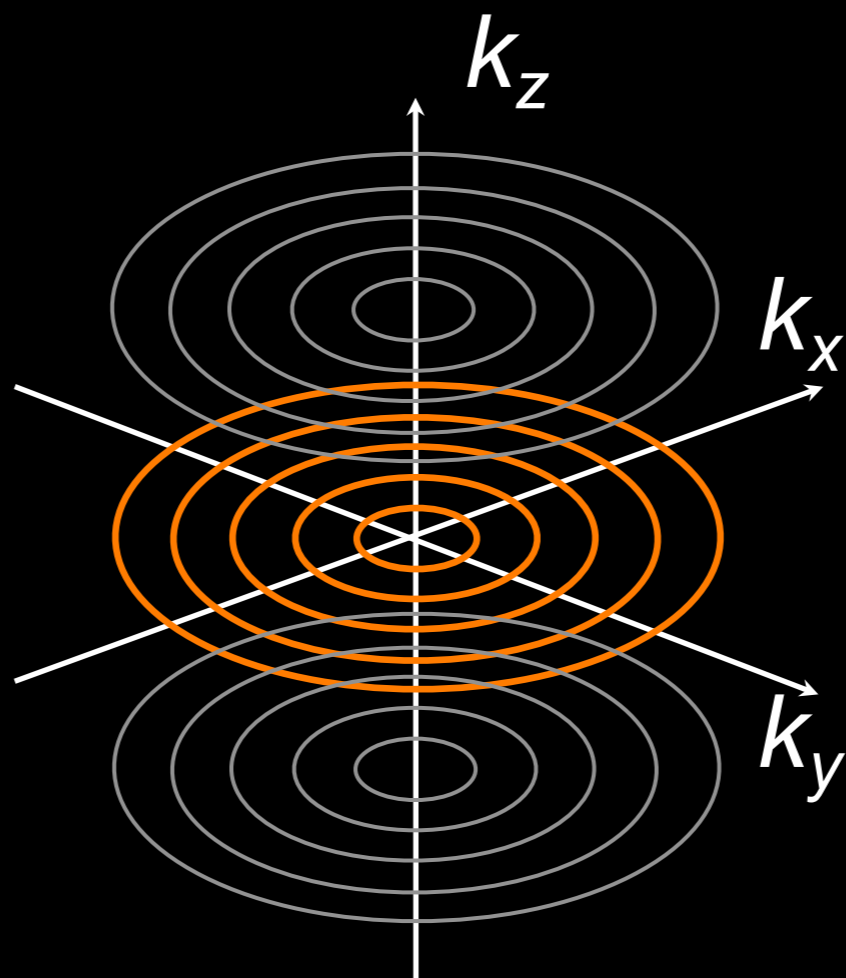
3D Stack of Rings
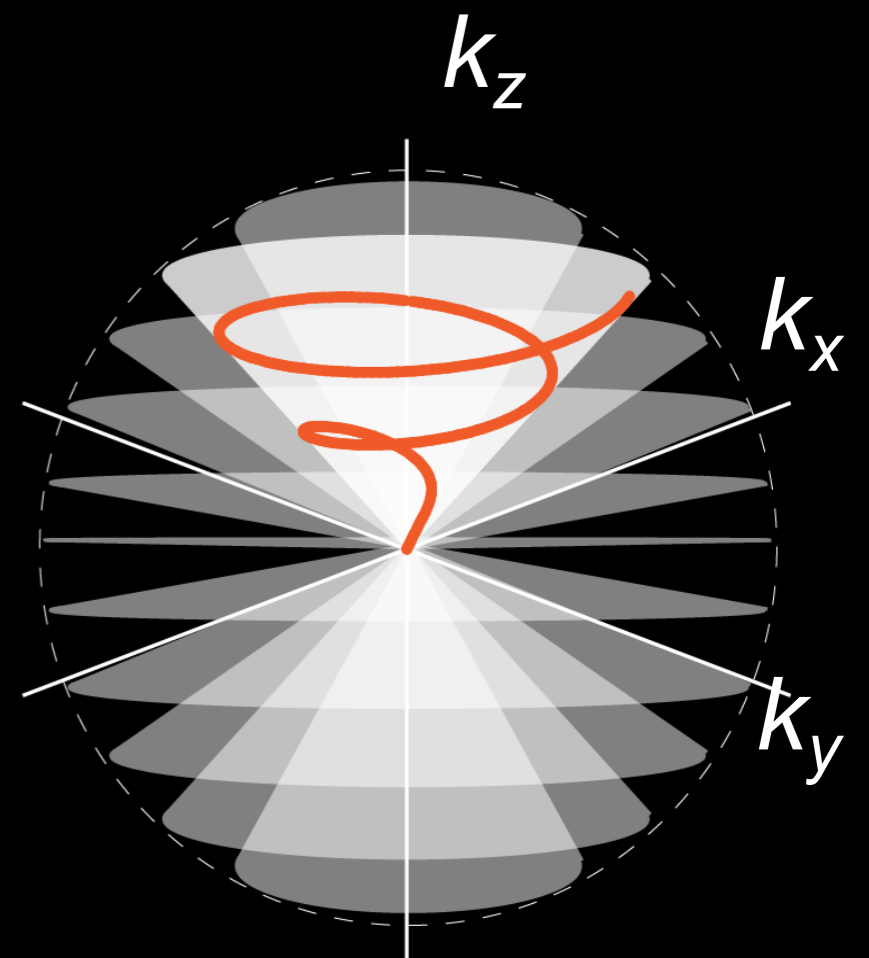
3D Cones

and much more ...

3D Stack-of-Radial

k_z

k_x

k_y
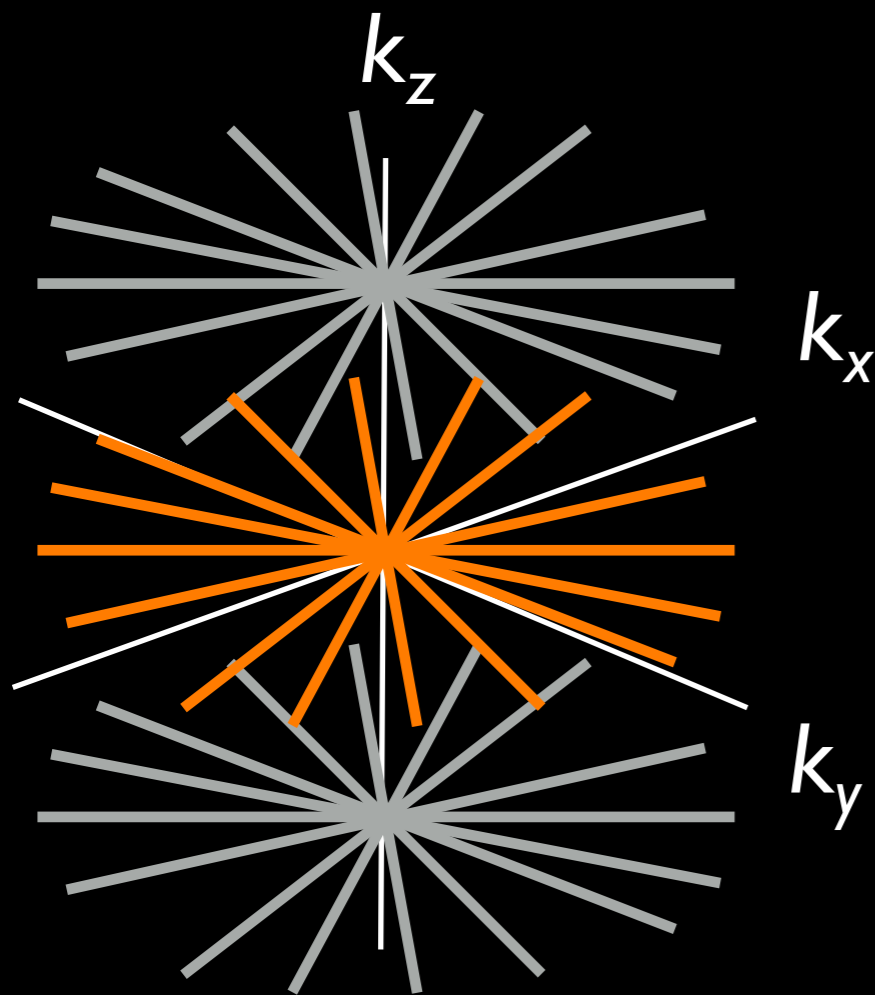
aka Stack-of-Stars

Pros
 - Straightforward extension of radial
 - Robust to motion
 - Can tolerate a lot of undersampling

Cons
 - May have mixed contrast
 - Sensitive to gradient delays
 - Sensitive to off-resonance effects

3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

Free-breathing 3D Stack-of-Radial MRI
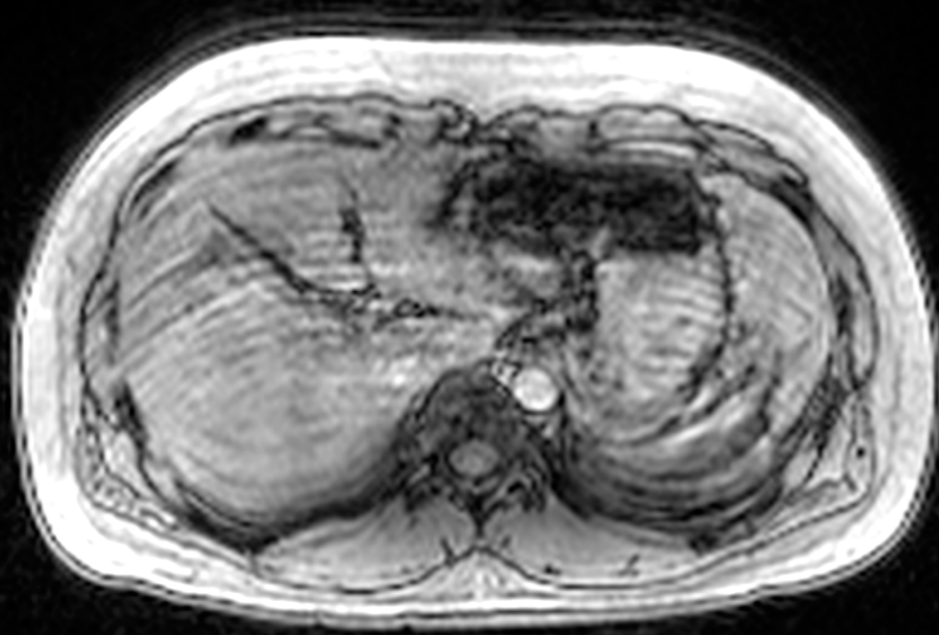
Insufficient breath-holding

Axial

Coronal

Sagittal

courtesy of Tess Armstrong

3D Radial

k_z

k_x

k_y
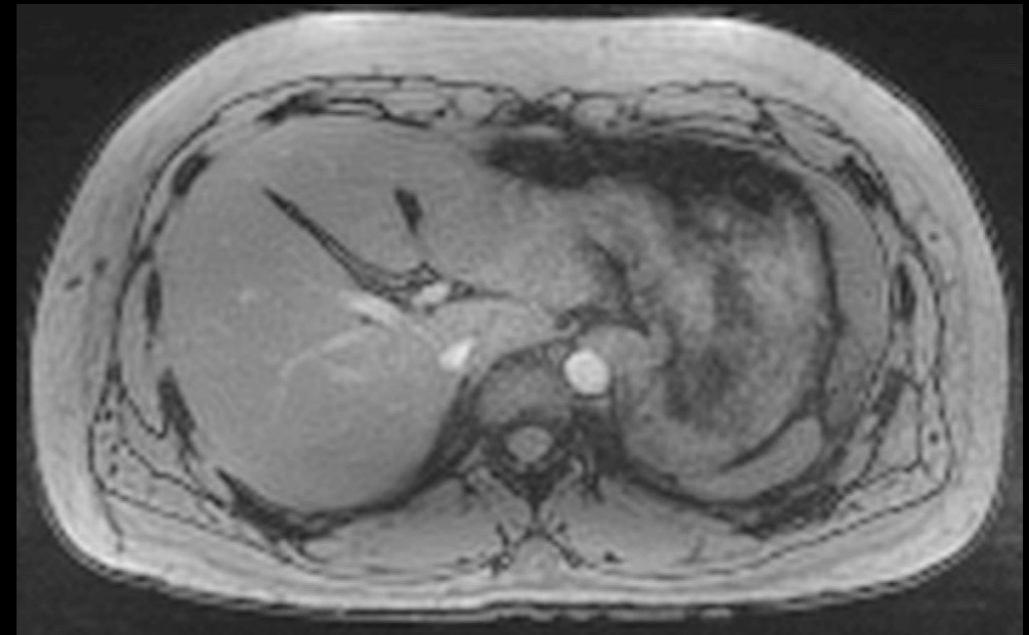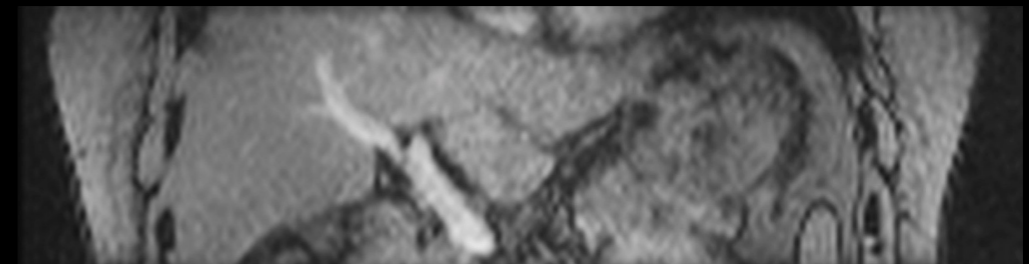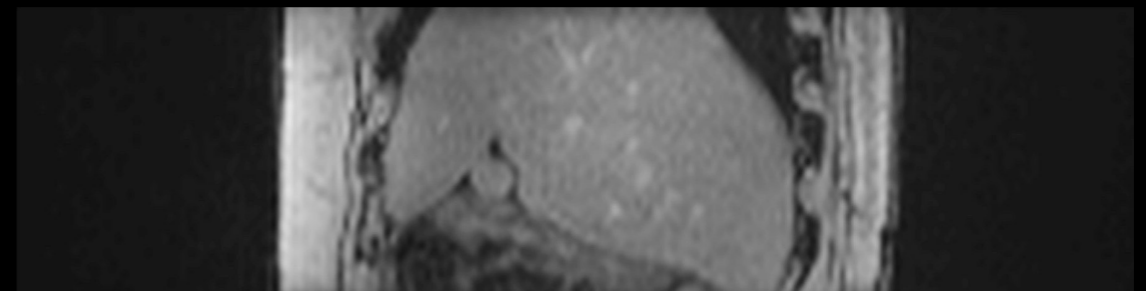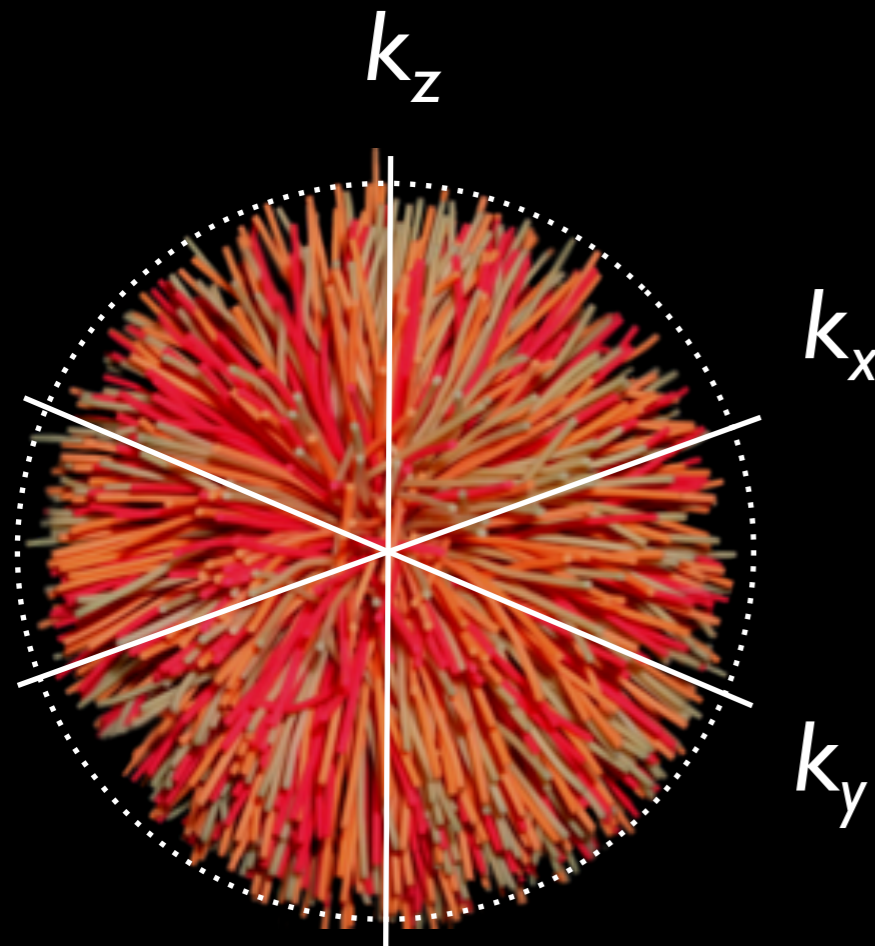
Pros

- Robust to motion (get DC every TR)
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

3D Radial: Coronary MRA

Contrast-Enhanced MRA at 3.0T

ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence

$(1.0 \text{ mm})^3$ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

3D Cones
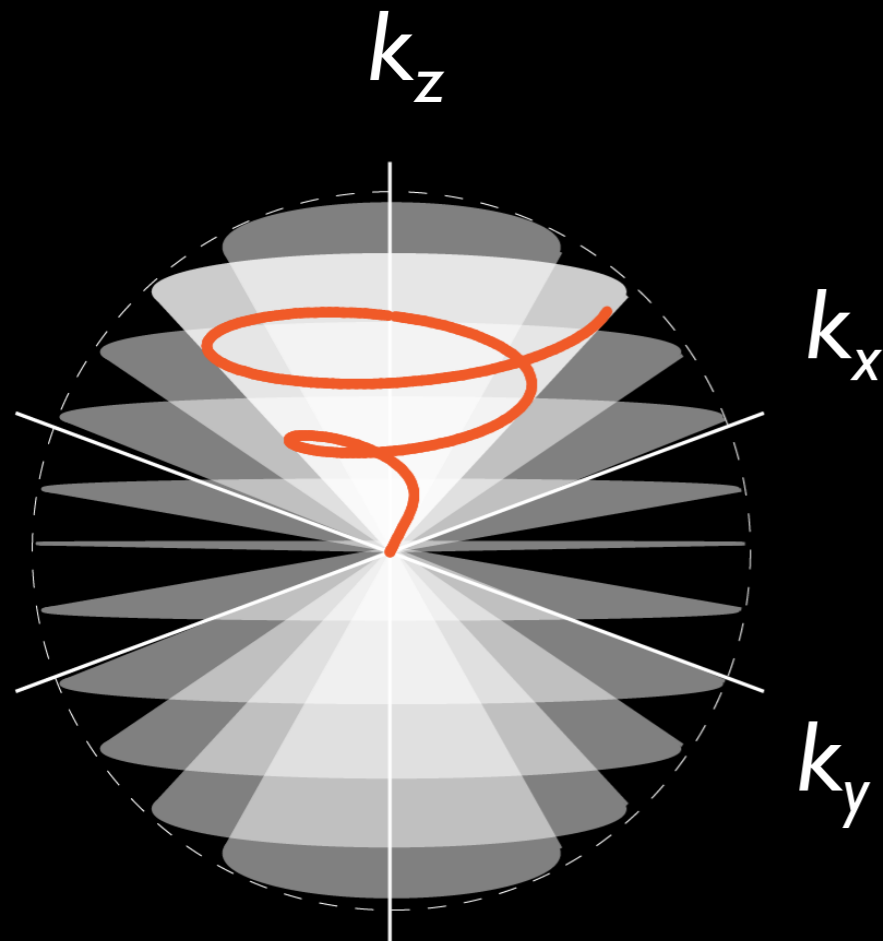
Pros

- Very fast (3-8x vs. Cartesian)
- Very short TE
- Flexible readout length
- Robust to motion/flow effects

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

3D Cones: Coronary MRA

Multi-Phase Thin-Slab MIP Reformats
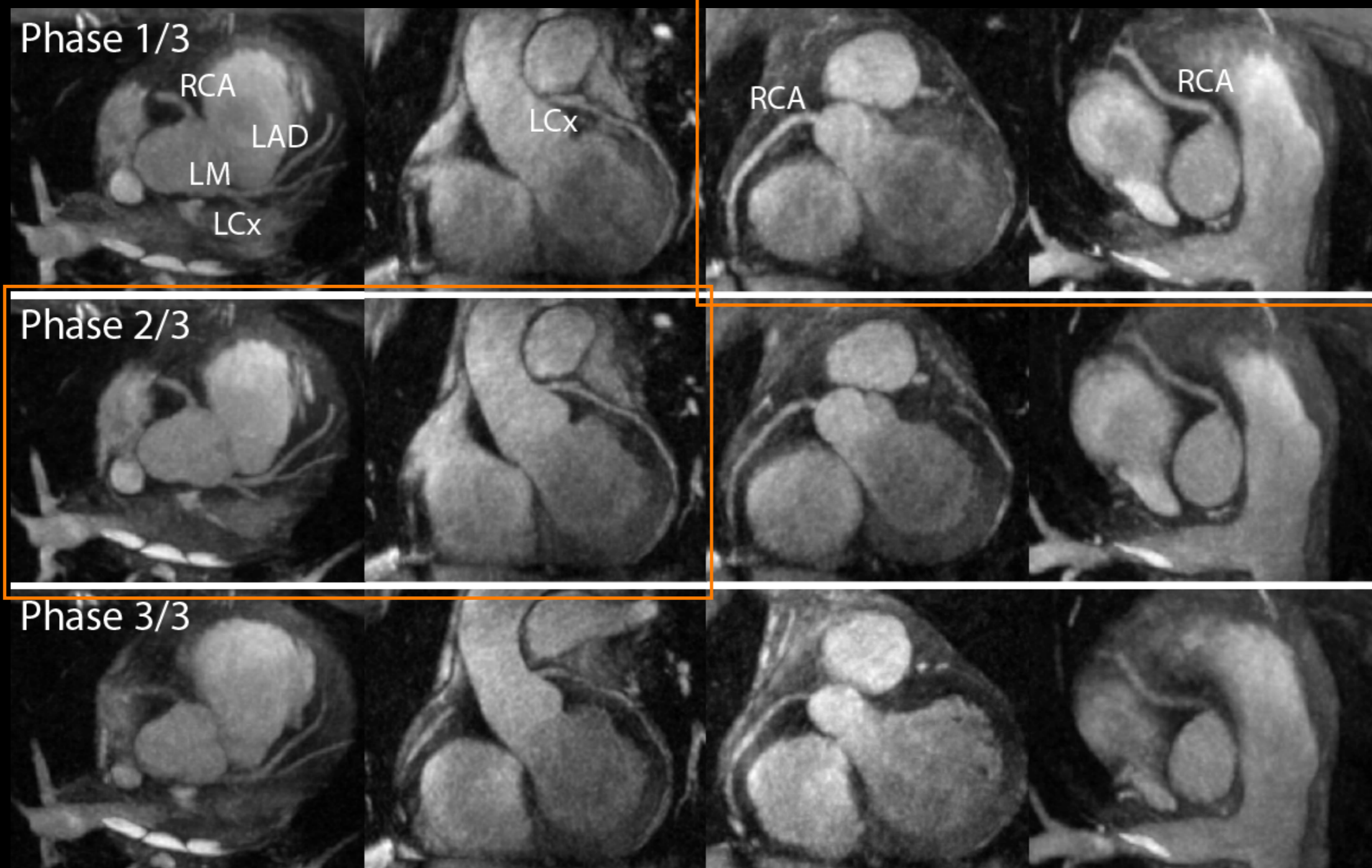
3D Cones: Hi-res CMRA

Thin-Slab MIP Reformats: 0.8 mm isotropic

Subject A

Subject B

Subject C

1.2 mm
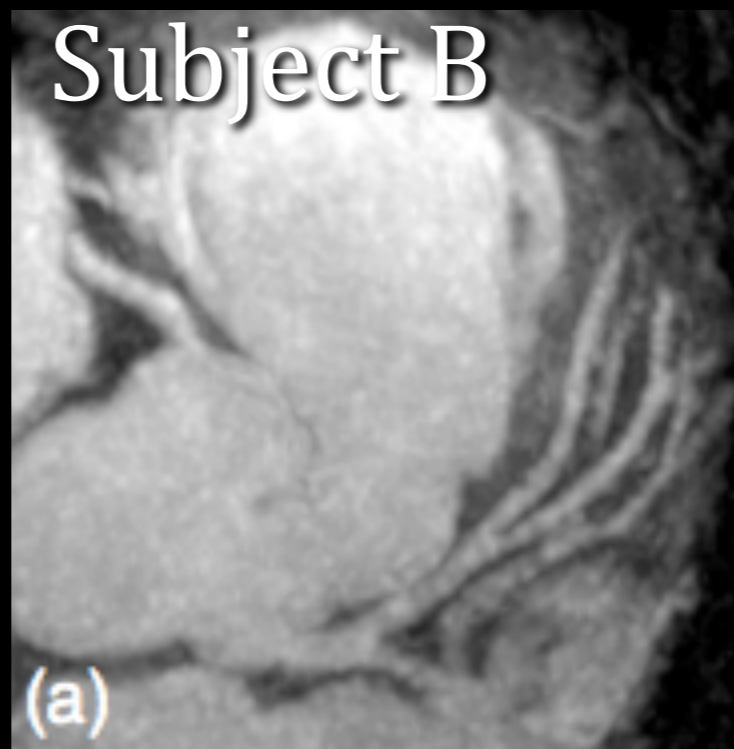
0.8 mm
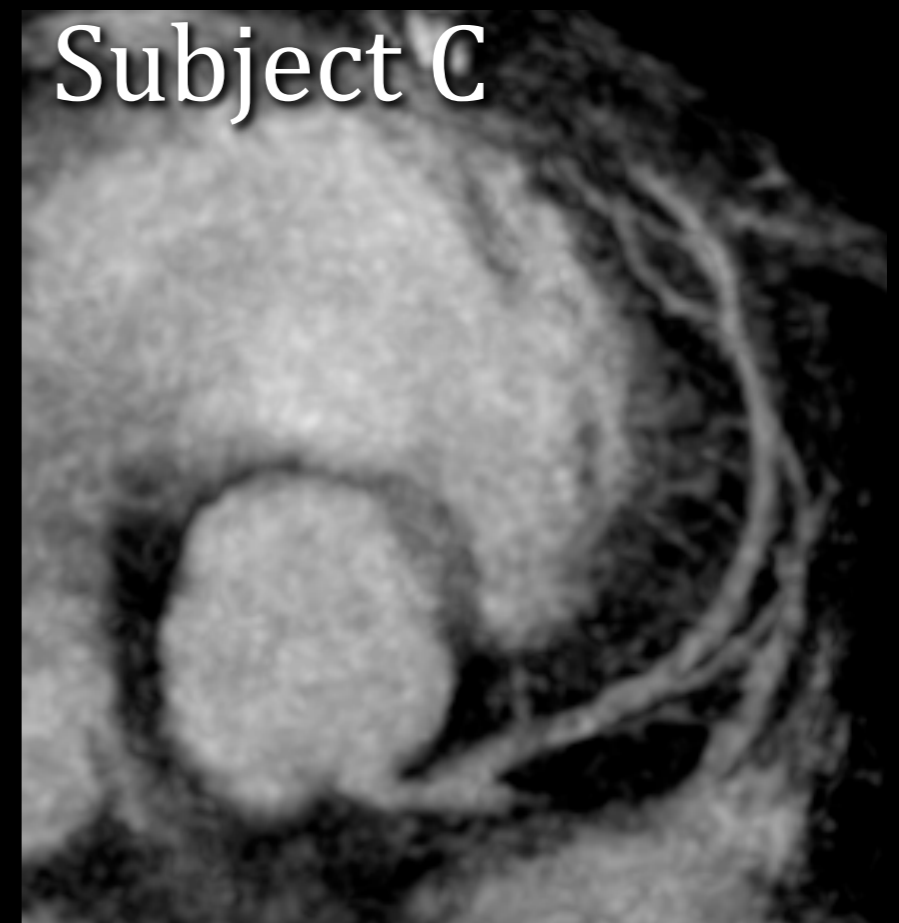
Right coronary
artery cross
section

(a)

(b)
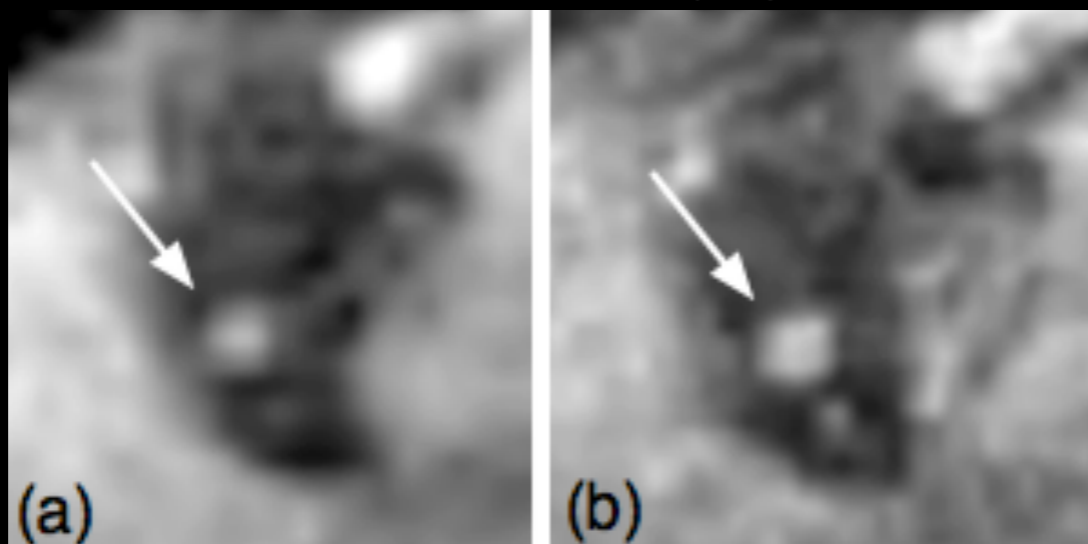
1.5 T; 8-channel cardiac coil

Addy NO, et al., MRM 2015; 74:614-621

Non-Cartesian Image Reconstruction

- Gridding reconstruction

- Gradient measurement

- Off-resonance correction

MRI Signal Equation

$$s(t) = \iint\limits_{X,Y} m(x,y) \cdot \exp(-i2\pi \cdot [k_x(t)\,x + k_y(t)\,y])\,\mathrm{d}x\,\mathrm{d}y$$

$$= \mathcal{FT}(\,m(x,y)\,) = M(\,k_x(t), k_y(t)\,)$$

General definition of *k*-space:

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau)\,\mathrm{d}\tau, \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau)\,\mathrm{d}\tau$$

MRI Reconstruction

$$m(x, y) = \mathcal{FT}^{-1}(\,M(k_x, k_y)\,)$$

$$m(x, y) = \iint\limits_{k_x, k_y} M(k_x, k_y) \cdot \exp(i2\pi \cdot [k_x x + k_y y])\, \mathrm{d}k_x\, \mathrm{d}k_y$$

k-space   *image space*

uniform $\longleftrightarrow$ *uniform*

non-uniform   *non-uniform*
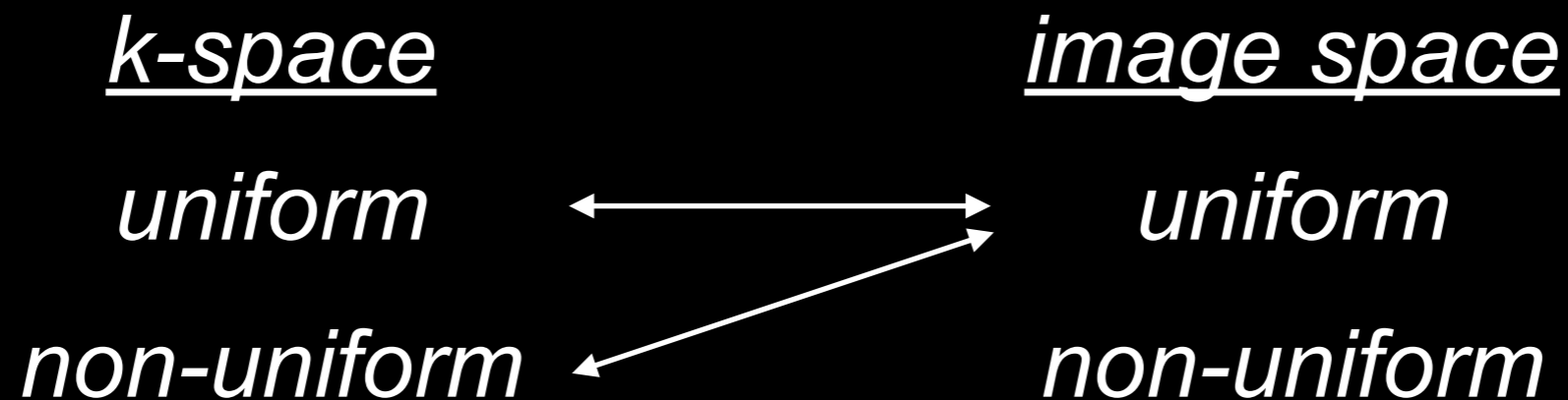
simple for Cartesian (k_x, k_y) to Cartesian (x, y): 2D FFT

time consuming for non-Cartesian (k_x, k_y) to Cartesian (x, y)

Non-Cartesian Reconstruction

- Inverse Fourier transform
  - aka conjugate phase reconstruction

- Gridding (+FFT)[1]
  - grid driven interpolation
  - data driven interpolation (more popular)
  - forward and reverse (inverse)

- Non-uniform FFT (NUFFT)[2]

- Block Uniform ReSampling (BURS)[3]

[1] *O'Sullivan JD, IEEE TMI 1985; 4: 200-207*

[2] *Fessler JA et al., IEEE TSP 2003; 51: 560-574*

[3] *Rosenfeld D, MRM 2002; 48: 193-202*

Gridding: Basic Idea

k-space
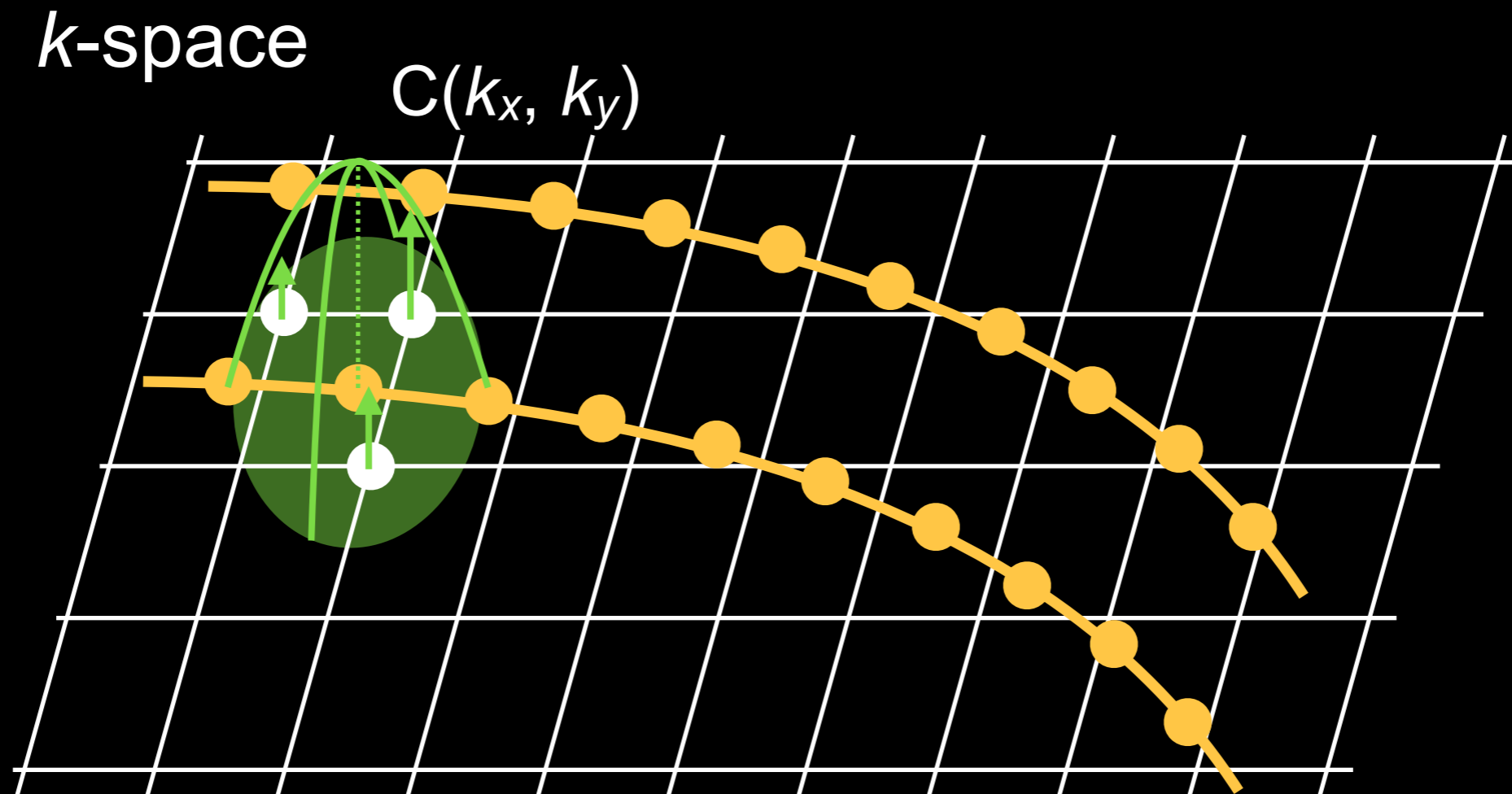
C(k_x, k_y)

convolve each acquired data point with kernel C(k_x, k_y)

resample the convolution onto Cartesian grid points

2D inverse FFT; de-apodization and FOV cropping

Gridding: Basic Math

Sampling pattern: $S(k_x, k_y) = \sum_j^2 \delta(k_x - k_{x,j}, k_y - k_{y,j})$

Convolution kernel: $C(k_x, k_y)$ $\qquad$ Grid: $\mathrm{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y})$

Gridding recon:

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \mathrm{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y})$$

FFT $\downarrow$ $\qquad$ *non-Cartesian dataset* $\qquad$ *interpolation* $\qquad$ *resample to grid*

$$\hat{m}(x, y) = [(m(x, y) * s(x, y)) \cdot c(x, y)] * \mathrm{III}(\frac{x}{\mathrm{FOV}_x}, \frac{y}{\mathrm{FOV}_y})$$

remove by deap $\qquad$ *remove by cropping*

$$\rightarrow m(x, y)$$

Gridding: Design Issues

- Convolution kernel
  - apodization; aliasing

- Sampling grid density (Cartesian)
  - aliasing

- Sampling pattern (non-Cartesian)
  - impulse response and side lobes
  - density characterization / compensation

Gridding: Design - Kernel

- Ideal convolution kernel: SINC
  - don't need de-apodization
  - infinite extent impractical to implement
  - windowed version has limited performance

- Desired kernel characteristics
  - compact support (finite width) in k-space
  - minimal aliasing effects in image (sharp transition)

Gridding: Design - Kernel

Combine with grid oversampling

$$\Delta k_x = \frac{1}{\text{FOV}_x}, \Delta k_y = \frac{1}{\text{FOV}_y}$$

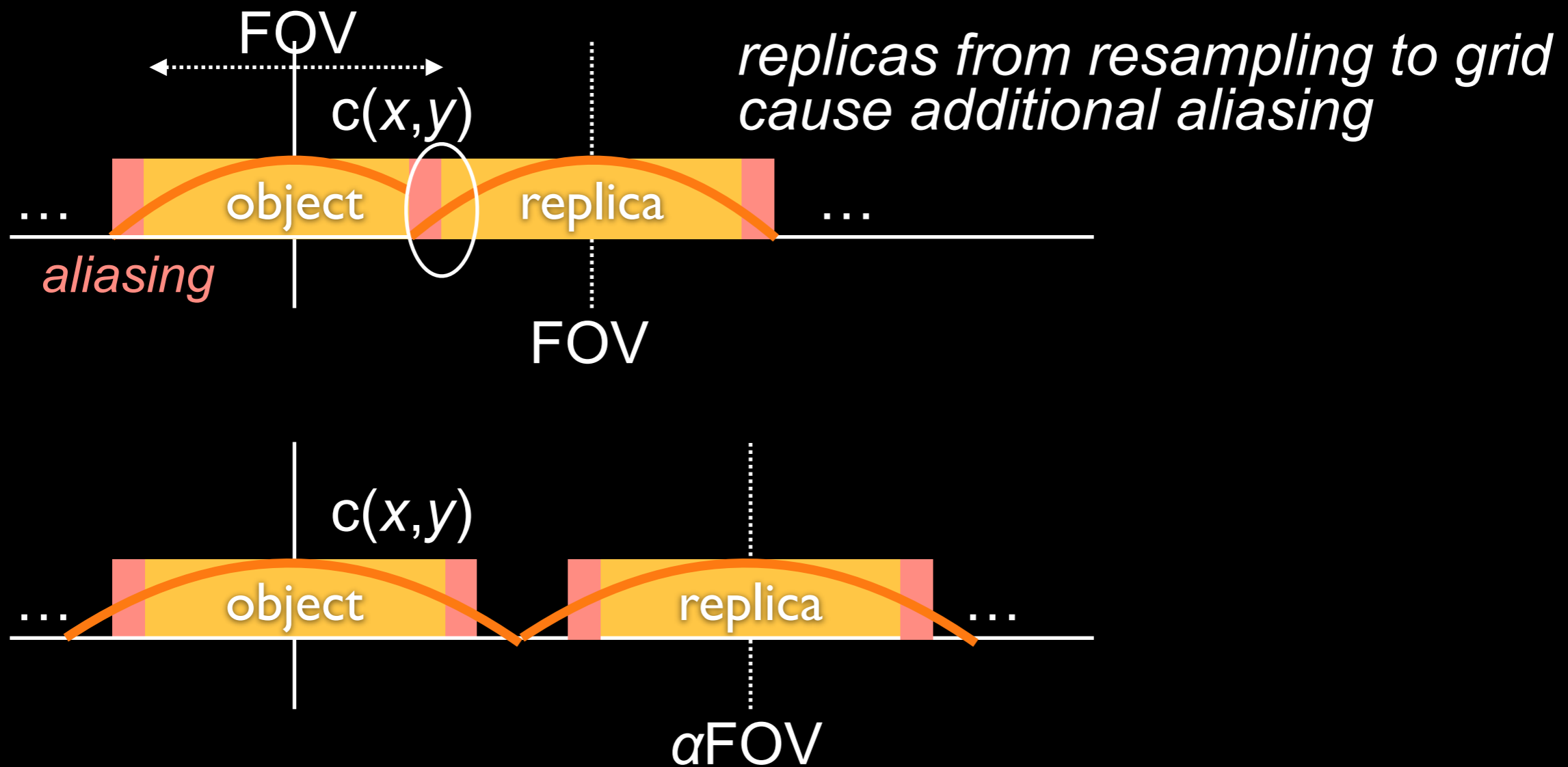$$\frac{\Delta k_x}{\alpha} = \frac{1}{\alpha \text{FOV}_x}, \frac{\Delta k_y}{\alpha} = \frac{1}{\alpha \text{FOV}_y} \qquad \alpha > 1$$

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \text{III}(\frac{k_x}{\Delta k_x/\alpha}, \frac{k_y}{\Delta k_y/\alpha})$$

$$\hat{m}(x, y) = [(m(x, y) * s(x, y)) \cdot c(x, y)] * \text{III}(\frac{x}{\alpha \text{FOV}_x}, \frac{y}{\alpha \text{FOV}_y})$$

Gridding: Design - Kernel

Combine with grid oversampling

FOV

c(*x*,*y*)

replicas from resampling to grid cause additional aliasing

… object   replica …

aliasing

FOV

c(*x*,*y*)

… object   replica …

*α*FOV

α = 2 very forgiving; many kernels work well; apodization minimal expensive … especially for 3D gridding

Gridding: Design - Kernel

- Jointly consider α and kernel
  - minimize aliasing energy
  - characterize trade-offs
  - numerical designs possible
  - Kaiser-Bessel window works very well, with proper choice of β and kw[1,2]; precompute a lookup table to speedup calculations[2]

$$C_{KB}(k_x) = \text{I}_0\left(\beta\sqrt{1-(\frac{k_x}{kw/2})^2}\right)$$

[1]*Jackson et al., IEEE TMI 1991; 10: 473-478*

[2]*Beatty et al., IEEE TMI 2005; 24: 799-808*

Gridding: Design - Density

Sampling density of $S(k_x, k_y)$ not uniform: $\quad \rho(k_x, k_y)$

Pre-compensation of sampling density:

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot \frac{S(k_x, k_y)}{\rho(k_x, k_y)}) * C(k_x, k_y)] \cdot \mathrm{III}$$

density corrected on a data point basis before convolution

need to know $\rho(k_x, k_y)$

from geometrical analysis, numerical analysis (Voronoi), etc.

inverse of ρ known as the density compensation function (DCF)

Gridding: Design - Density

Post-compensation of sampling density:

$$\hat{M}(k_x, k_y) = \frac{[(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \mathrm{III}}{\rho(k_x, k_y)}$$

density corrected on a grid point basis after convolution

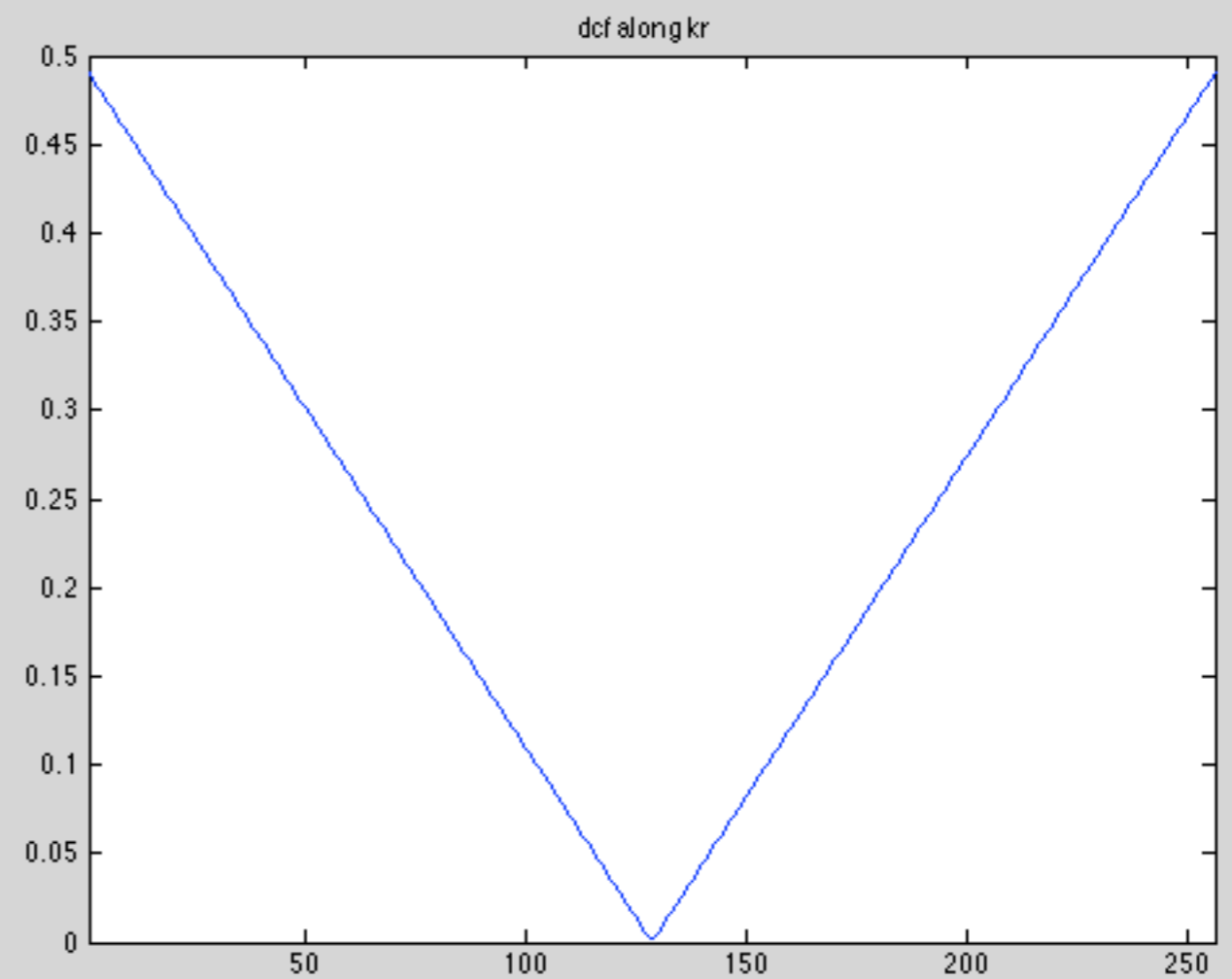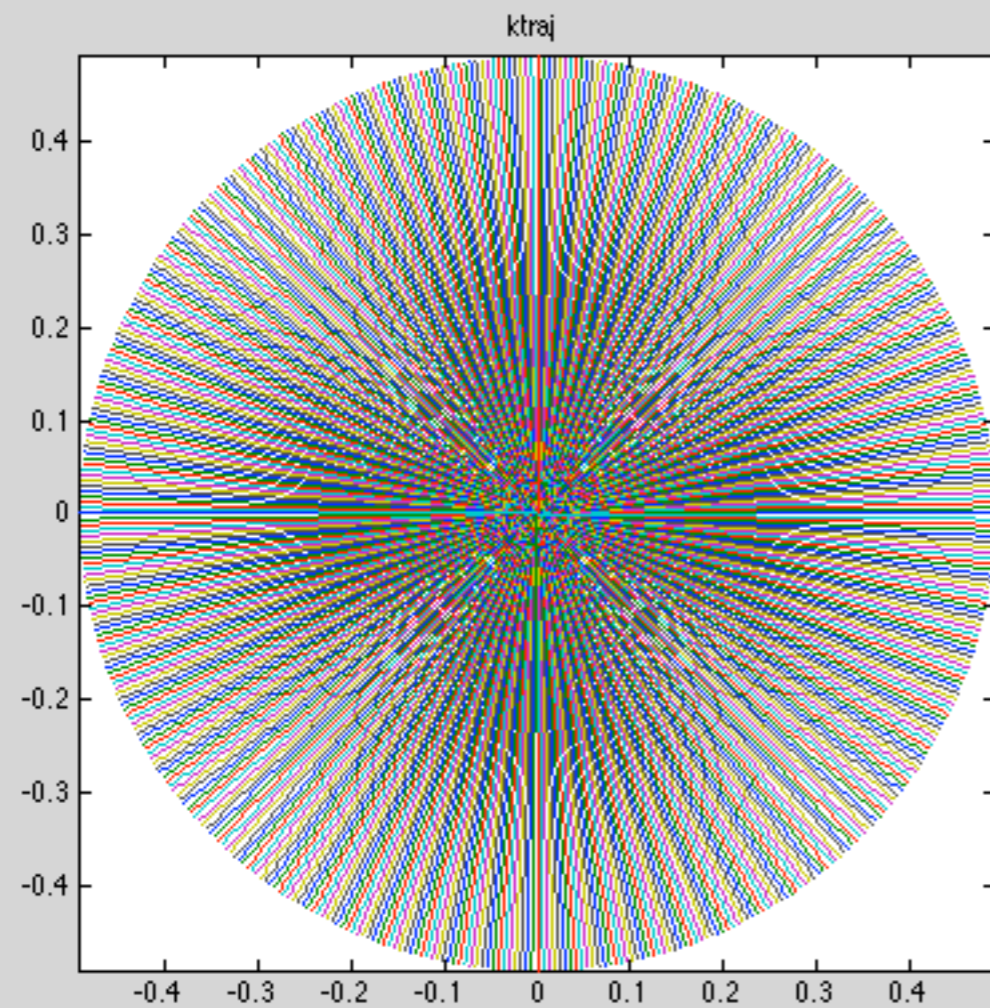can estimate ρ along with gridding; grid all 1s:

$$\hat{\rho}(k_x, k_y) = [S(k_x, k_y) * C(k_x, k_y)] \cdot \mathrm{III}$$

may be okay if S changes slowly

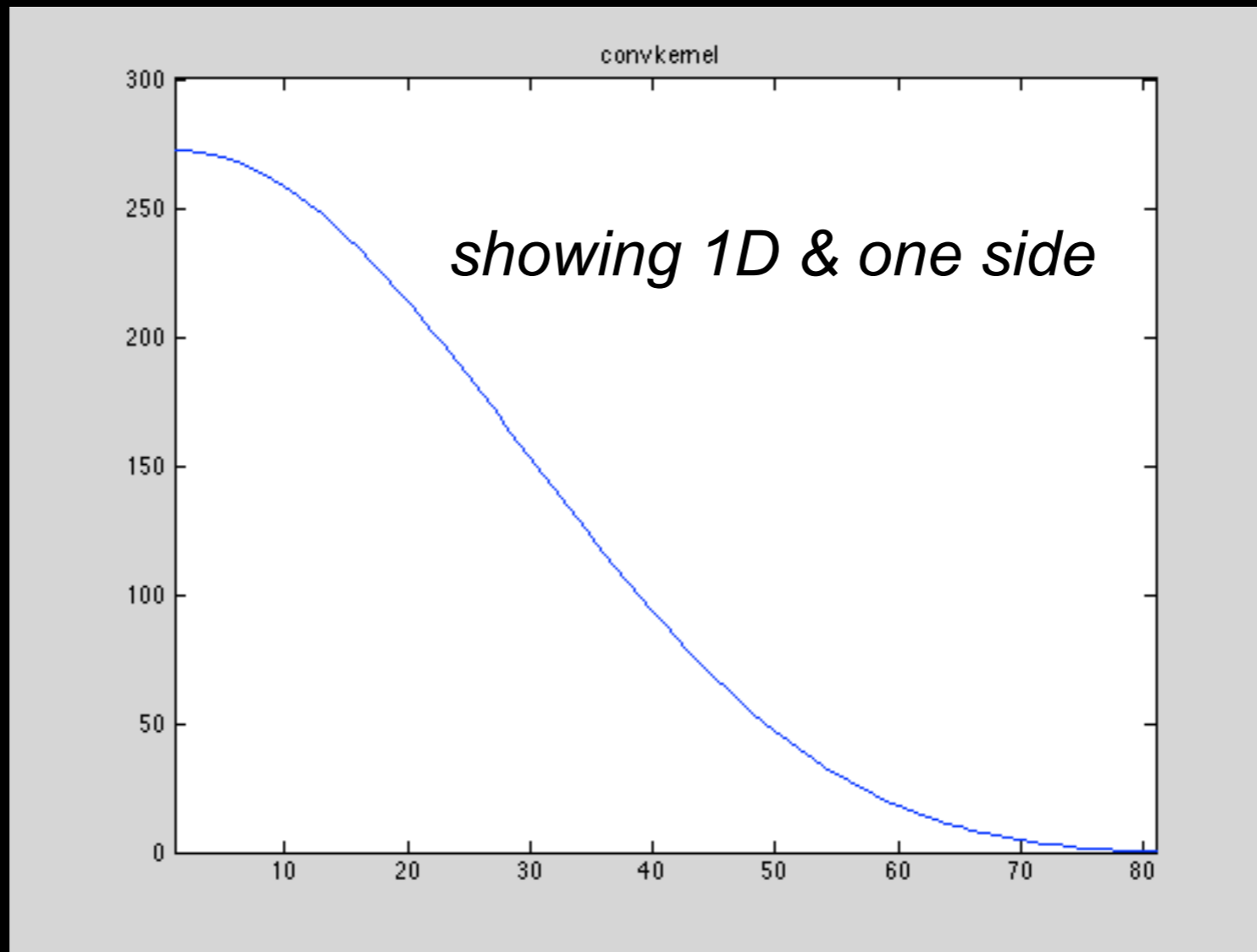… but only an approximation and fails when S changes rapidly

Gridding: 2D Radial Example

Radial trajectory [256x256] with ramp DCF

Gridding: 2D Radial Example
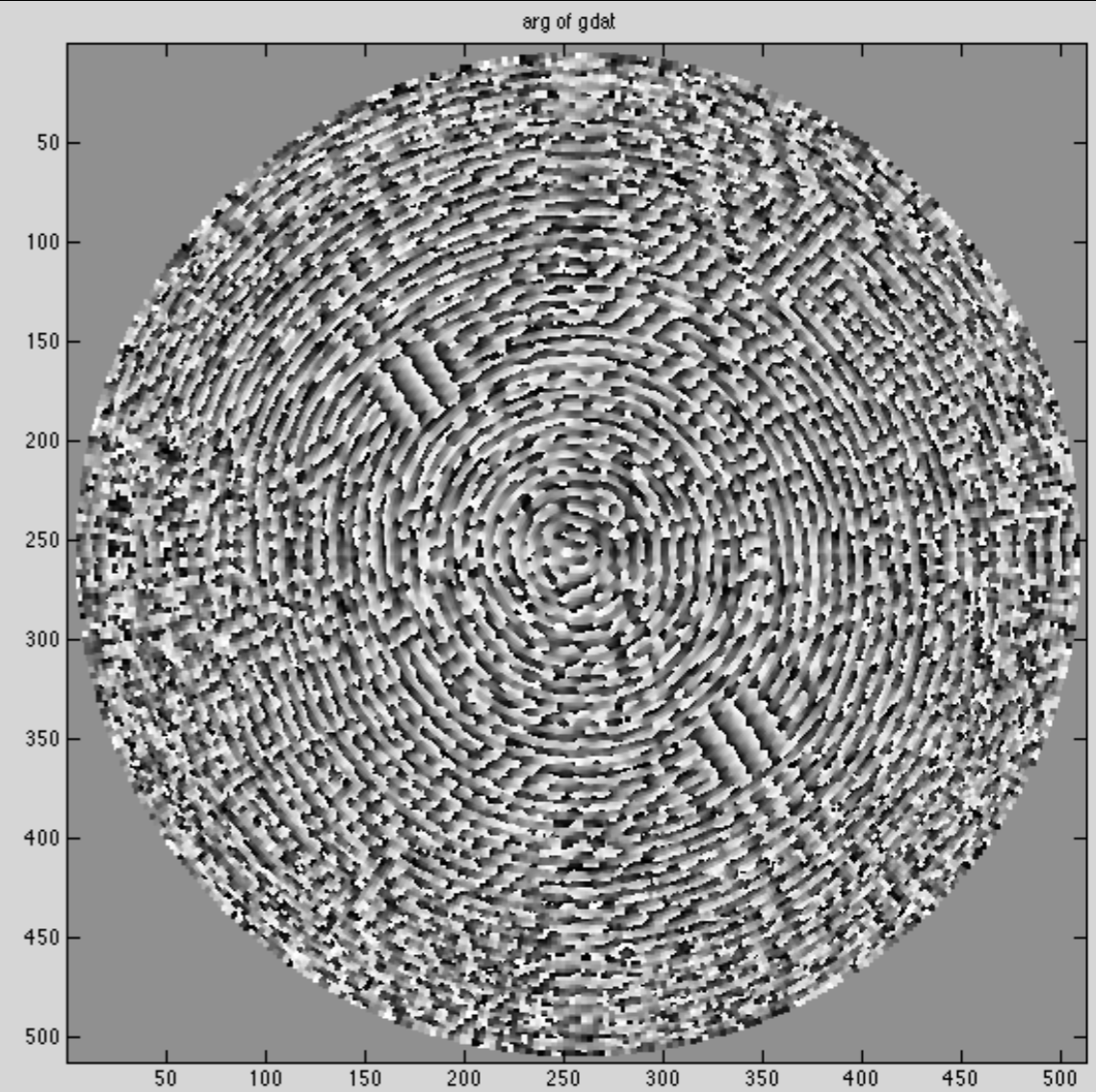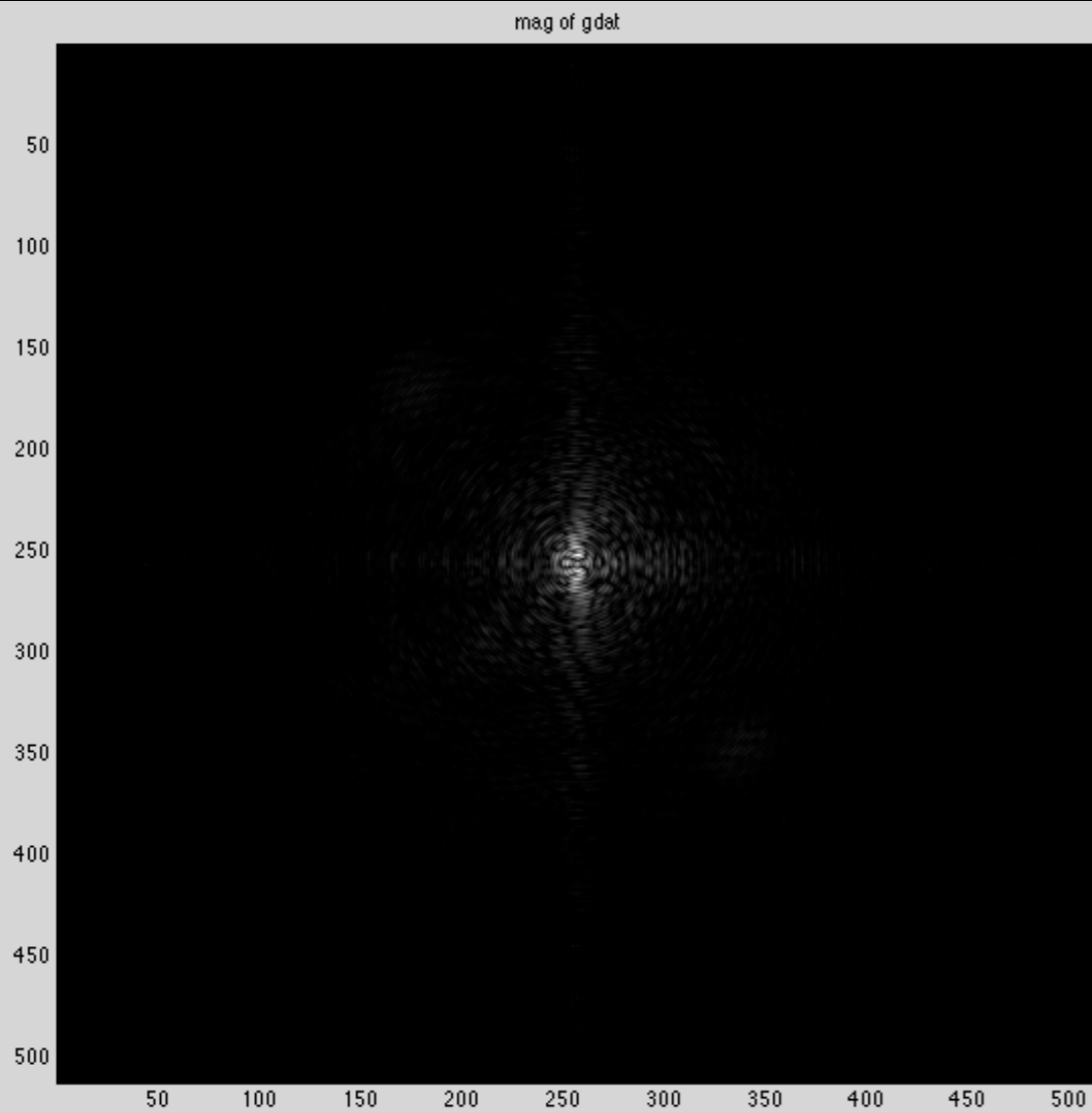
Kaiser-Bessel convolution kernel with linear lookup table[1]

showing 1D & one side

α = 2; grid size = 2x[256 256]; kw = 4;
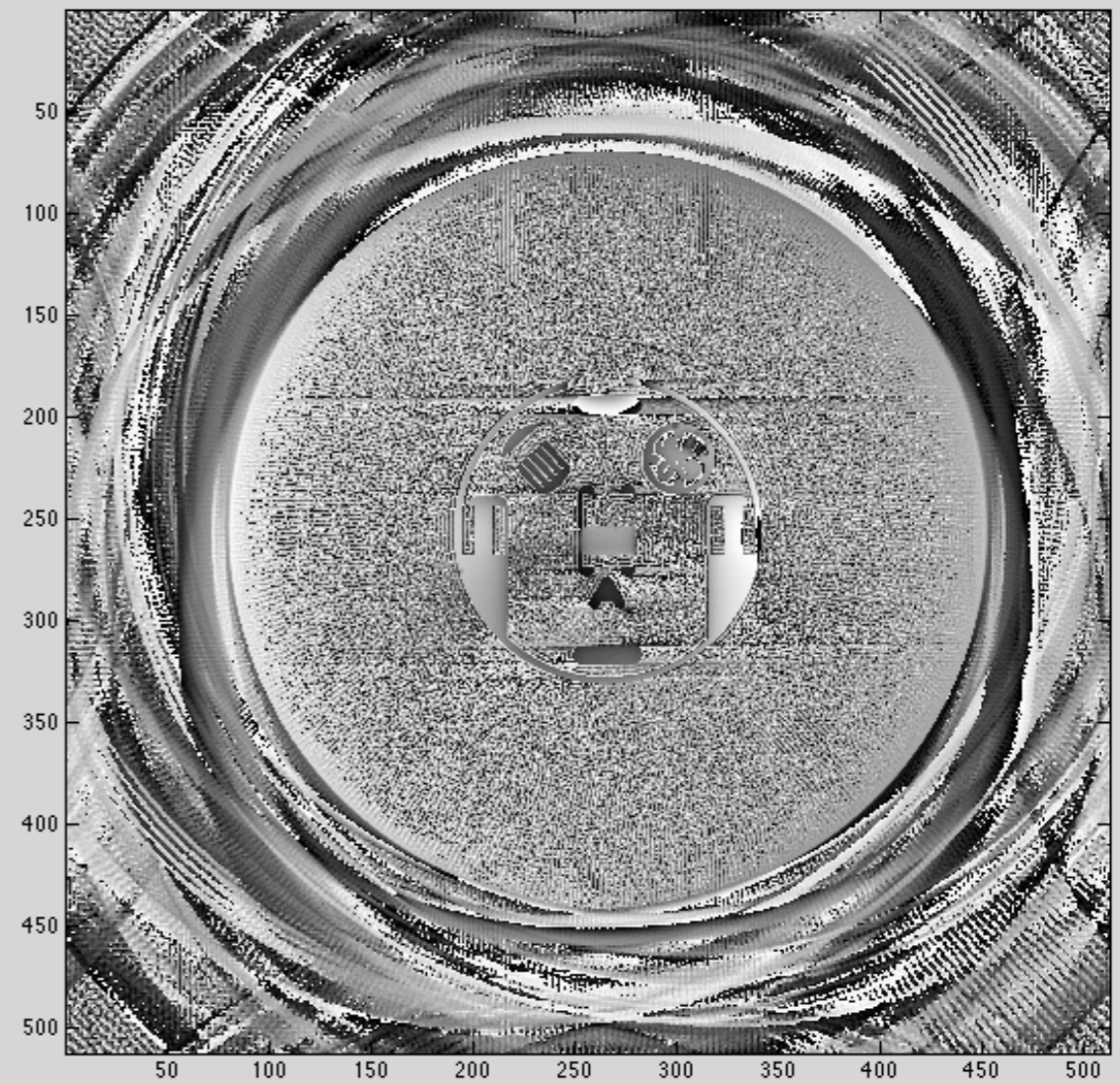
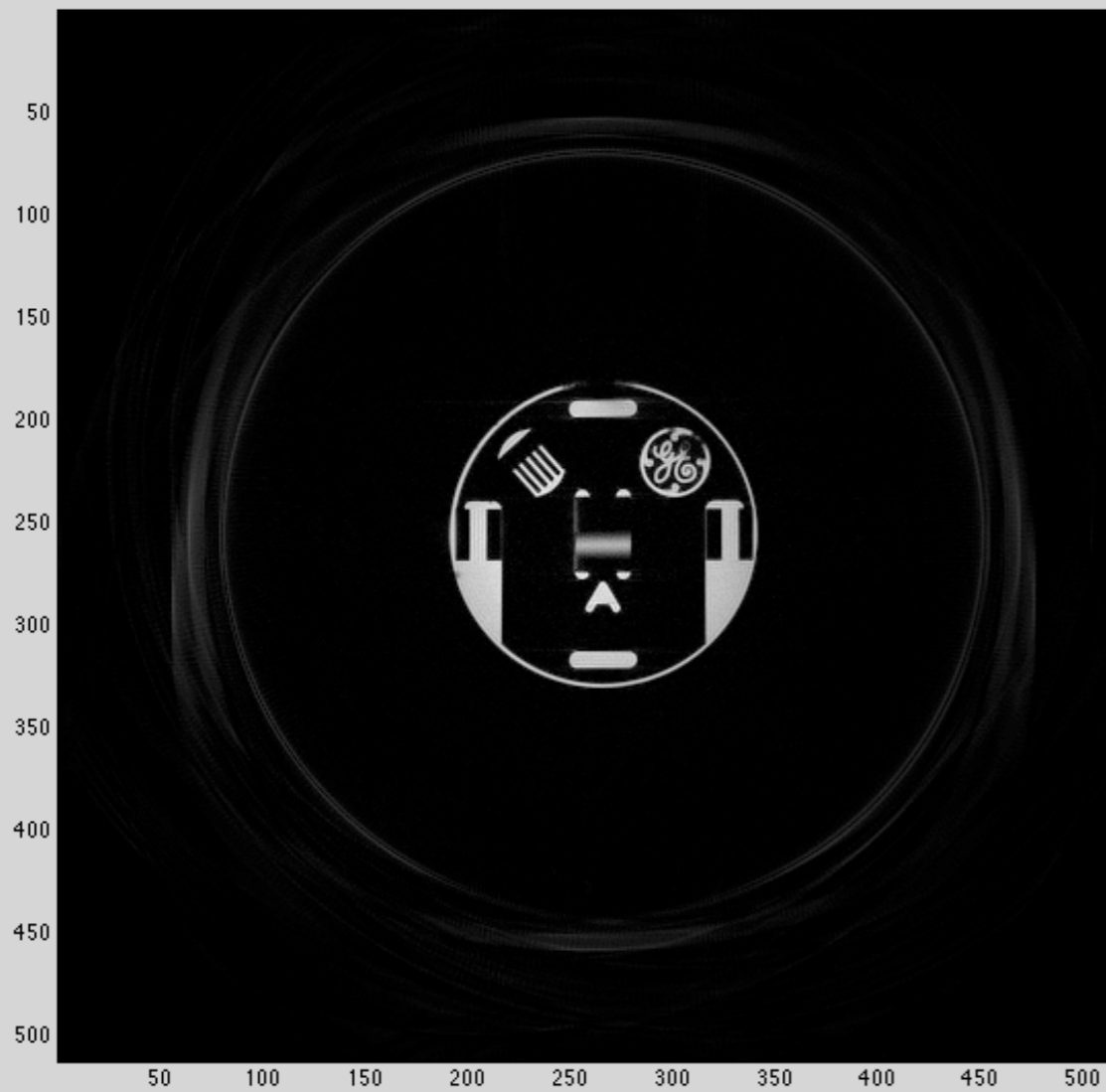[1]*Beatty et al., IEEE TMI 2005; 24: 799-808*

Gridding: 2D Radial Example

Gridded data on [512x512] grid

Gridding: 2D Radial Example

Inverse 2D FFT produces image with 2x FOV

Gridding: 2D Radial Example

Deapodization function is FT of KB convolution kernel
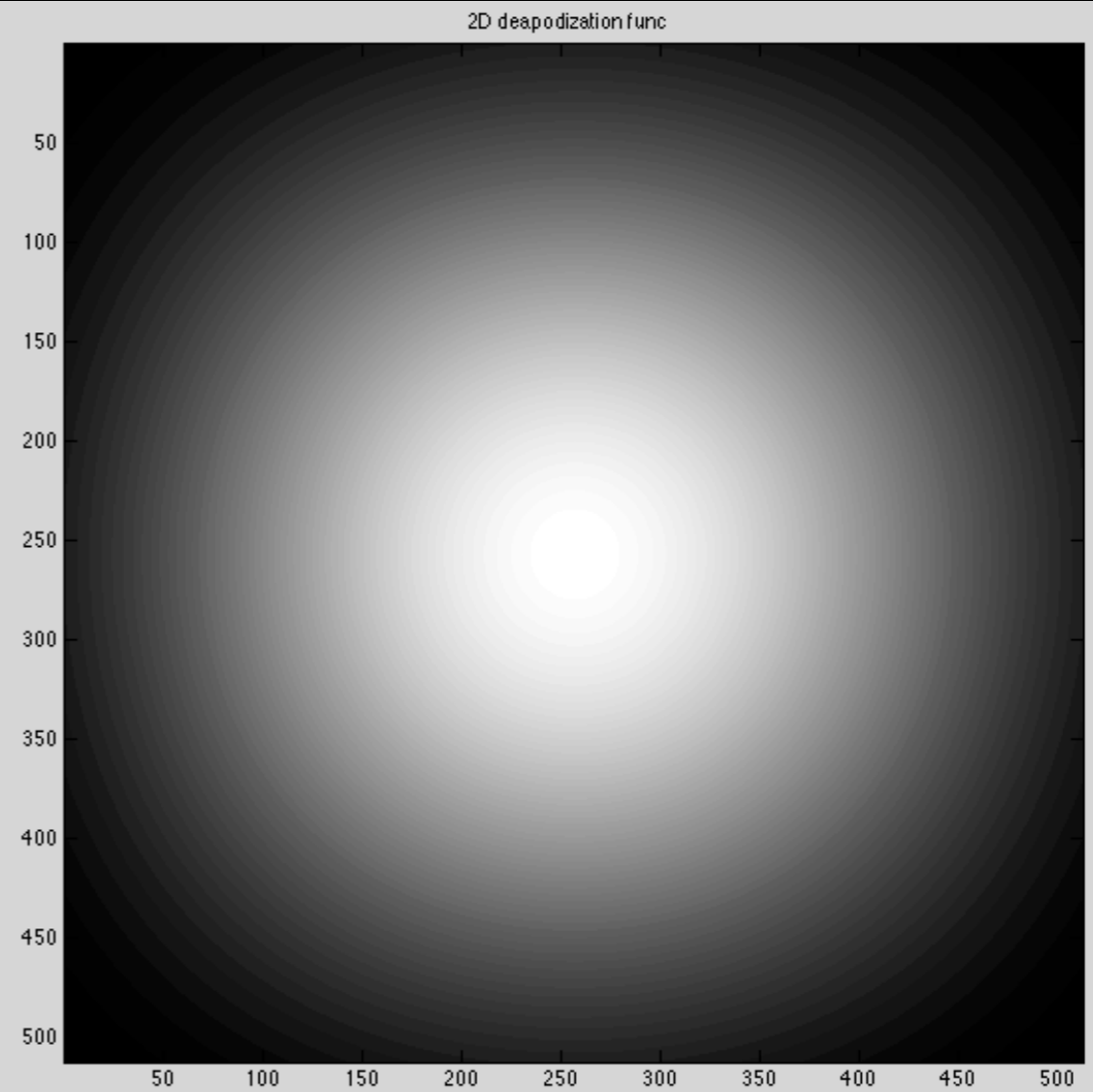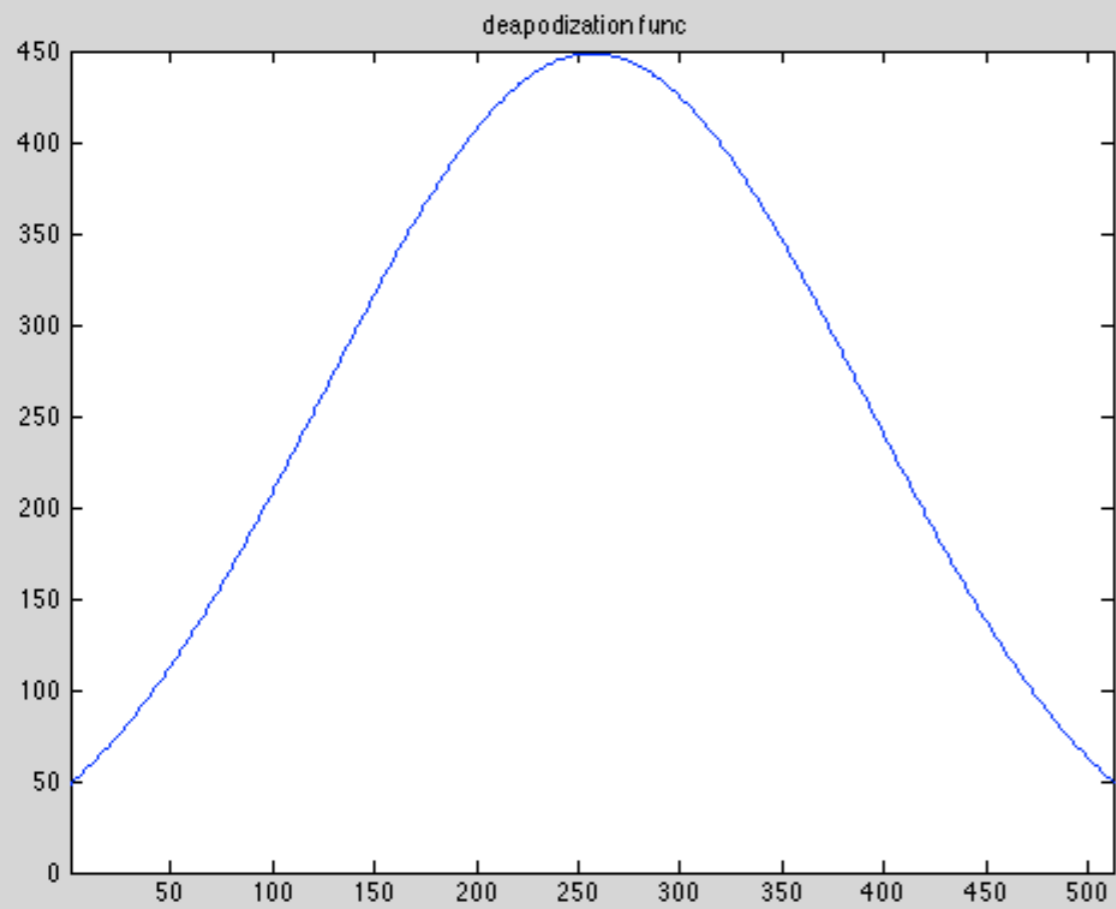
Gridding: 2D Radial Example

Deapodized image
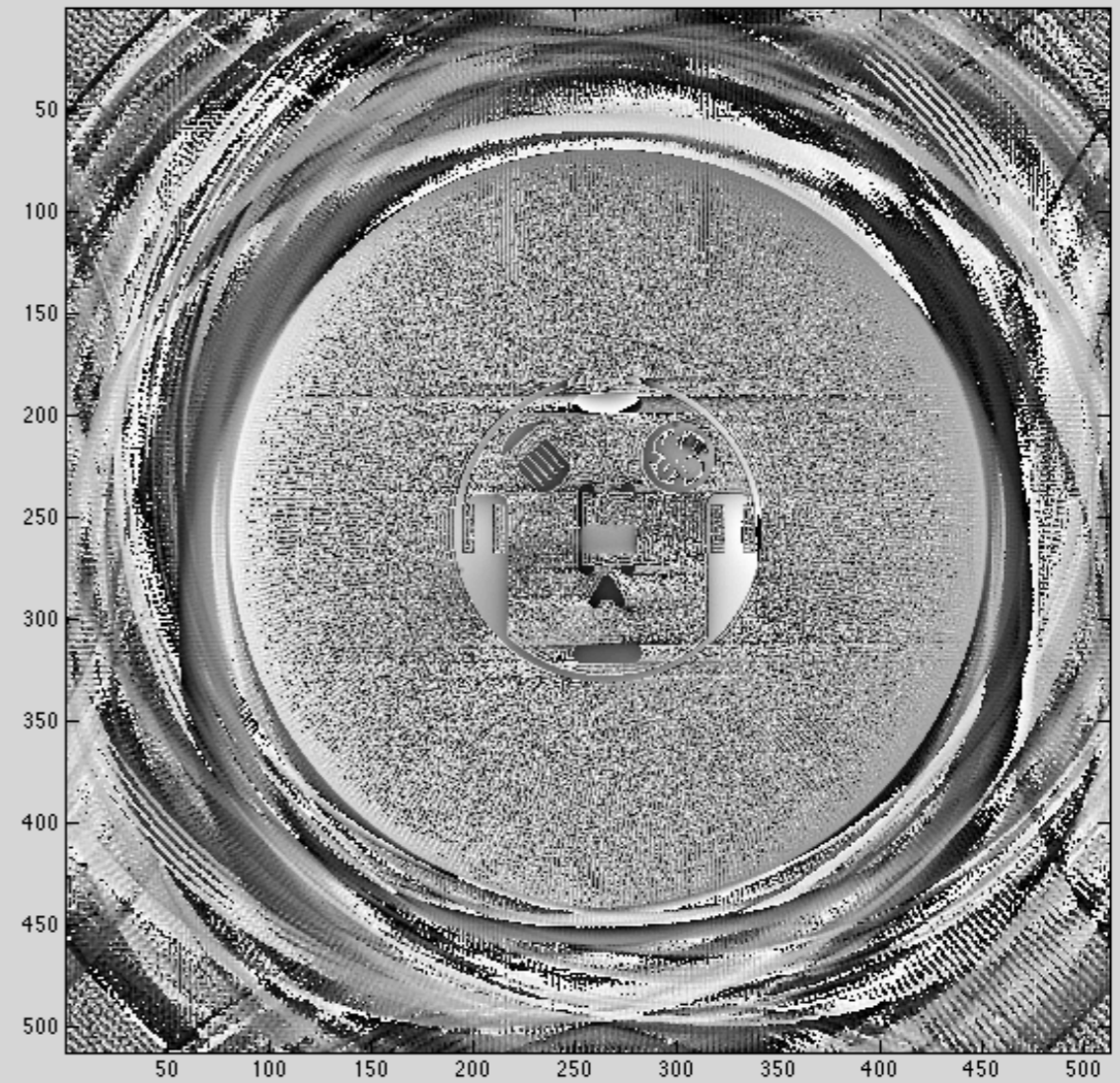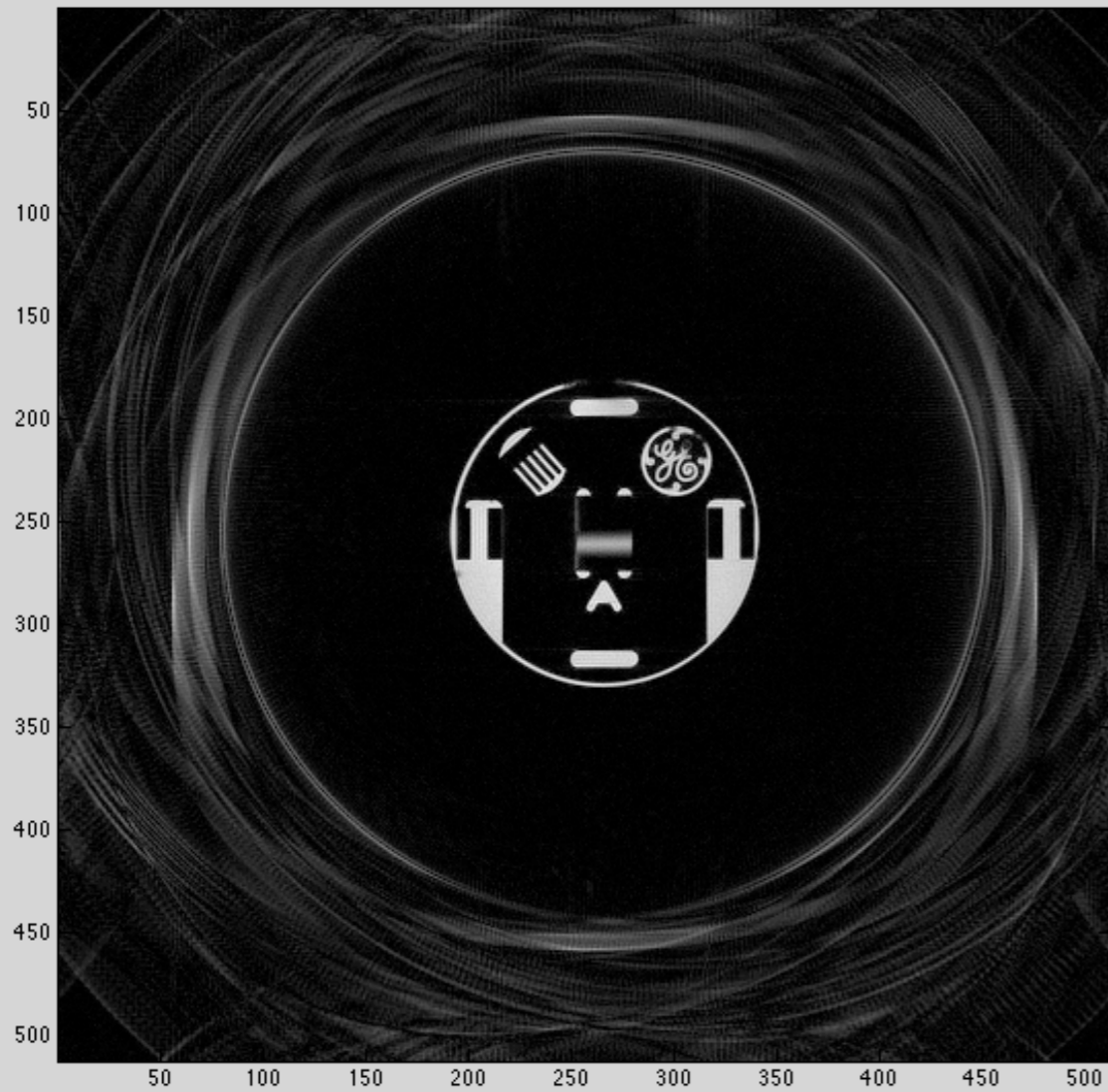
Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

$\alpha = 2$, kw = 4
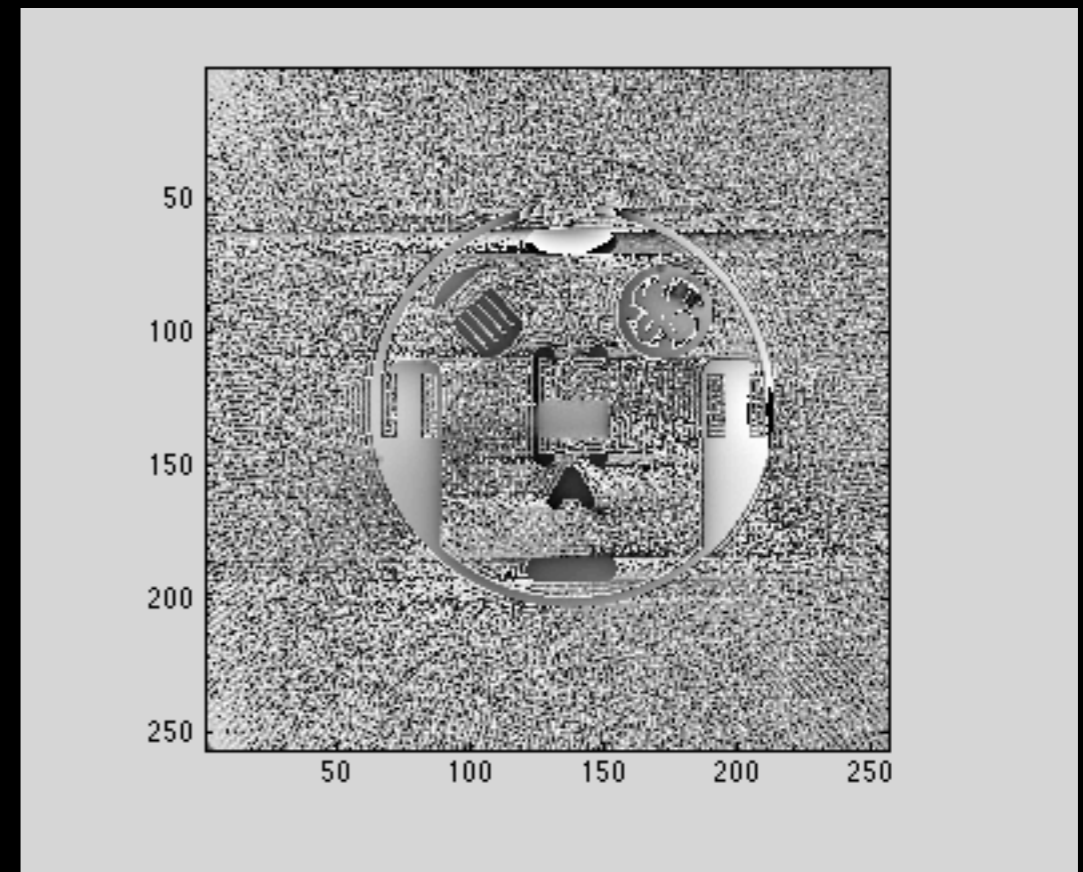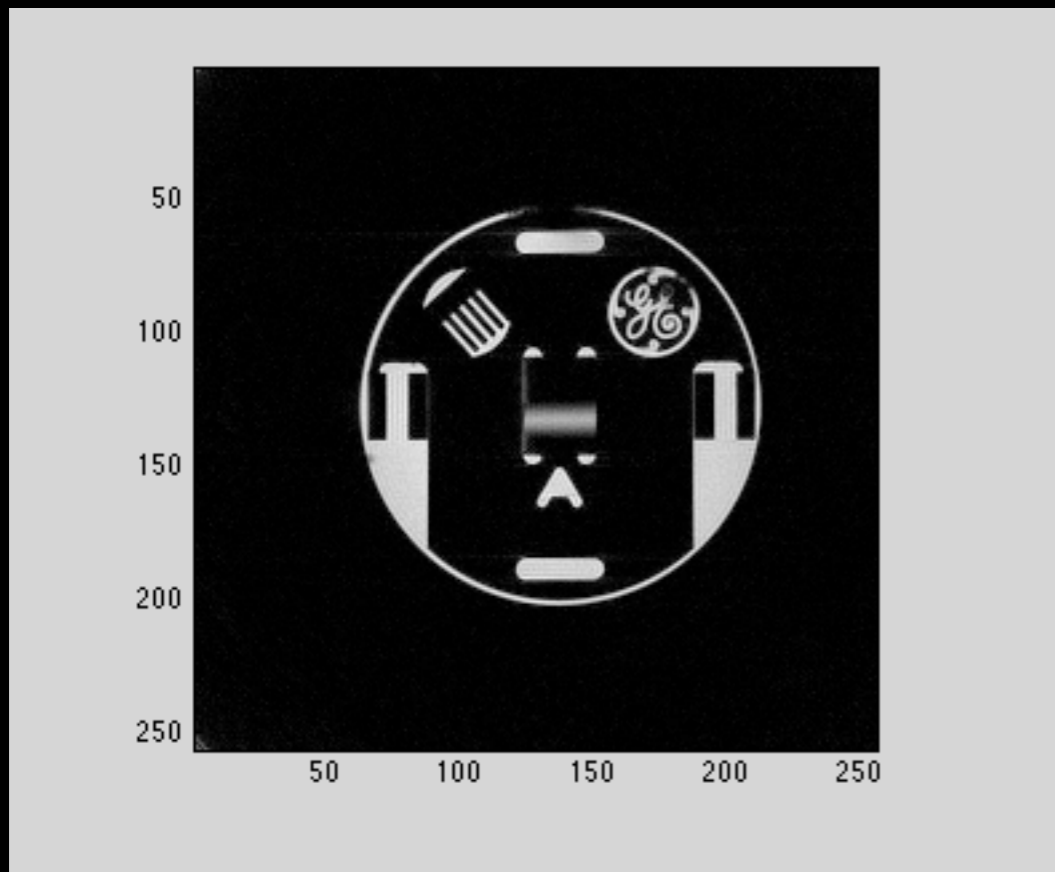
Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

$\alpha = 1.375$, kw = 5[1]

[1]*Beatty et al., IEEE TMI 2005; 24: 799-808*
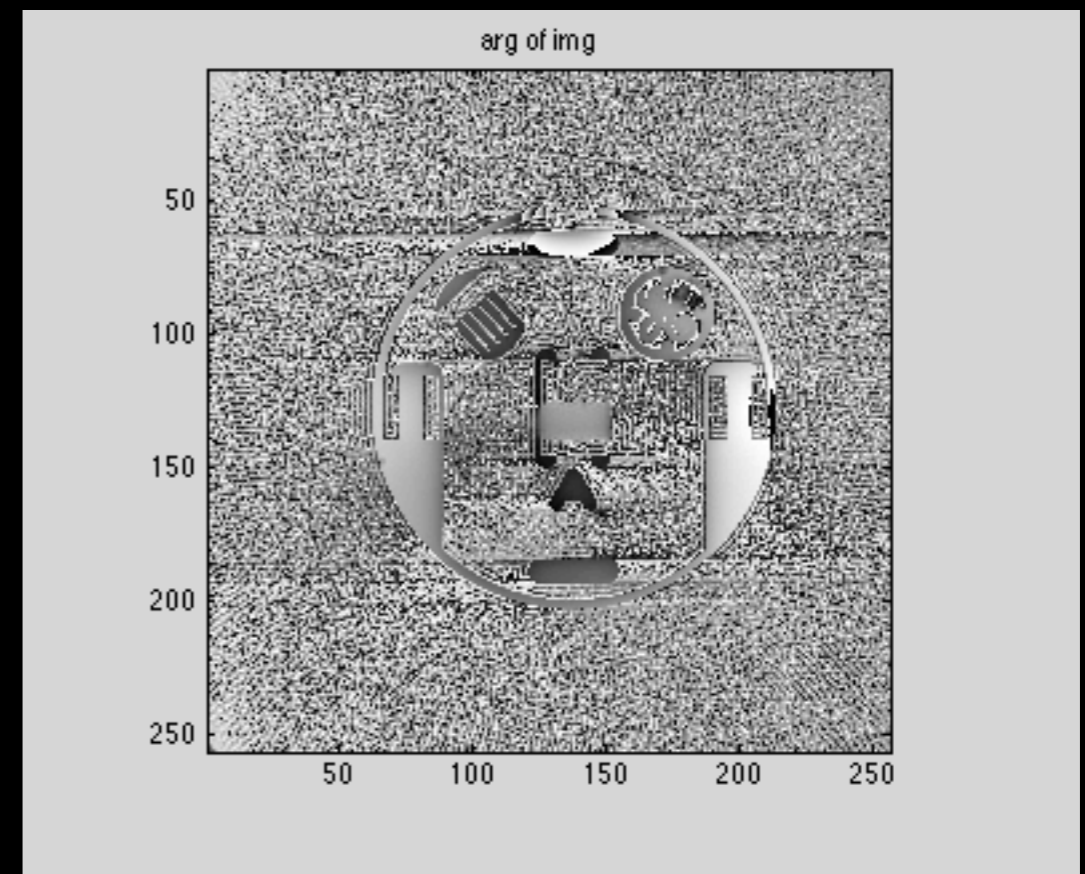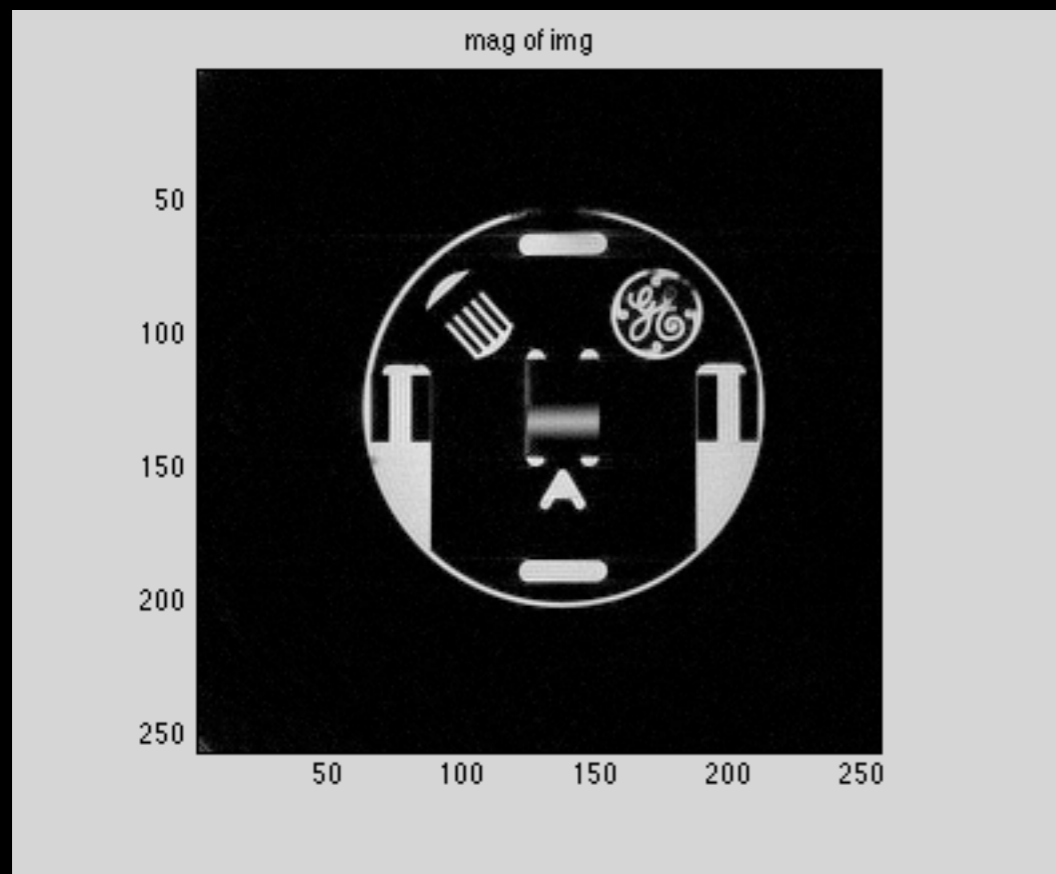
Gridding: Summary

- Data input
  - k-space data
  - k-space traj (usually normalized), DCF

- Gridding params
  - target image dimensions [MxN]
  - grid oversampling factor α
  - kernel type and width

- Data output
  - gridded Cartesian k-space
  - reconstructed image

Gradient Measurement

- Non-Cartesian recon requires
  - k-space trajectory
  - density compensation function

- Both depend on actual gradient waveforms on scanner
  - can deviate from desired

- Knowledge of k-space trajectory also important for RF design

Gradient Measurement

- Gradient imperfections cause artifacts
  - FOV scaling, shifting
  - signal loss, shading
  - image blurring, geometric distortion

- Sources of gradient errors
  - eddy currents (B_0, linear)
  - group delays (RF filters, A/D)
  - amplifier limitations (BW, freq response)
  - gradient warping
  - other ...

Gradient Measurement

- General techniques
  - off-iso slice technique[1,2], and more

- Trajectory-specific techniques
  - radial[3], spiral[4], and more

- Characterize gradient system
  - assume linear time-invariant model[5]

1 Duyn JH et al., JMR 1998; 132: 150-153

2 Beaumont M et al., MRM 2007; 58: 200-205

3 Peters DC et al., MRM 2003; 50: 1-6

4 Robison RK et al., MRM 2010; 63: 1683-90

5 Addy NO et al., MRM 2012; 68: 120-129

Gradient Measurement

Off-isocenter slice measurement technique
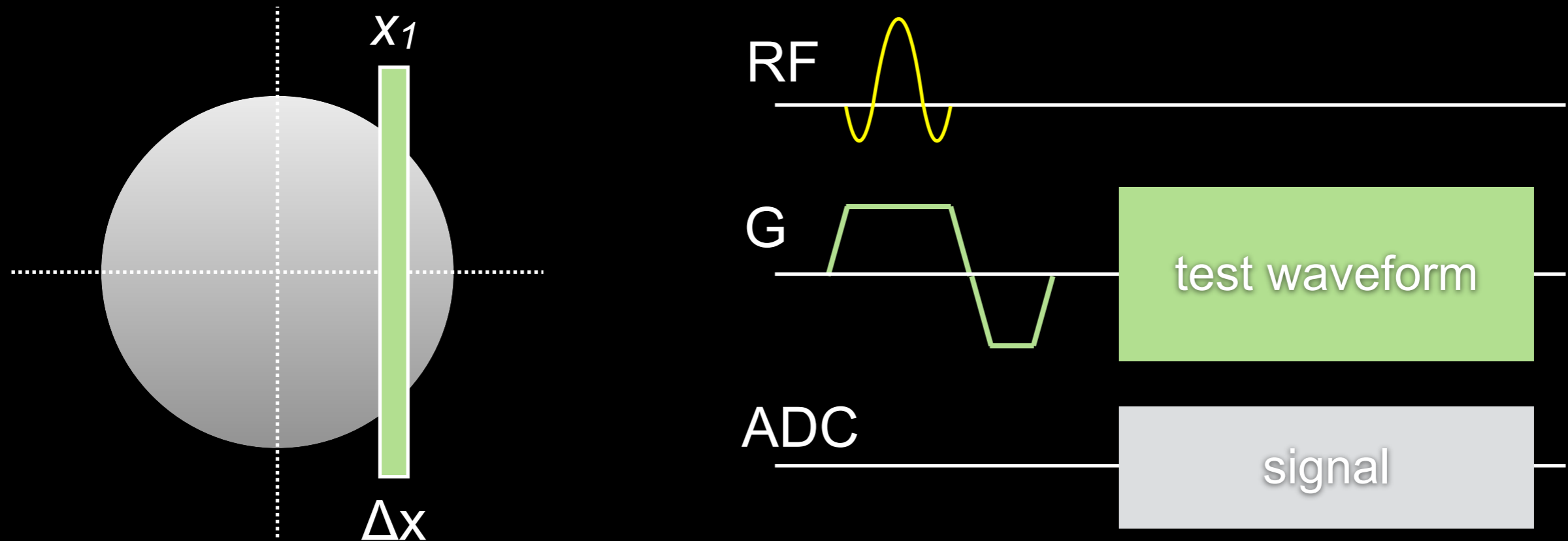
Can repeat on all three axes G_x, G_y, G_z

Duyn JH et al., JMR 1998; 132: 150-153

Gradient Measurement

Off-isocenter slice measurement technique

Waveform ON:

$$s_{x1,Gon}(t) = \iint\limits_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1,y,z,t)} \cdot e^{-i2\pi \cdot [\frac{\gamma}{2\pi} \int_0^t G(\tau)\mathrm{d}\tau] \cdot x_1} \, \mathrm{d}y \, \mathrm{d}z$$

Waveform OFF:

$$s_{x1,Goff}(t) = \iint\limits_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1,y,z,t)} \, \mathrm{d}y \, \mathrm{d}z$$

Phase difference:

$$\Delta\phi_{x1}(t) = \gamma \int_0^t G(\tau) \cdot x_1 \, \mathrm{d}\tau = x_1 \cdot k(t)$$

Gradient Measurement
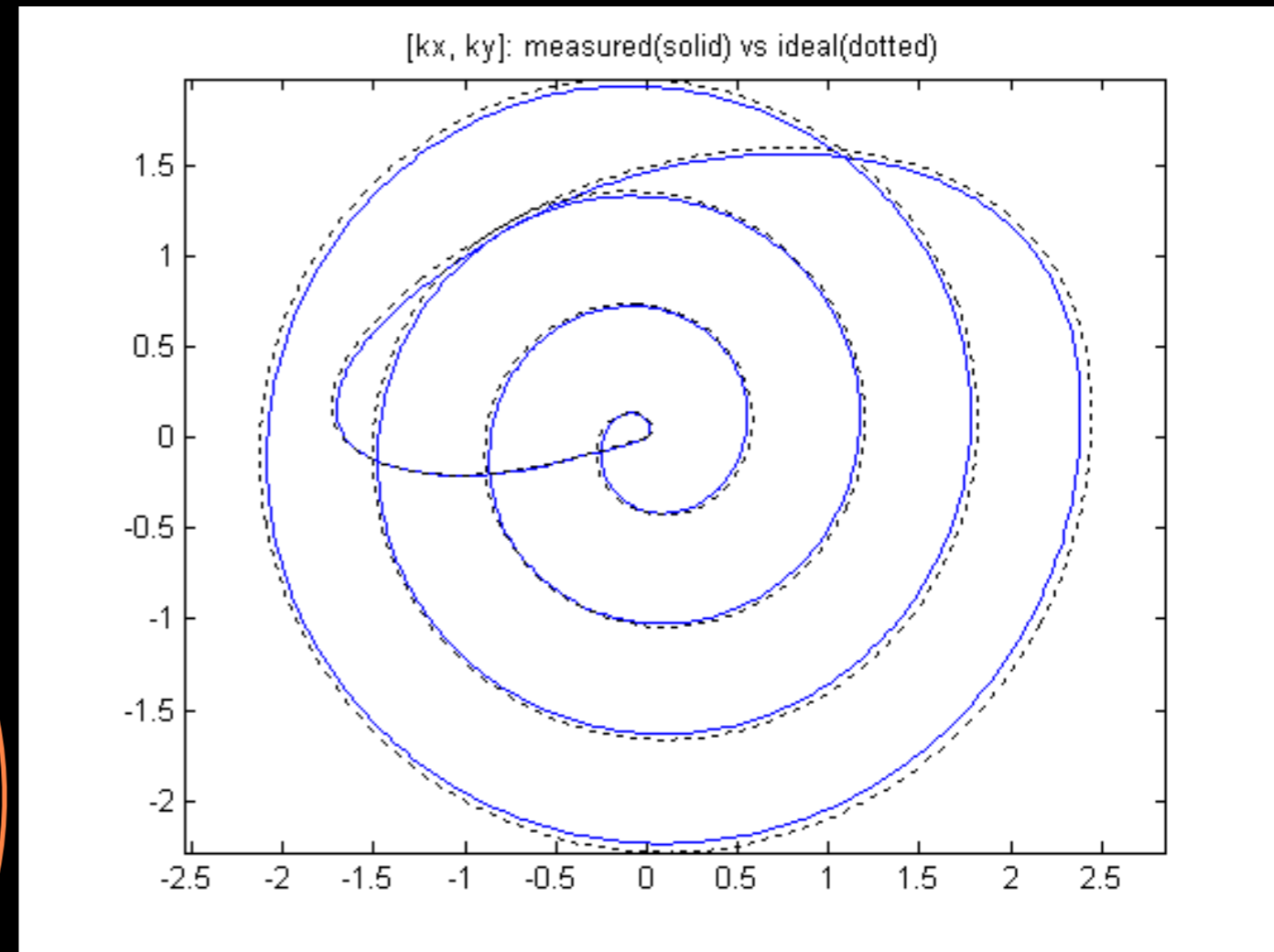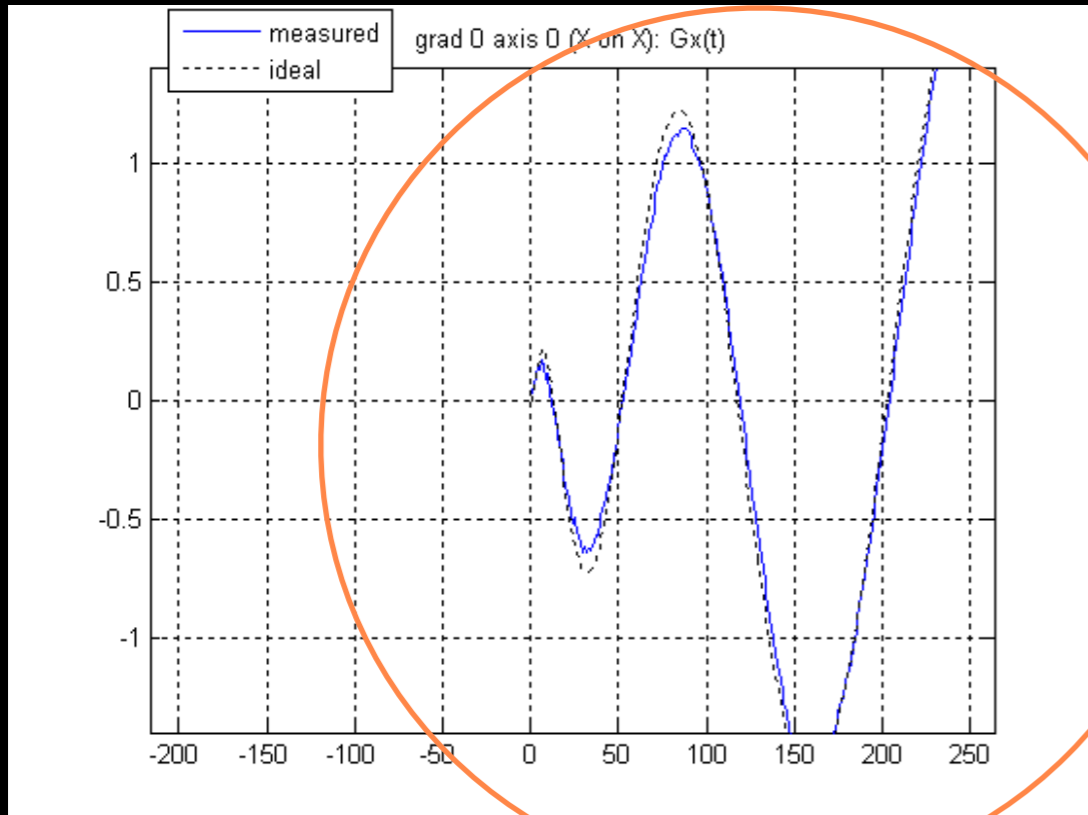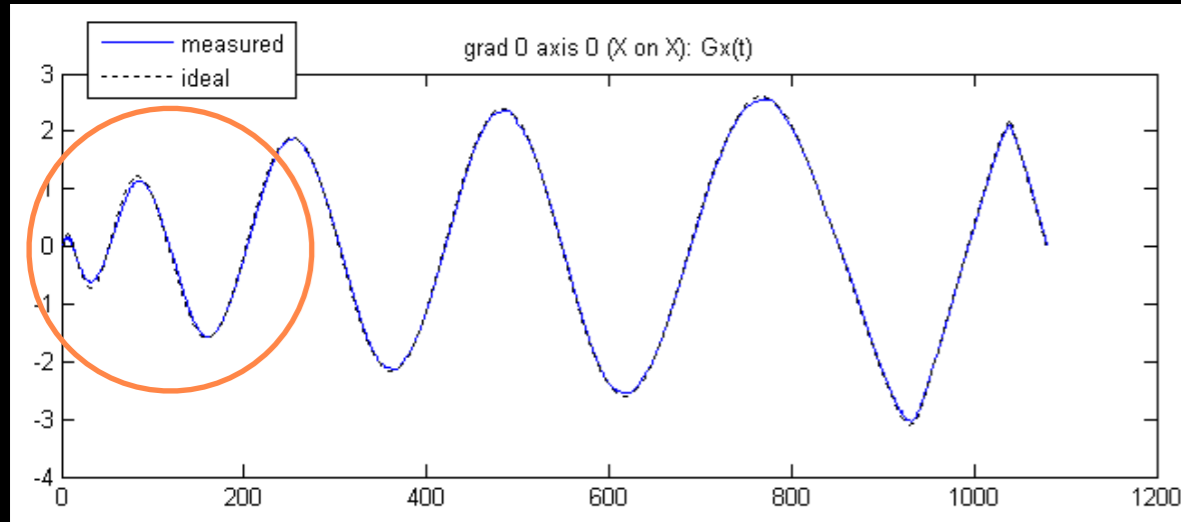
Gradient Measurement

- Gradient (trajectory) correction
  - use actual trajectory for recon
  - pre-tune bulk gradient delay

Example: Axial Spiral at 1.5 T

Nominal Trajectory | Calculated Trajectory | Difference (x8)

Addy NO et al., MRM 2012; 68: 120-129
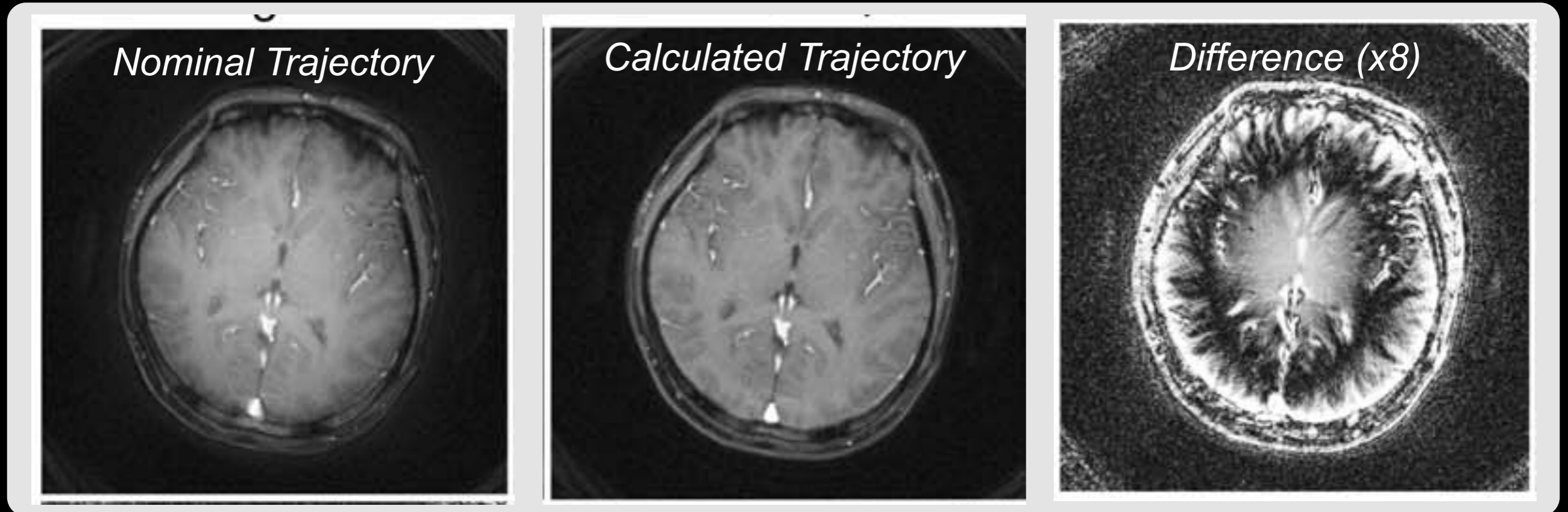
Gradient Measurement

- Off-iso slice measurement technique
  - two measurements per axis
  - can measure X on X, Y on Y, Z on Z, and also cross terms; linearly combine
  - Δx should be small (may need avging)
  - need to account for phase wrapping
  - use spin echo for long waveforms
  - can acquire multiple slice offsets and gradient polarities to model individual gradient error terms

Gradient Measurement

- Delay calibration
  - gradient errors (e.g., linear eddy currents) mainly cause an apparent bulk delay
  - adjust ADC window w.r.t. gradients
  - delays may be different for each axis

Off-resonance Correction

- Off resonance effects (ΔB_0, fat, etc.)

$$s(t) = \iint\limits_{X,Y} m(x,y) \cdot e^{-i\phi(x,y,t)} \cdot e^{-i2\pi \cdot [k_x(t)\,x + k_y(t)\,y]} \,\mathrm{d}x\,\mathrm{d}y$$

$$\phi(x,y,t) = 2\pi\psi(x,y)t$$

- patient (scan) dependent
- pre-scan shim calibration helps
- usually negligible for Cartesian MRI
- non-Cartesian MRI: signal loss, spatial blurring, geometric distortion

Off-resonance Correction

Effects of off-res for concentric rings: PSF blurring
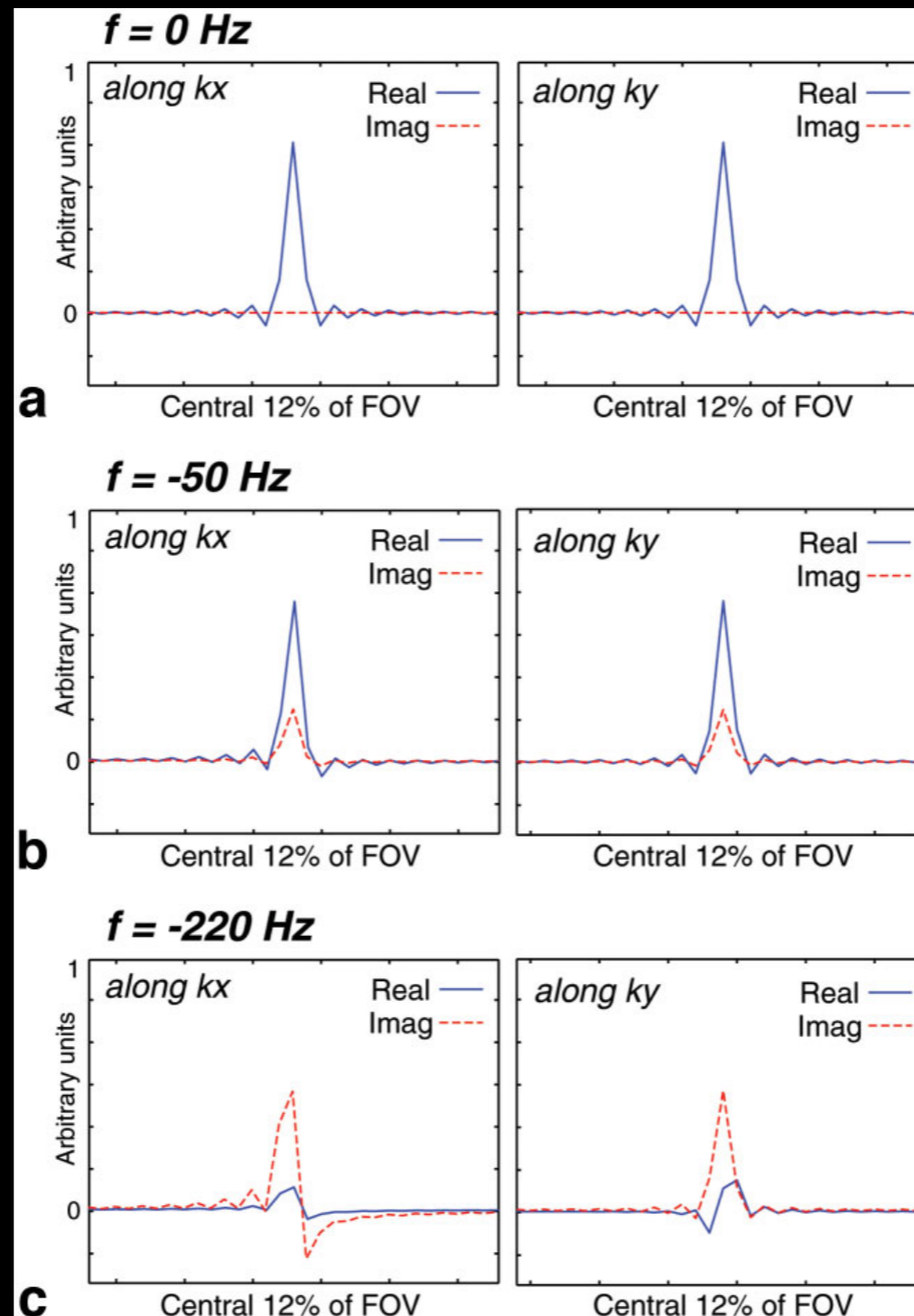
Off-resonance Correction

- Account for field inhomogeneity
  - use shorter readouts
  - measure/estimate field map

$$s(\mathrm{TE}_1) \longrightarrow I_1 = M'(x,y) \cdot e^{-i2\pi\psi(x,y)\mathrm{TE}_1}$$

$$s(\mathrm{TE}_2) \longrightarrow I_2 = M'(x,y) \cdot e^{-i2\pi\psi(x,y)\mathrm{TE}_2}$$

$$\hat{\psi}(x,y) = \arg(I_1 \cdot I_2^*)/2\pi(\Delta\mathrm{TE}) \quad [\pm 1/2\pi\Delta\mathrm{TE}]$$

and then correct (during recon)[1,2,3]
time-segmented, freq-segmented, etc.

1 Noll DC et al., IEEE TMI 1991; 10: 629-637

2 Noll DC et al., MRM 1992; 25: 319-333   *3 Chen JY et al., MRM 2011; 66: 390-401*

Off-resonance Correction

Linear Correction

$$\psi(x,y) = f_0 + f_x x + f_y y \quad \text{(can fit to this model)}$$

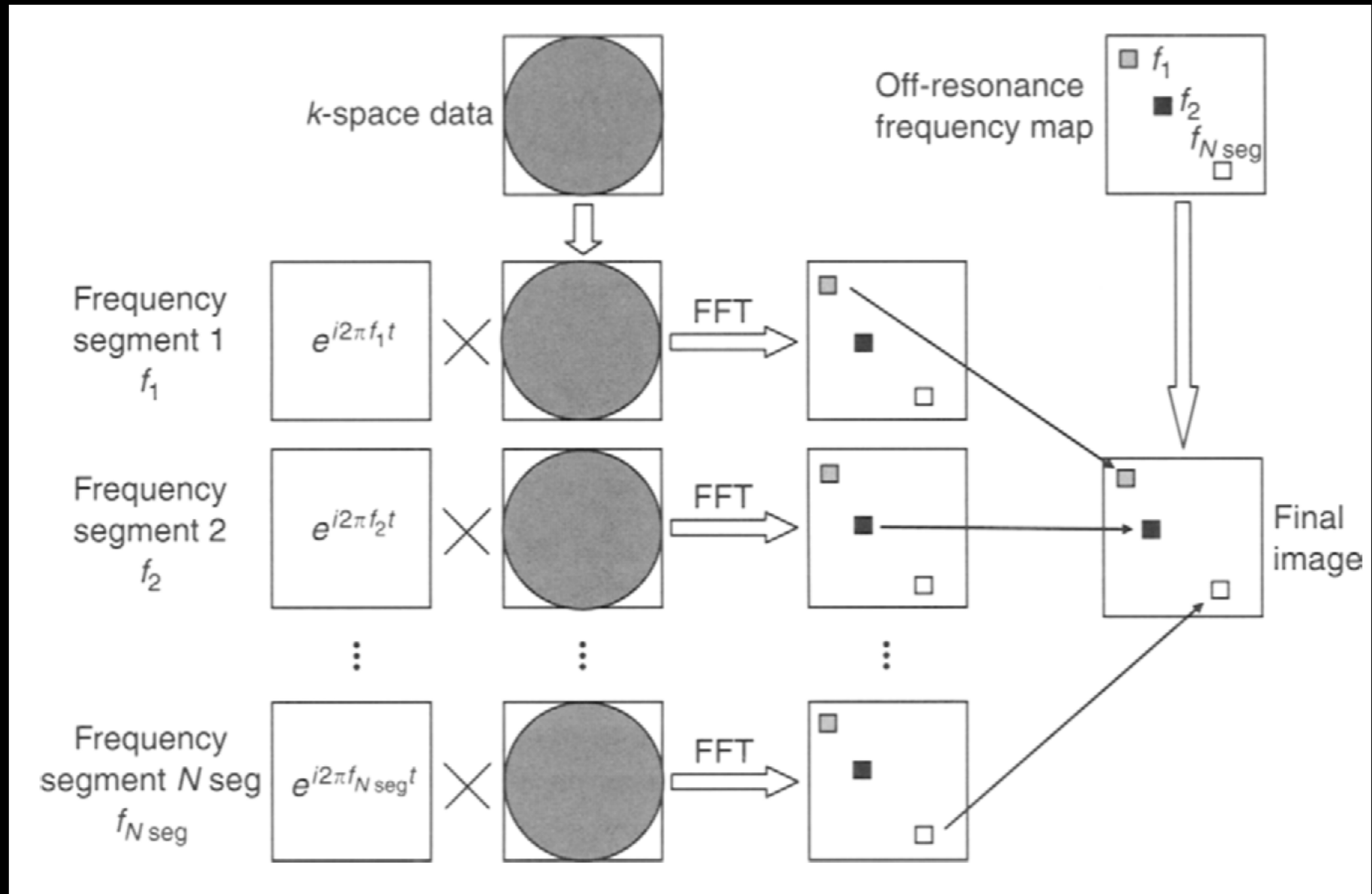$$\phi(x,y) = 2\pi f_0 t + 2\pi \Delta k_x(t) x + 2\pi \Delta k_y(t) y$$

$$\Delta k_x(t) = f_x t, \quad \Delta k_y(t) = f_y t$$

$$s(t) = \underbrace{e^{-i2\pi f_0 t}}_{demod} \iint\limits_{X,Y} m(x,y) \cdot e^{-i2\pi \cdot [\underline{(k_x(t)+\Delta k_x(t))\,x} + \underline{(k_y(t)+\Delta k_y(t))\,y}]} \,\mathrm{d}x\,\mathrm{d}y$$

shift k-space trajectory

Can follow with frequency-segmented off-res correction
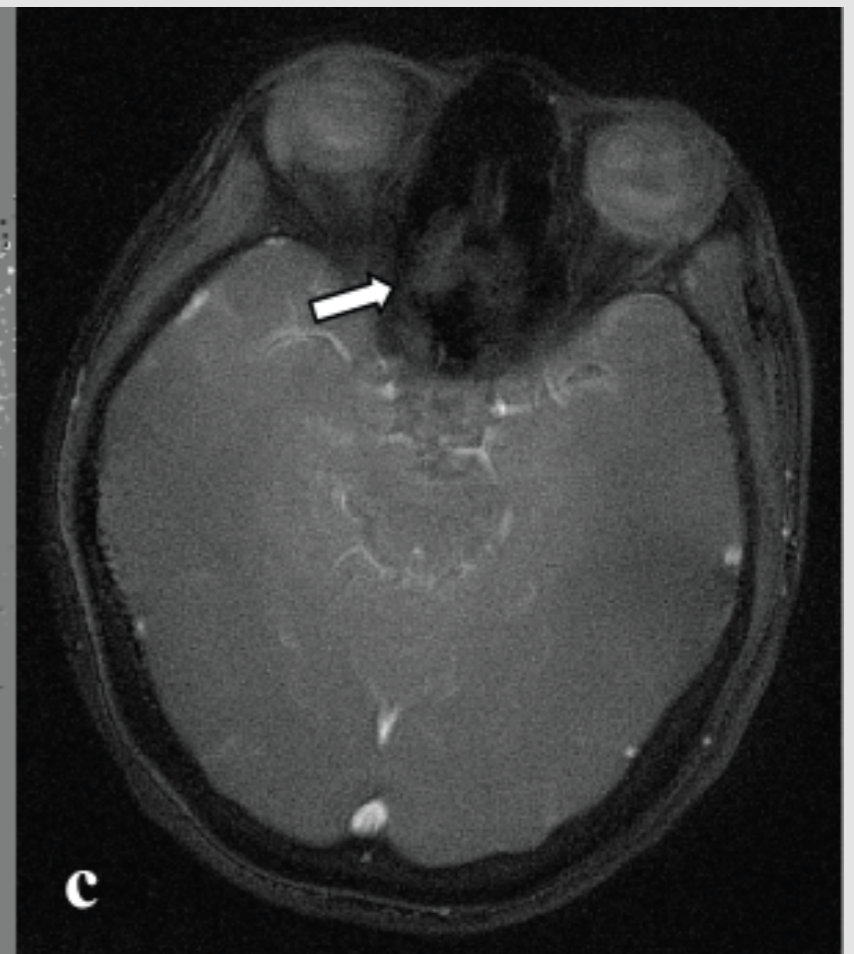
Off-resonance Correction
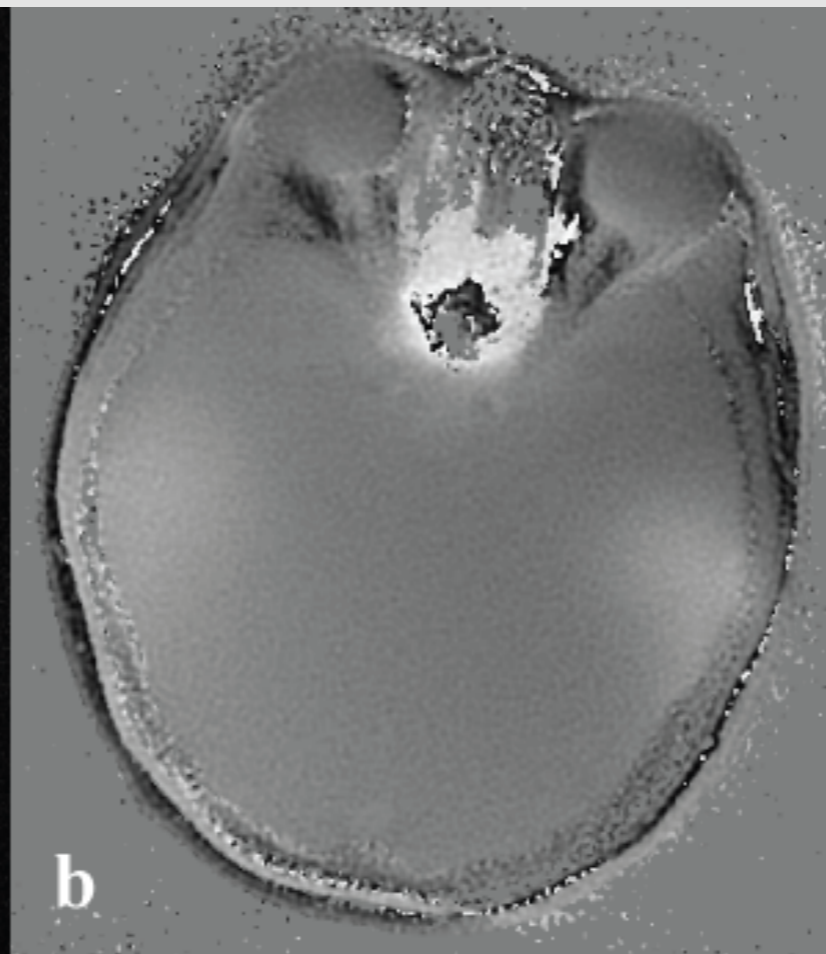
Frequency-segmented correction

Bernstein et al., Handbook of MRI Sequences, Fig. 17.63

Off-resonance Correction

Example: Axial Concentric Rings at 1.5 T

Regular Recon   Field Map   ORC Image

Wu HH et al., MRM 2008; 59: 102-112

Off-resonance Correction

- Field map measurement

- Segmented correction methods
  - Need to recon multiple images,
    $N_{bins} \sim 4(f_{max} - f_{min})T_{acq}$

- Other sources of off resonance
  - concomitant gradients
  - chemical shift (*e.g.,* fat)

- Other ORC algorithms
  - autofocusing (field map optional)
  - combine with image reconstruction

Thanks!

- Further reading
  - references on each slide
  - further reading section on website

- Acknowledgments
  - John Pauly's EE369C class notes (Stanford)

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