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# Pulse Sequences: Rapid Gradient Echo

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M229 Advanced Topics in MRI

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2023.04.06

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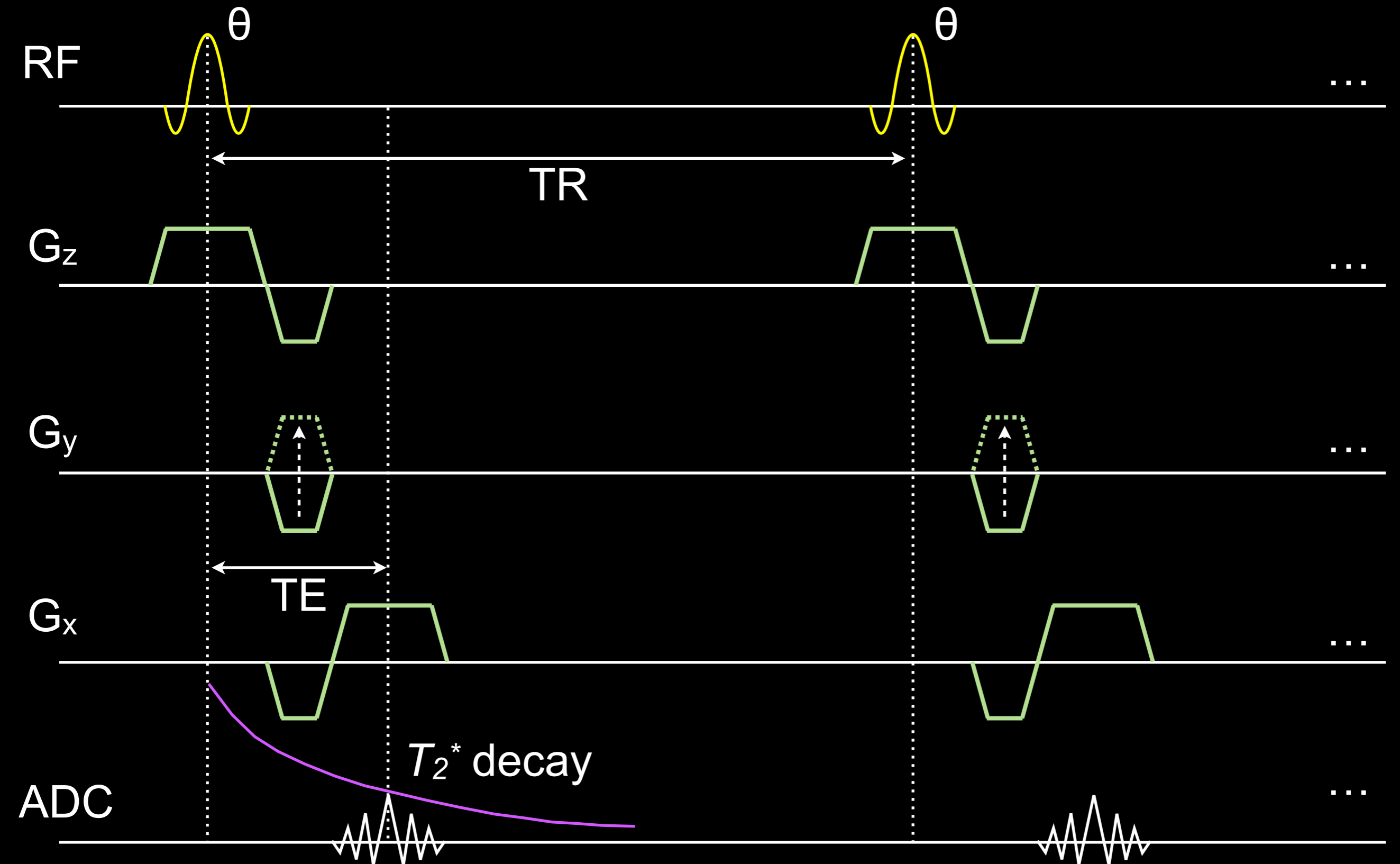
# Class Business

- Textbook, email list, and class website
- Office hours
  - Holden: Fri 10-11 am; appointment
  - Shu-Fu: Mon 2-4 pm; appointment
  - Timo: TBD
  - Email beforehand would be helpful
- Follow Brian's Bloch sim tutorial
  - <http://www-mrsrl.stanford.edu/~brian/bloch/>
  - *(Archived version to be emailed.)*

# Outline

- Gradient Echo (GRE)
- Rapid Gradient Echo
  - Balanced SSFP
  - Gradient-spoiled GRE
  - RF-spoiled GRE
- Comparison
- Extensions and Variations
- Applications

# Gradient Echo



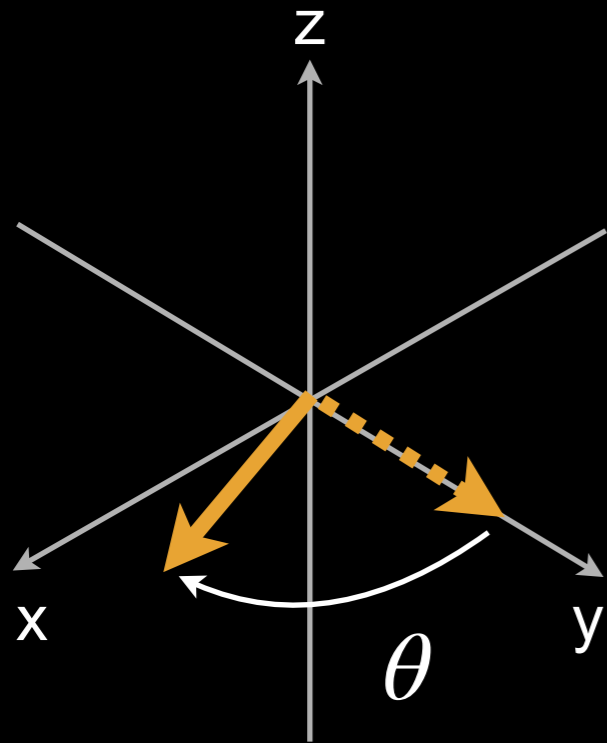
# Gradient Echo

- Gradient reversal on the readout axis forms the echo (vs. RF spin echo)
- A.k.a. gradient-recalled echo, gradient-refocused echo, field echo
- Flip angle  $\theta$  typically  $< 90^\circ$
- $M_{xy}$  has  $T_2^*$  instead of  $T_2$  decay
- Advantageous for fast 3D imaging

# Gradient Echo

- Basic steps
  - RF excitation (flip angle  $\theta$  and phase  $\phi$ )
  - Free precession (from  $G$  and  $\Delta B$ )
  - $T_1$  and  $T_2$  (or  $T_2^*$ ) relaxation
- Steady state
  - “Dynamic equilibrium”
  - Established after initial transient state
  - $M_z$  and  $M_{xy}$  remain the same, TR to TR
  - Need to meet certain conditions

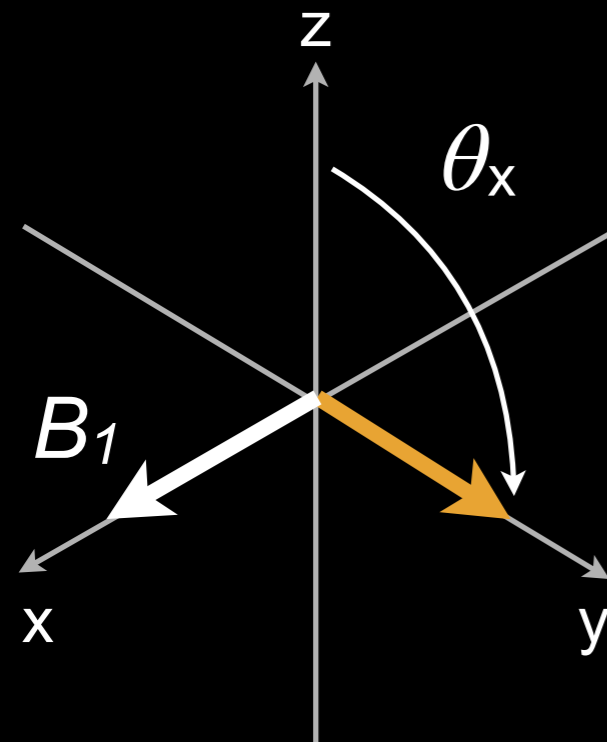
# Notations



Free Precession (FP)

$$R_z(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Excitation



$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

rotating frame at  $\omega_0$

# Notations

Relaxation + Free Precession:

$$M(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix} R_z(\Delta\omega t) M(0) + \begin{bmatrix} 0 \\ 0 \\ M_0(1 - e^{-t/T_1}) \end{bmatrix}$$

$AM(0) + B$



# Gradient Echo

- When  $TR > 5 \cdot T_2^*$ ,  $M_{xy}$  naturally “spoiled”
  - *i.e.*, consider  $M_{xy} = 0$  at the end of each TR

# Gradient Echo

Steady-state signal equation:

$$M_{xy,ss}(\text{TE}) = \frac{M_0 \sin \theta (1 - E_1)}{1 - \cos \theta E_1} e^{-\text{TE}/T_2^*}$$

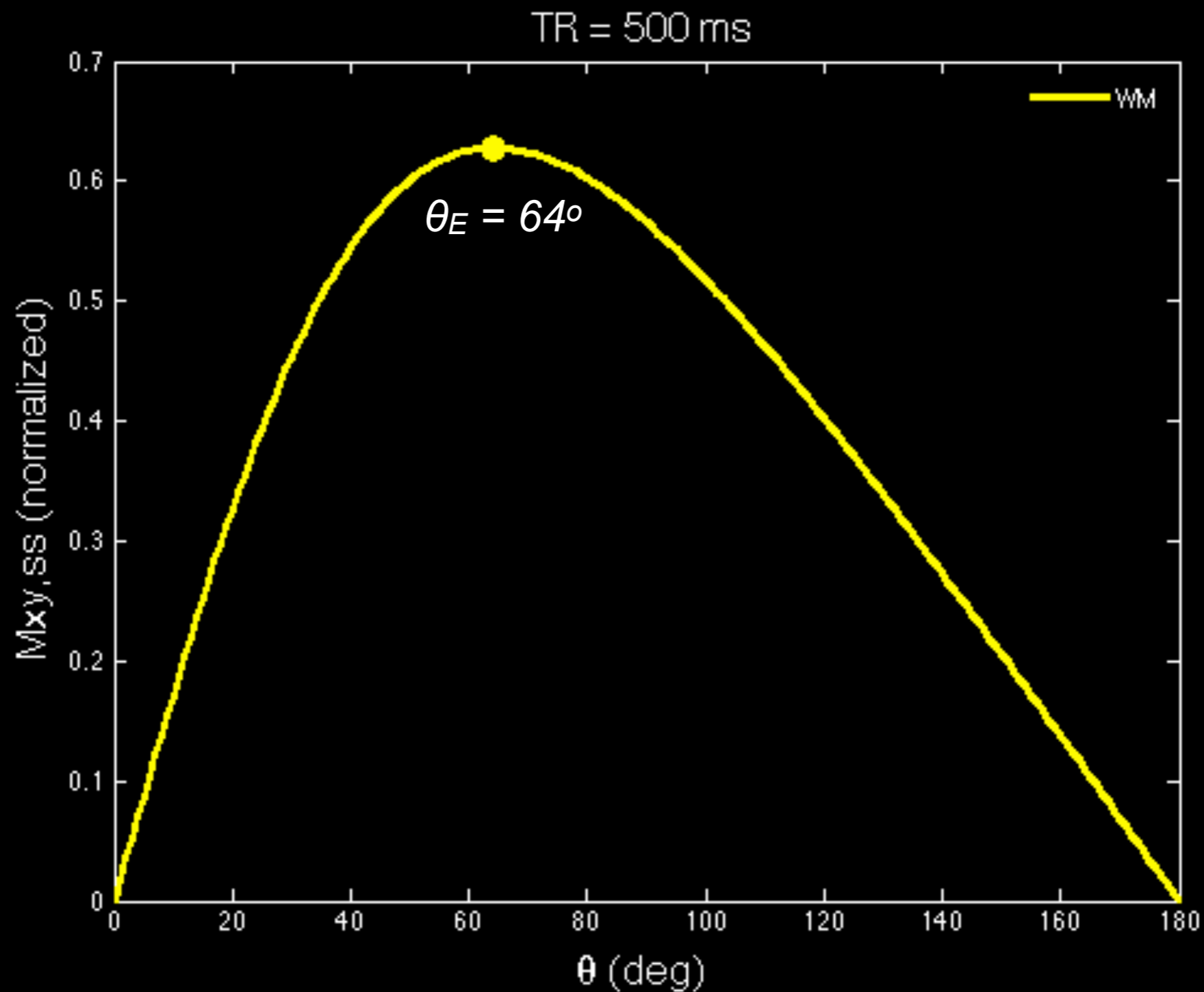
Ernst angle:

$$\theta_E = \cos^{-1}(E_1)$$

$$E_1 = e^{-\text{TR}/T_1}$$

# Gradient Echo

Ernst angle:



WM  $T_1 = 600$  ms,  $T_2 = 80$  ms

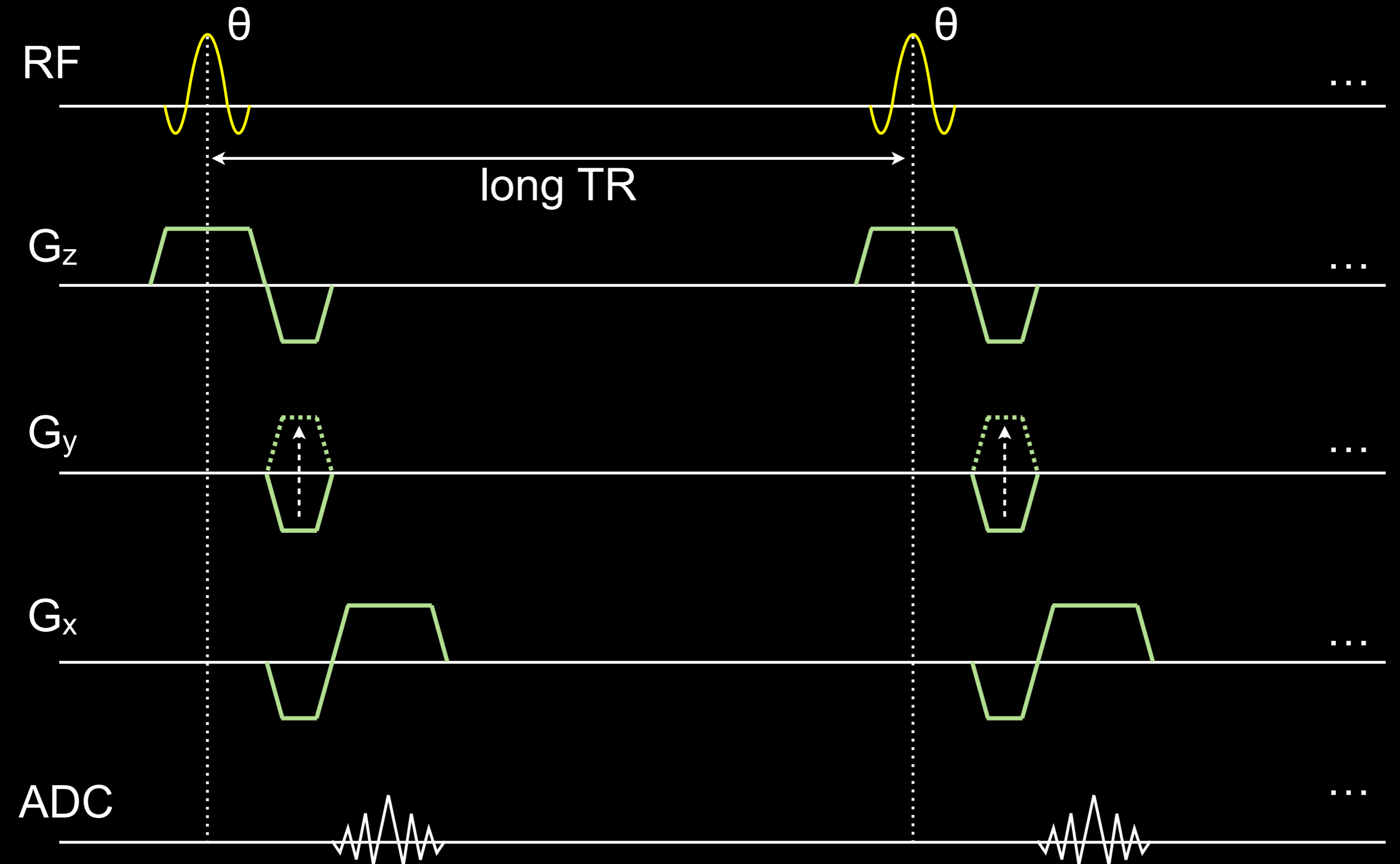
# Gradient Echo

- $T_1$ -weighted image contrast
  - $M_{xy}$  gone at end of each TR
  - TE controls  $T_2^*$  weighting
- Typical  $T_2^* \sim 50$  ms
  - need TR  $\sim 250$  ms for “natural” spoiling
- Reduce TR and maintain T1w contrast?
  - rapid GRE with appropriate spoiling

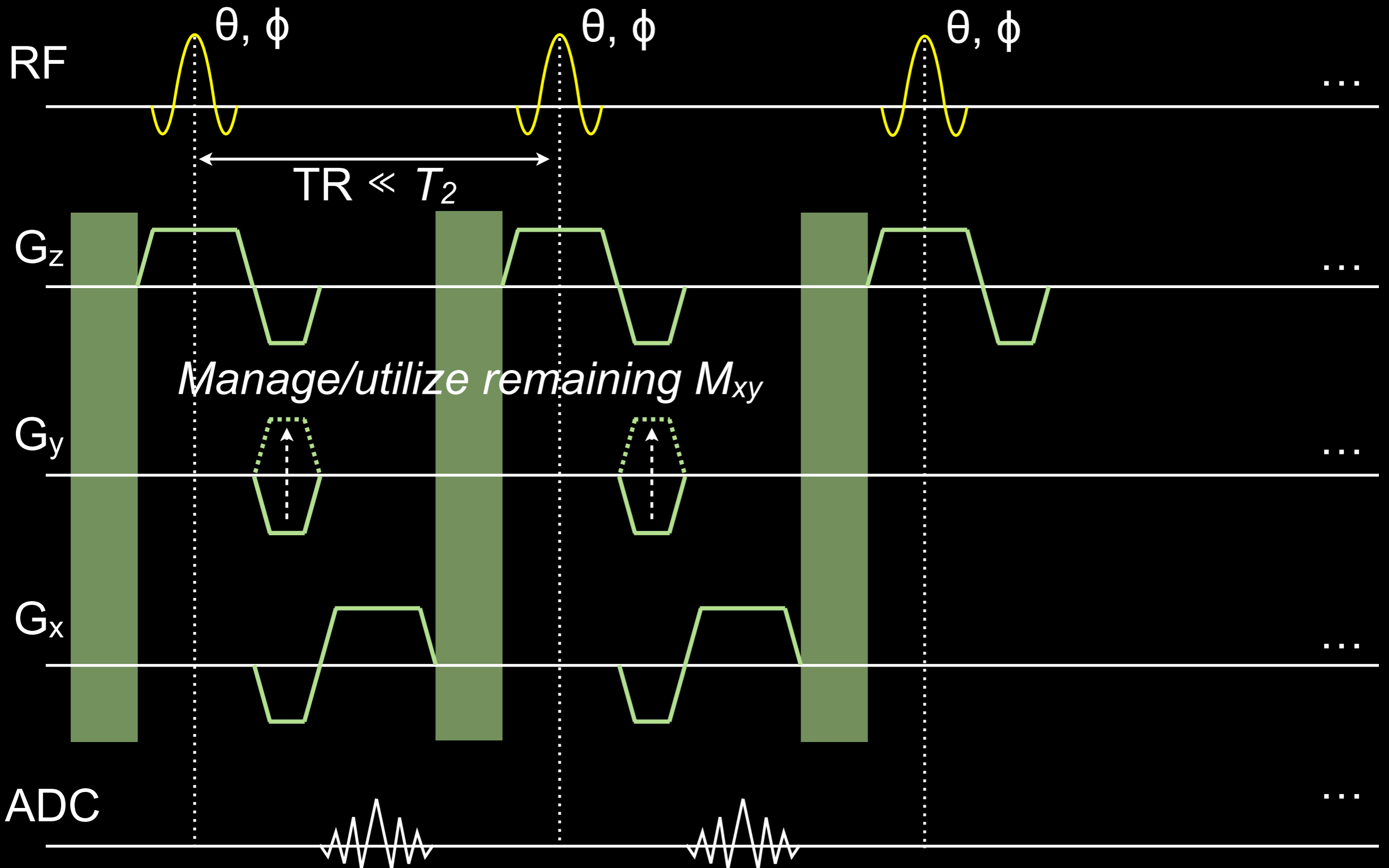
# Rapid Gradient Echo

- Rapid imaging with  $TR \ll T_2 < T_1$
- Steady state
  - Involves a mixture of  $M_z$  and  $M_{xy}$
  - Necessary and sufficient conditions:
    1. Constant RF flip angle  $\theta$
    2. Constant TR
    3. Constant dephasing  $\beta$  between RF pulses
    4. RF phase  $\phi_n = a + bn + cn^2$

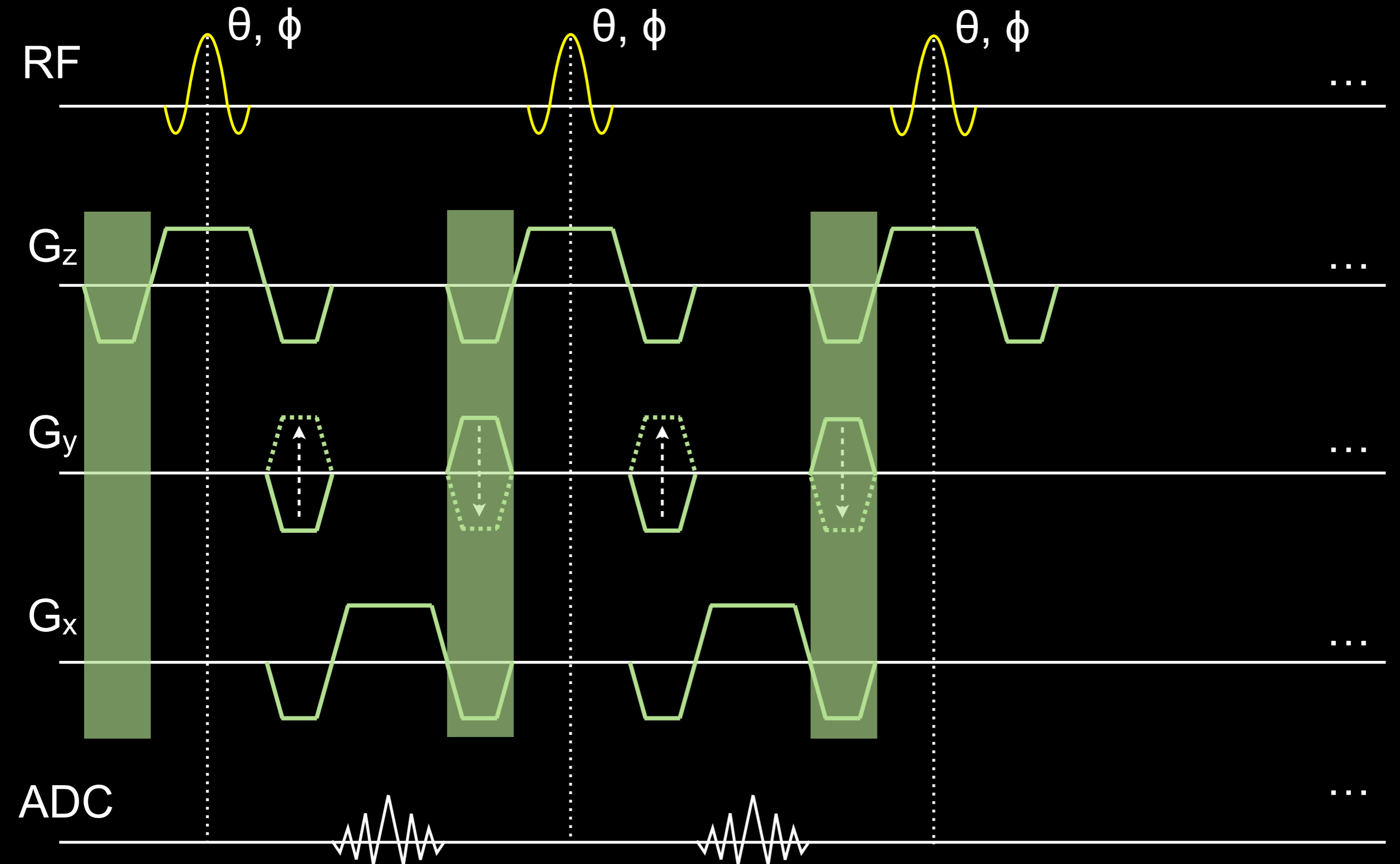
# Gradient Echo



# Rapid Gradient Echo

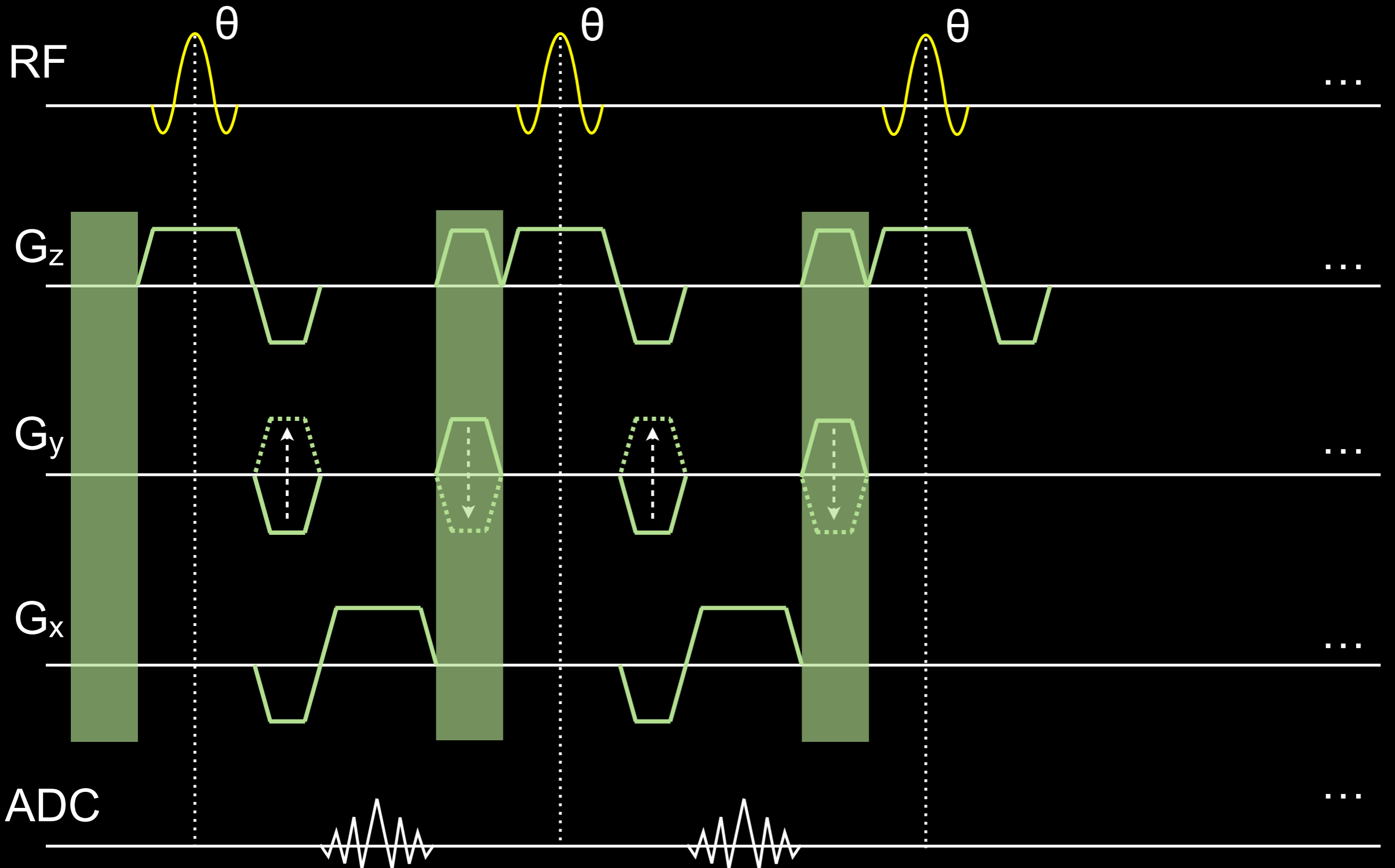


# Balanced SSFP

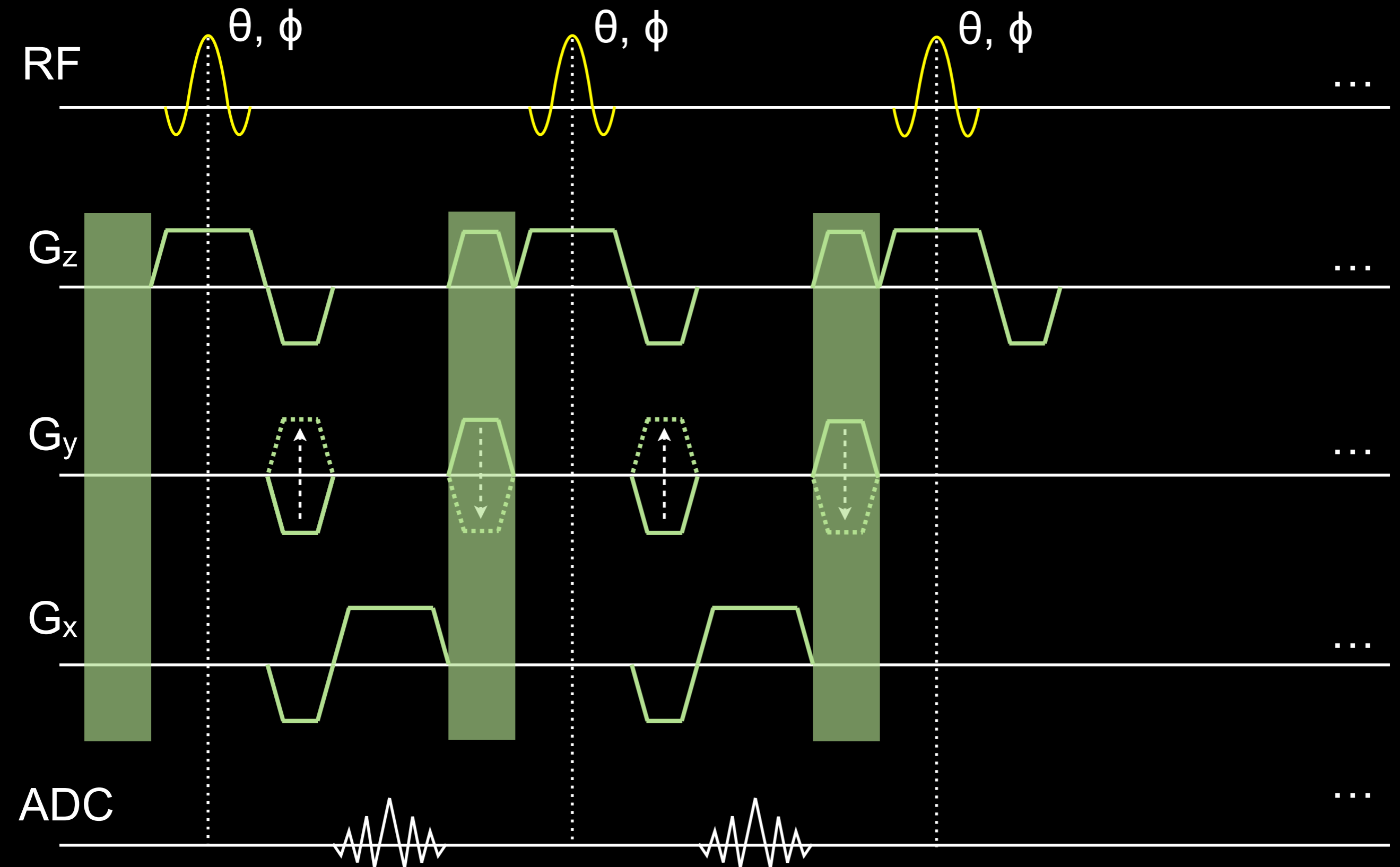




# Gradient-spoiled GRE



# Gradient & RF-spoiled GRE

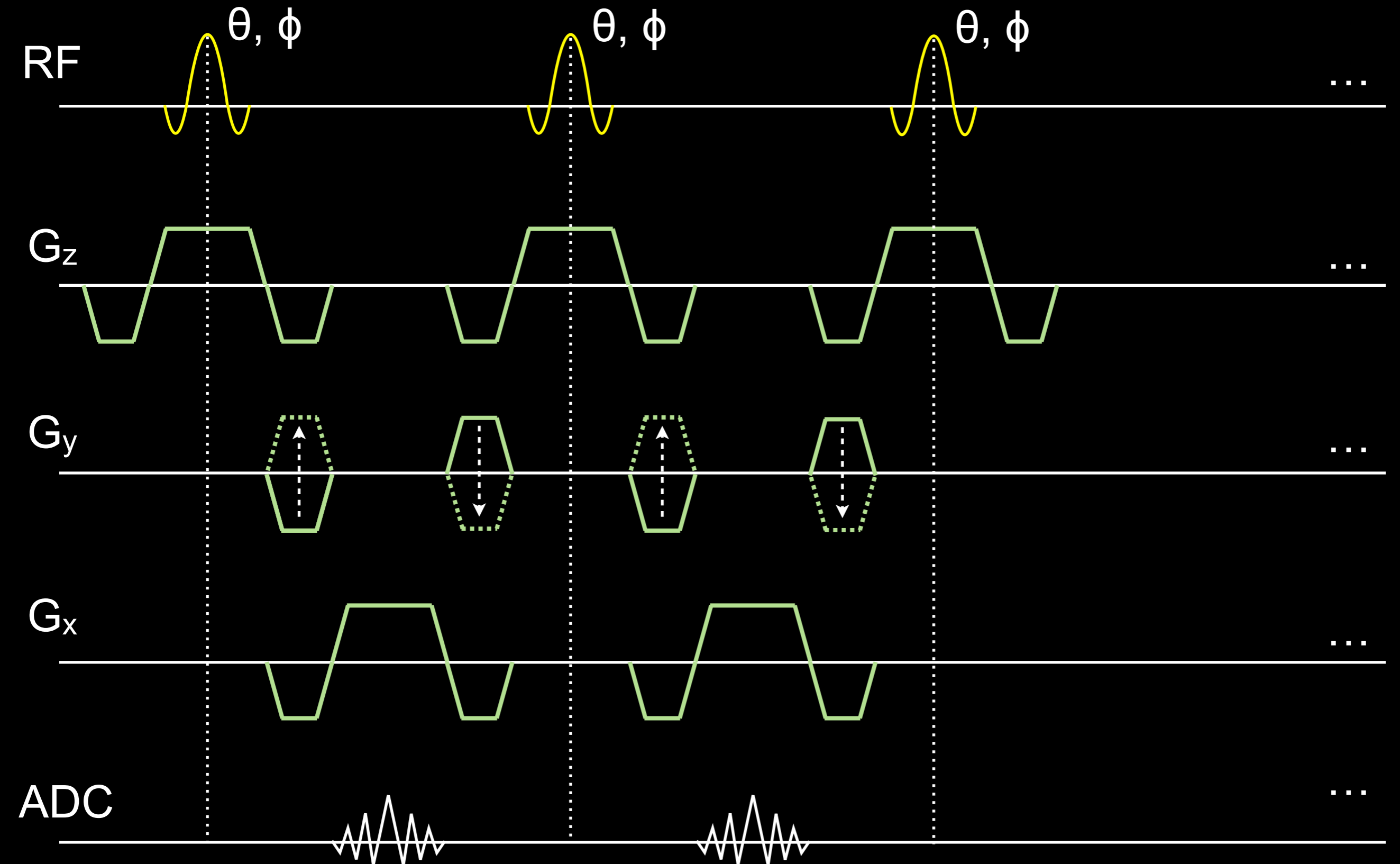


# Rapid Gradient Echo

General terminology		Siemens	GE	Philips
Balanced SSFP	bSSFP	TrueFISP	FIESTA	Balanced FFE
Gradient-spoiled GRE	SSFP-FID	FISP	GRASS	FFE
	SSFP-Echo	PSIF	SSFP	T2-FFE
Gradient and RF-spoiled GRE	Spoiled GRE	FLASH	SPGR	T1-FFE

*cf. Table 14.1, Handbook of MRI Pulse Sequences  
cf. "MRI Acronyms", Siemens Healthcare*

# Balanced SSFP



# Balanced SSFP

- All gradients are balanced
  - $\beta$  from  $G_x, G_y, G_z = 0$
  - $\beta$  only comes from  $\Delta B$
- Typically use  $\phi_n = n \cdot \pi$  ( $\Delta\phi = \pi$ )
- Typically use  $TE = TR/2$ 
  - $M_{xy}$  actually has  $T_2$  (not  $T_2^*$ ) decay<sup>1</sup>
- Contrast depends on  $T_1$  and  $T_2$

<sup>1</sup>Ganter C, MRM 2006; 56:687-691

# Balanced SSFP

Steady-state signal equation ( $\beta = 0$ ):

$$M_{xy,ss}(\text{TE}) = M_0 \sin \theta \frac{1 - E_1}{1 - (E_1 - E_2) \cos \theta - E_1 E_2} \sqrt{E_2}$$

$$E_1 = e^{-\text{TR}/T_1}$$

$$E_2 = e^{-\text{TR}/T_2}$$

$$\sqrt{E_2} = e^{-\text{TE}/T_2}$$

# Balanced SSFP

Steady-state signal equation ( $\beta = 0$ ):

If TR (3-5 ms)  $\ll T_2$ ,  $E_1 \sim 1 - TR/T_1$  and  $E_2 \sim 1 - TR/T_2$ :

$$M_{xy,ss}(\text{TE}) = \frac{M_0 \sin \theta}{(T_1/T_2)(1 - \cos \theta) + (1 + \cos \theta)} \sqrt{E_2}$$

$T_2/T_1$  contrast weighting

$$\theta_{max} = \arccos\left(\frac{T_1 - T_2}{T_1 + T_2}\right) \quad M_{xy,ss}(\theta_{max}) \sim \frac{M_0}{2} \sqrt{\frac{T_2}{T_1}}$$

When  $T_1 = T_2$ ,  $\theta_{max} = 90^\circ$ ,  $M_{xy,ss} \sim 0.5 M_0$  !

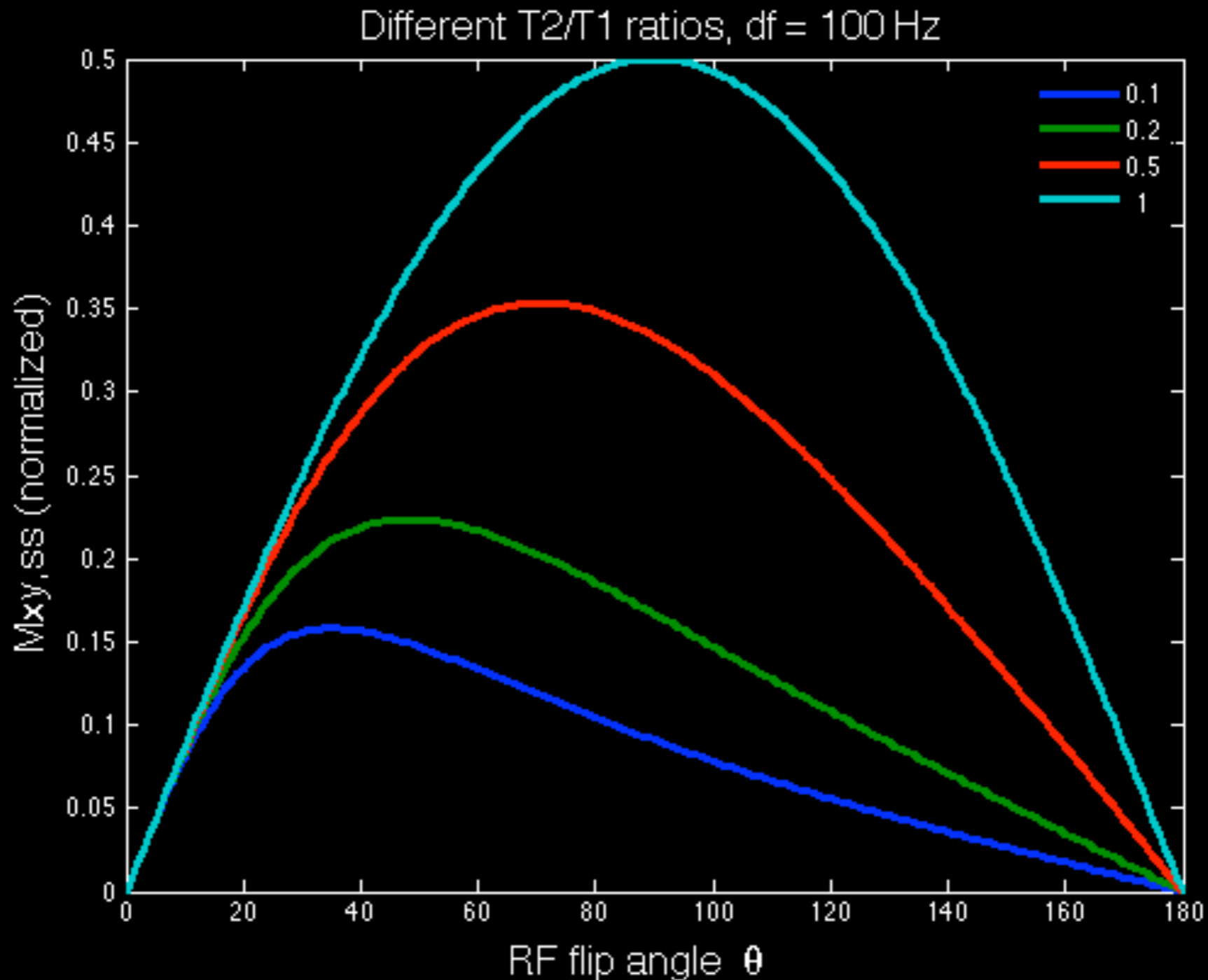
# Balanced SSFP

SS signal as a function of flip angle:

$TR = 5 \text{ ms}$

$\Delta\phi = 0$

$\beta = \pi$



$T_1 = 1000 \text{ ms}, T_2 = 100, 200, 500, 1000 \text{ ms}$

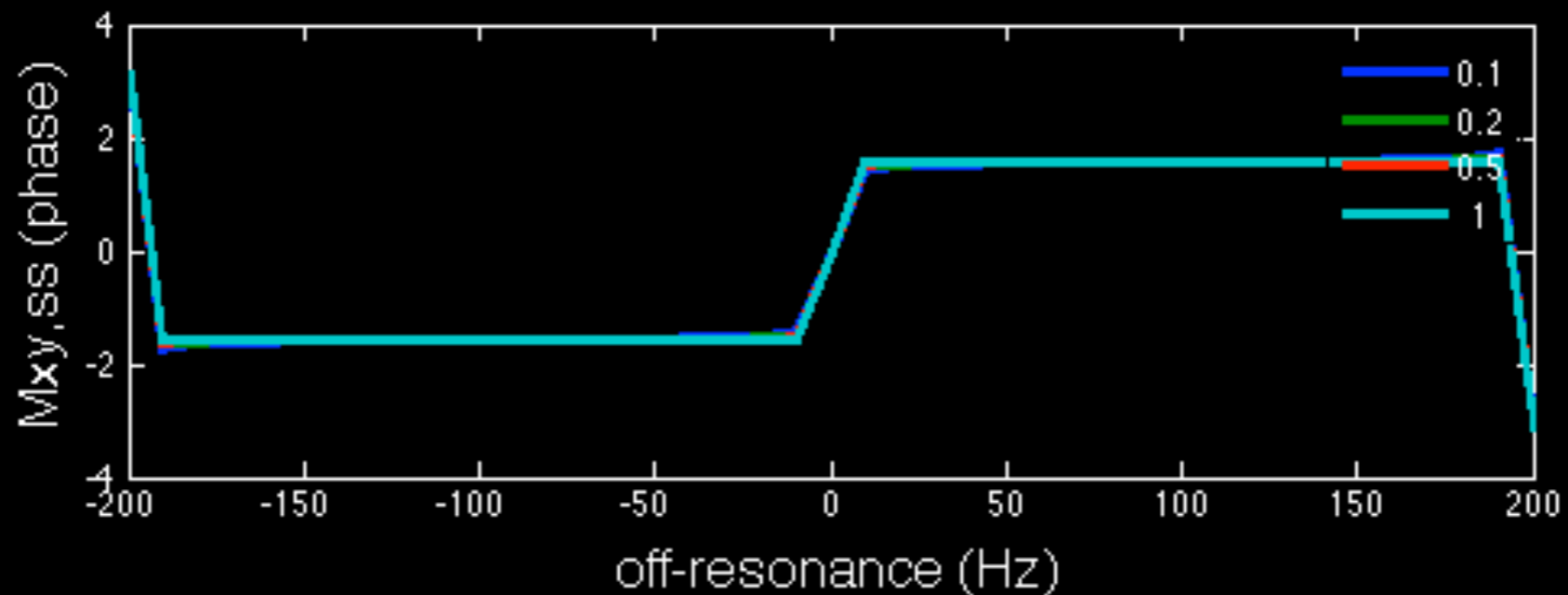
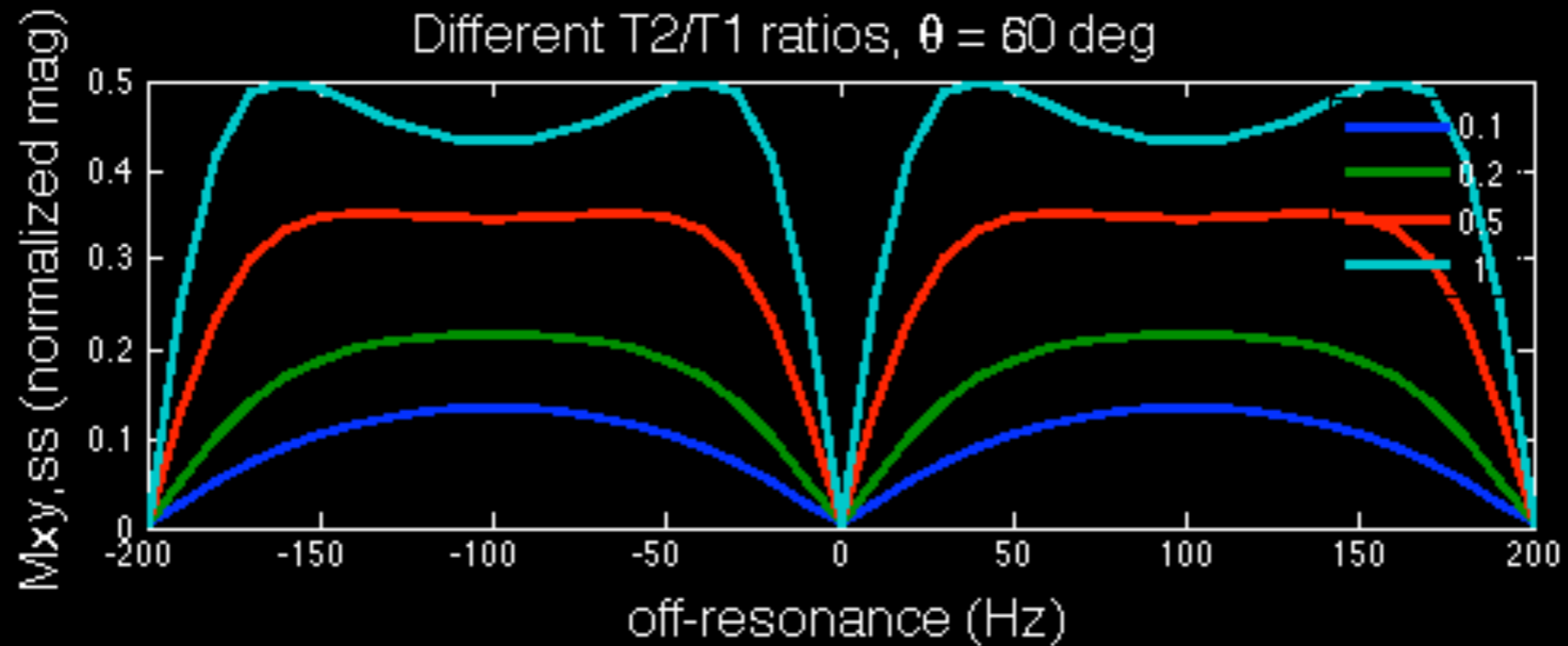


# Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



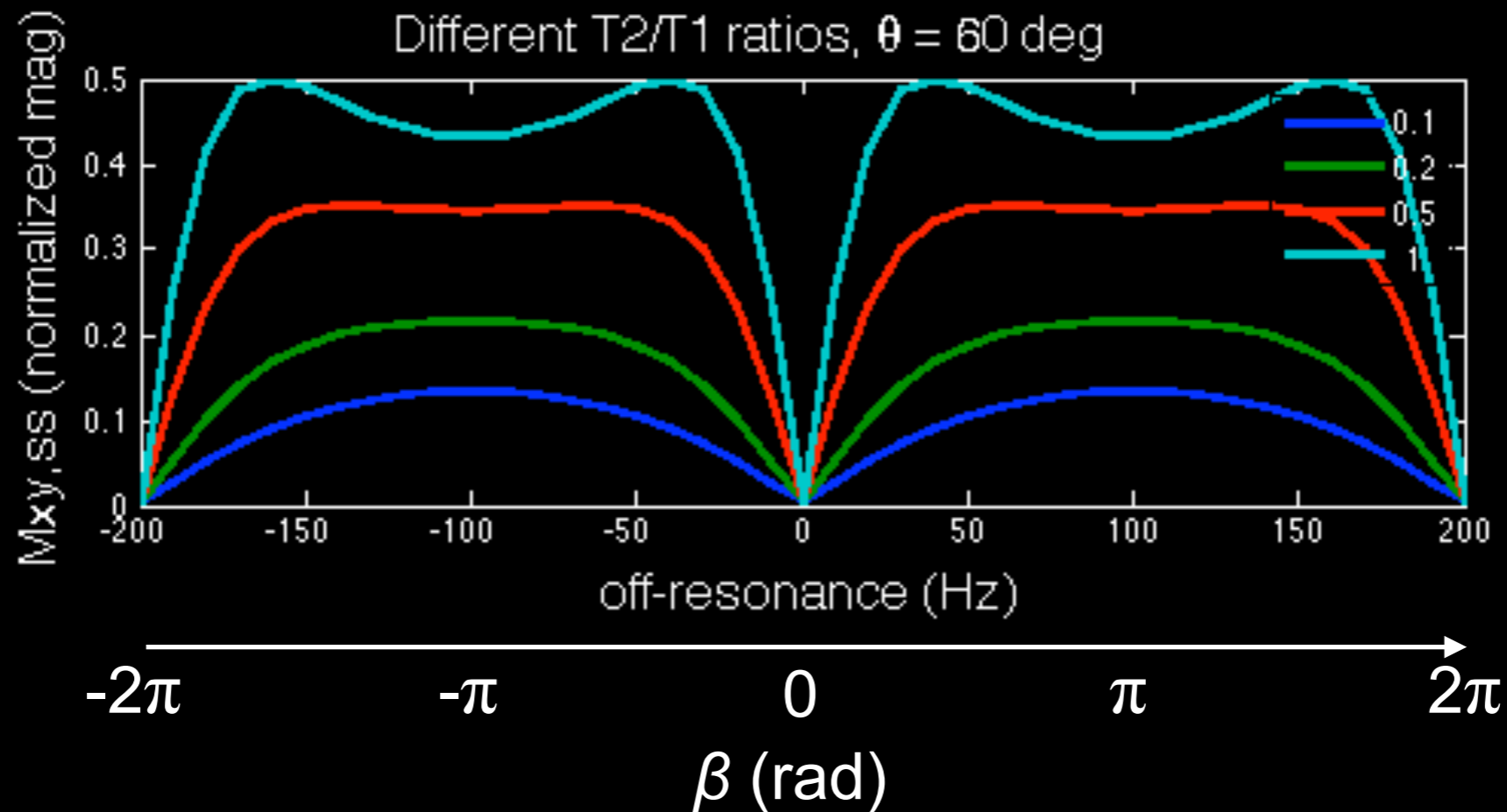
$T_1 = 1000$  ms,  $T_2 = 100, 200, 500, 1000$  ms

# Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



Recall  $\beta = 2\pi \Delta f \times \text{TR}$  and  $\Delta f = \gamma B / 2\pi$   
 $\beta = \pm\pi$  corresponds to  $\Delta f = \pm 1/(2 \text{TR})$  Hz

TR = 5 ms:  $\Delta f = \pm 100$  Hz

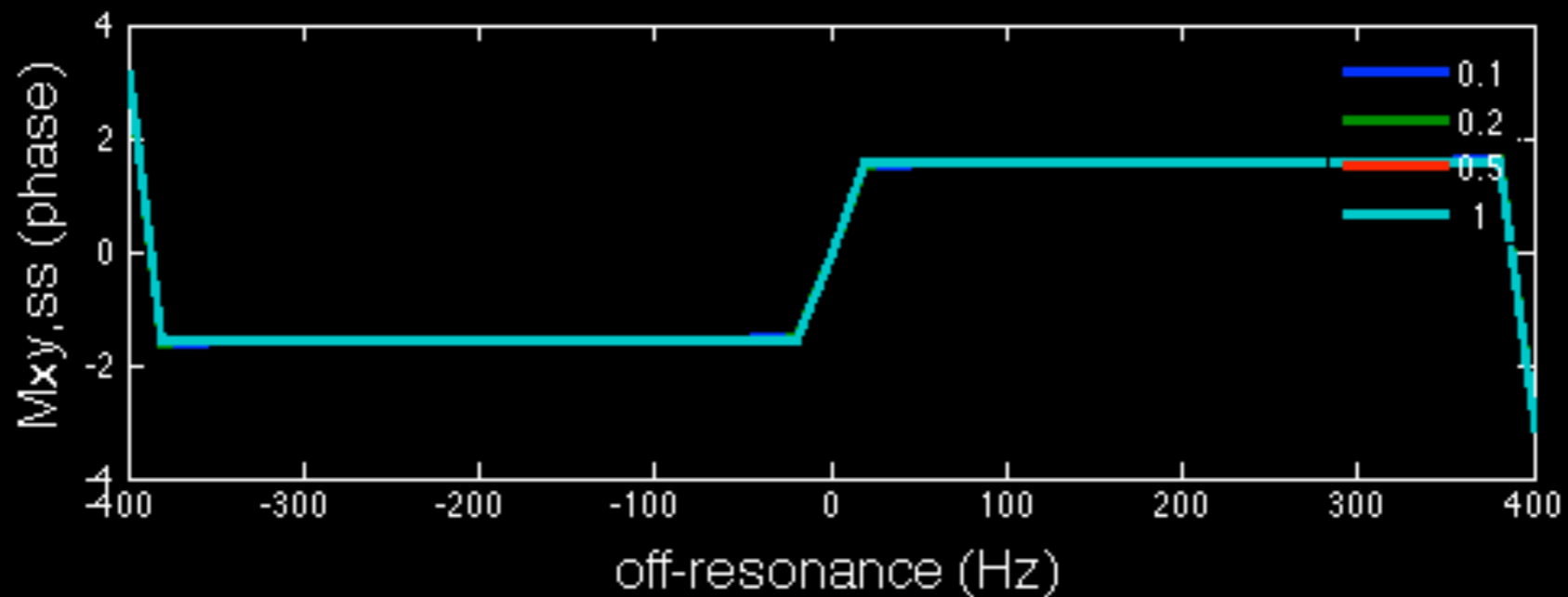
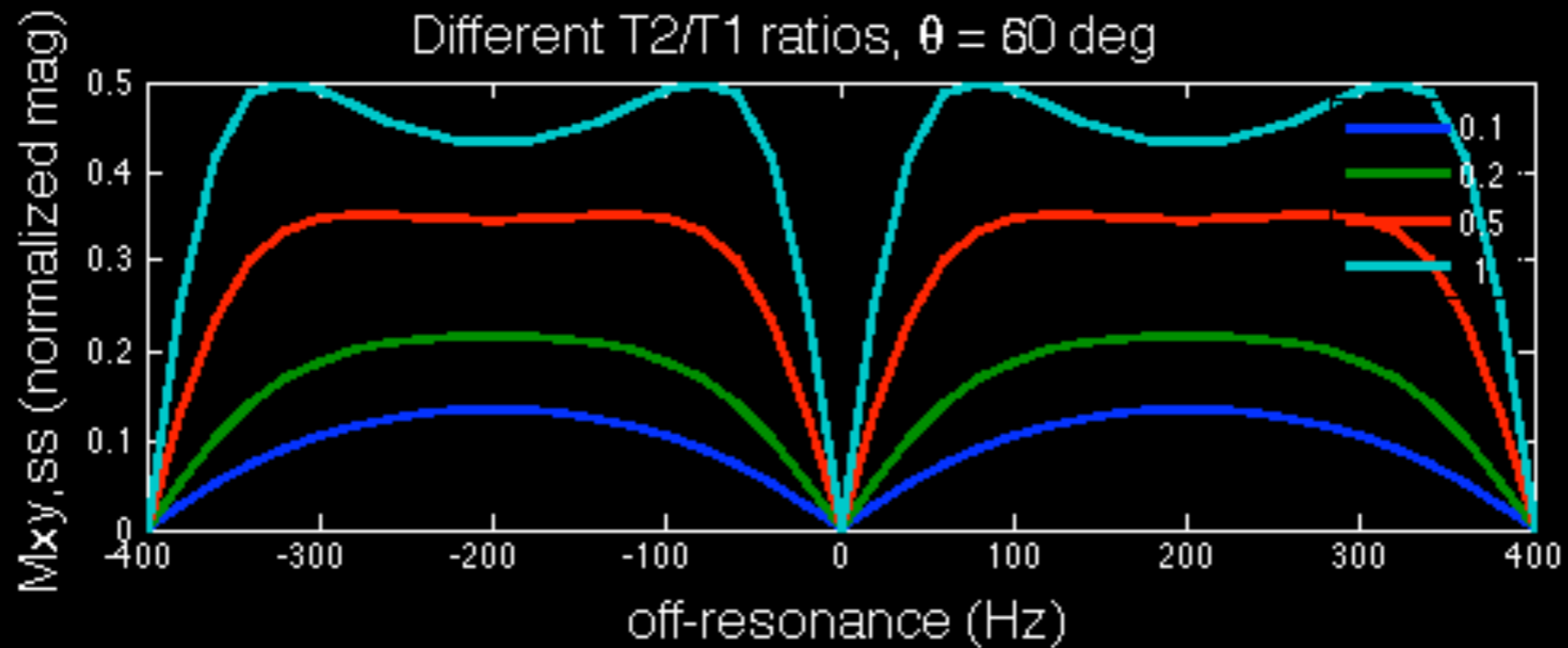
TR = 2.5 ms:  $\Delta f = \pm 200$  Hz

# Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

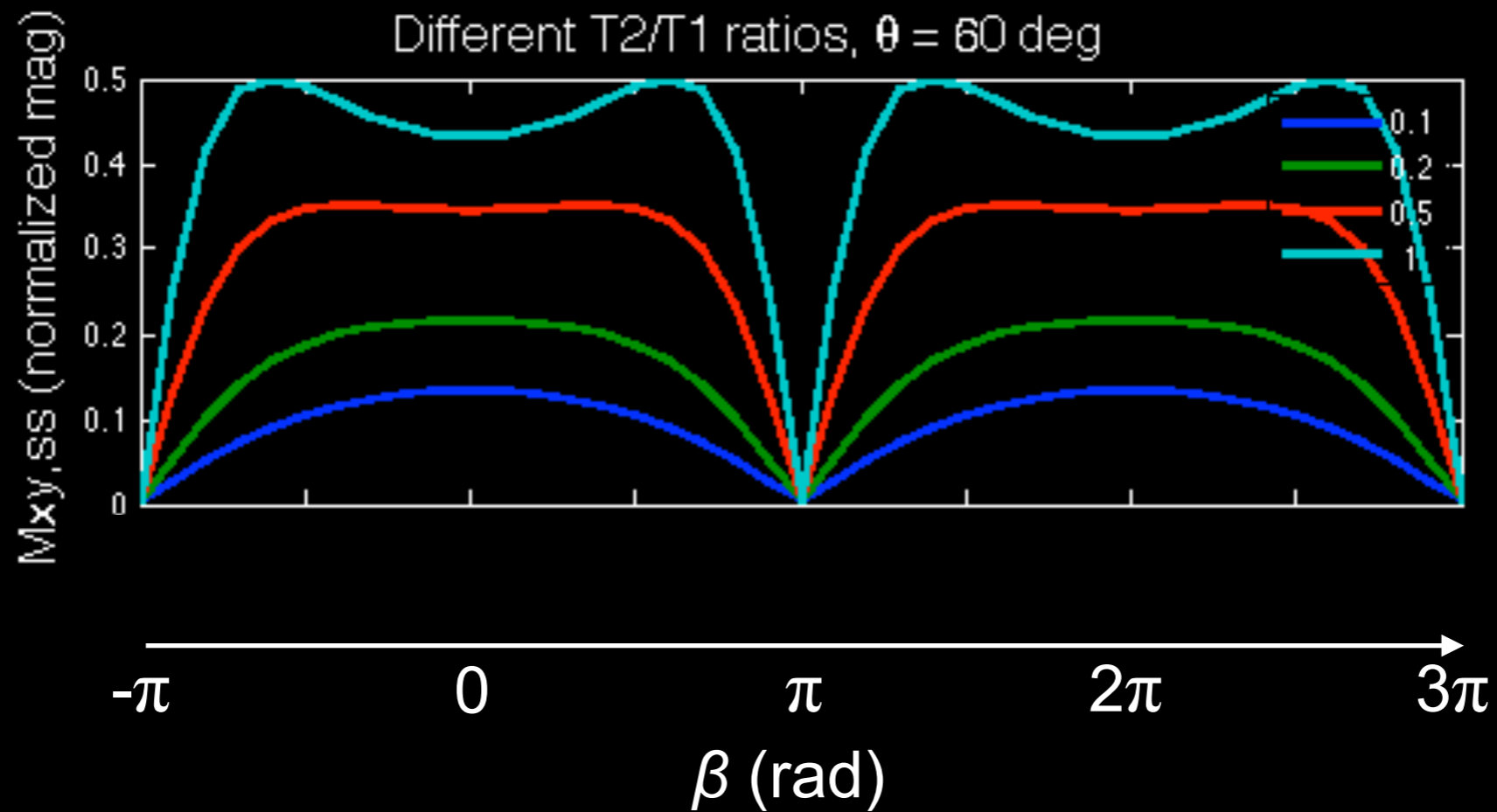
$\Delta\phi = 0$



$T_1 = 1000$  ms,  $T_2 = 100, 200, 500, 1000$  ms

# Balanced SSFP

SS signal as a function of off-resonance:



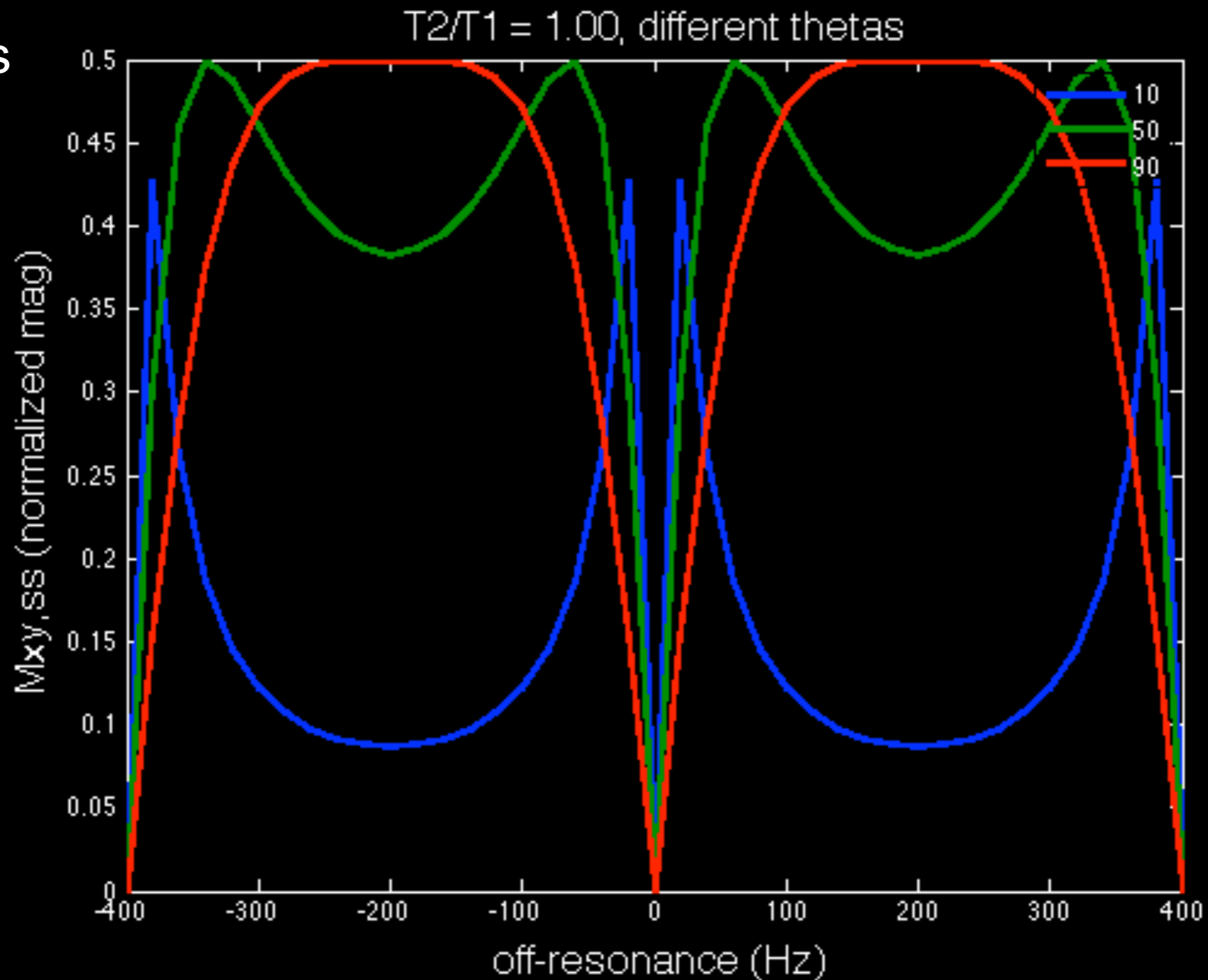
$\Delta\phi$  can shift the off-resonance response

# Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

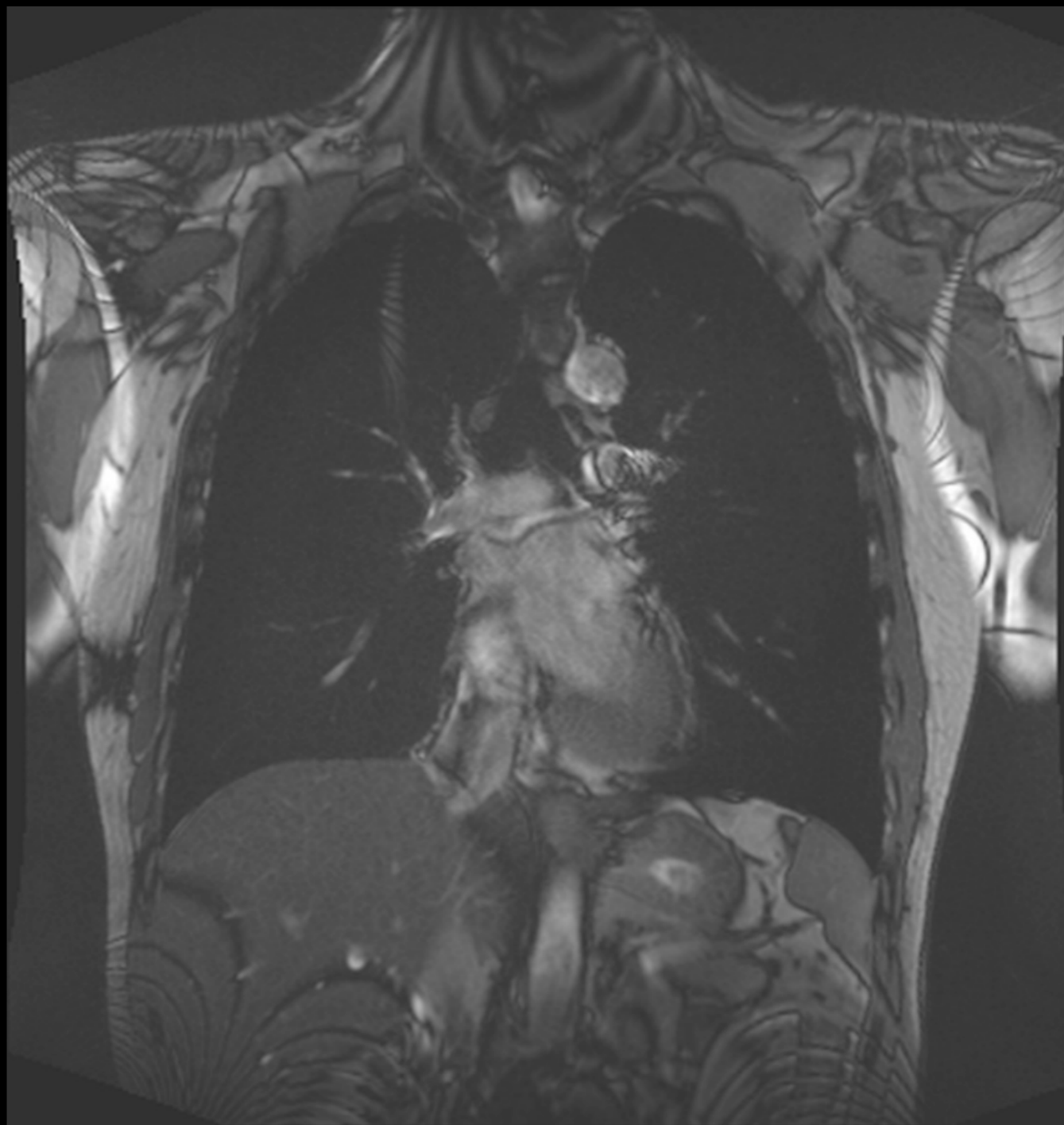
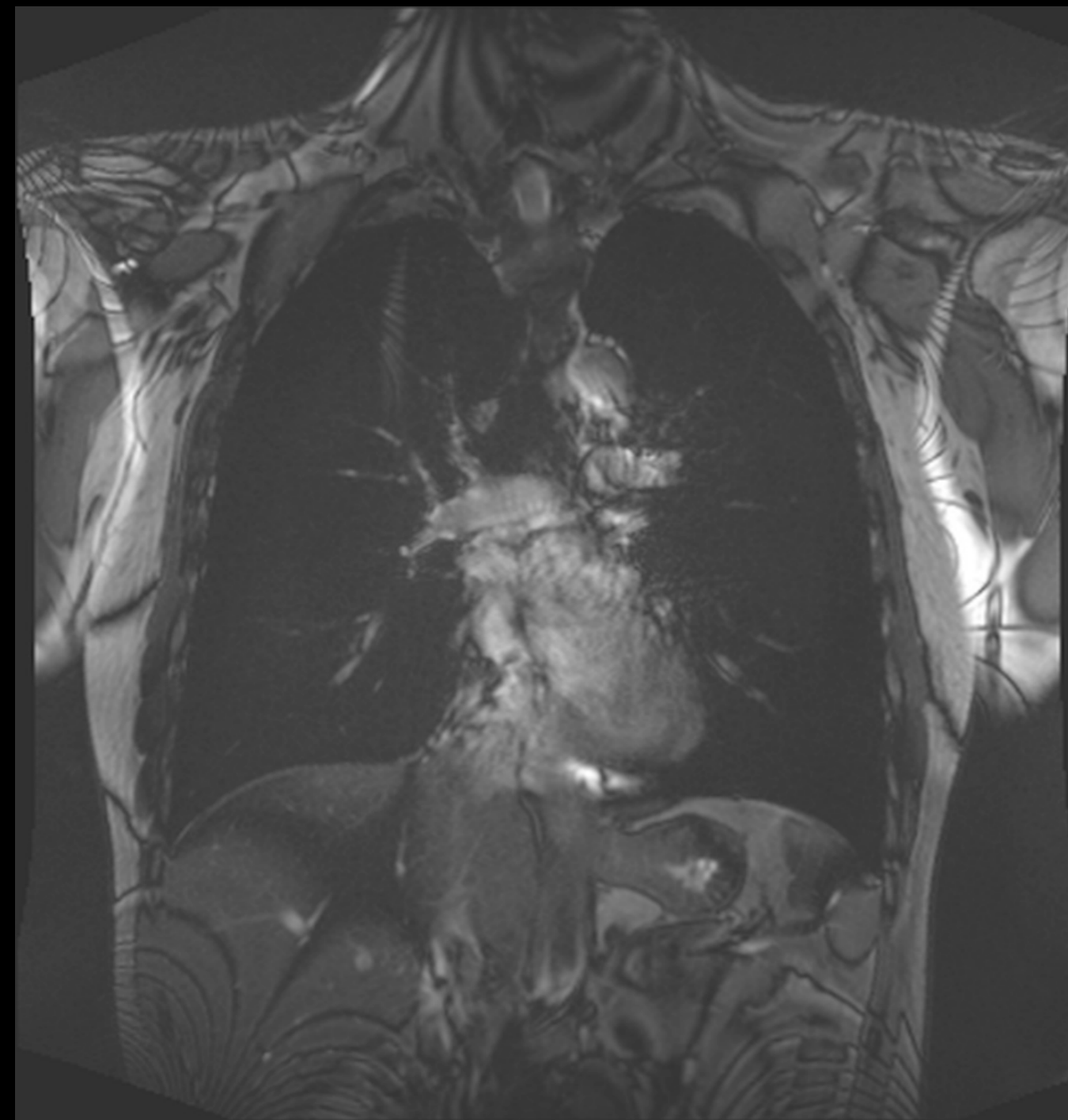
$\Delta\phi = 0$



$T_1 = 1000\text{ ms}$ ,  $T_2 = 1000\text{ ms}$

# Balanced SSFP

Banding artifacts at 3 T:

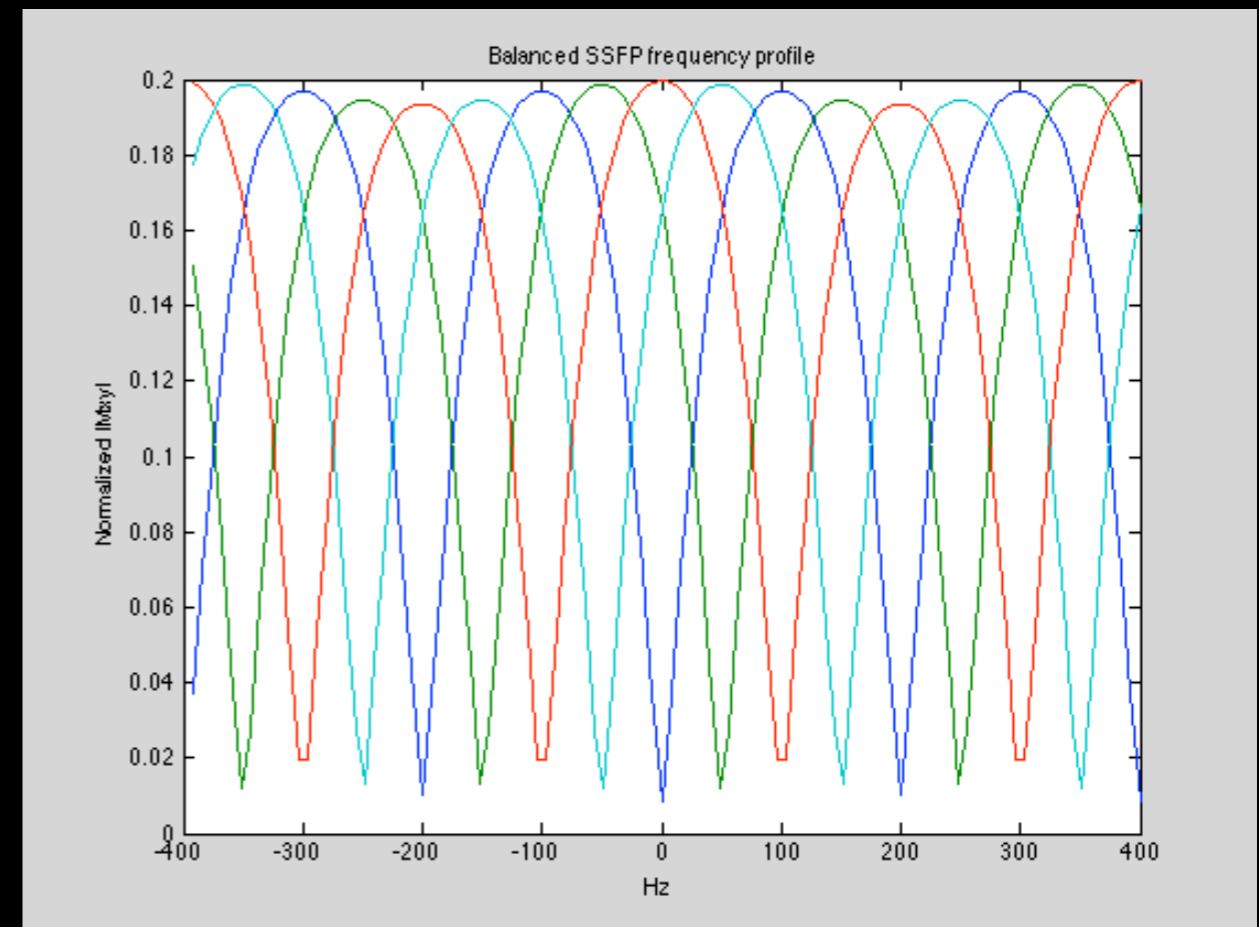
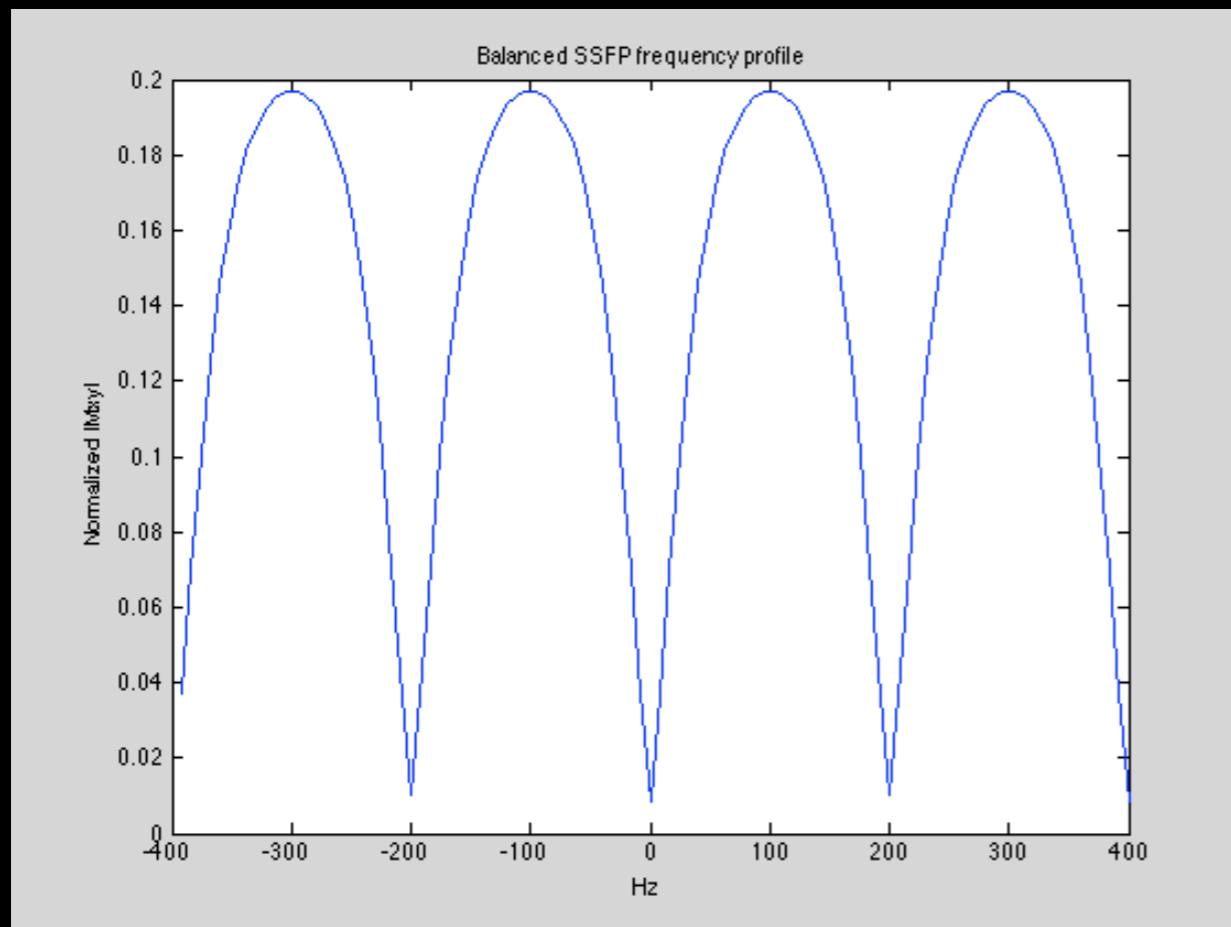


# Balanced SSFP

- Banding artifacts
  - bSSFP has freq-dep null bands
  - spatially varying field inhomogeneity
  - shim not perfect
  - worse at high field (e.g., 3 T vs 1.5 T)
- Mitigating banding artifacts
  - reduce TR
  - custom shim; shift center freq
  - phase cycling

# Balanced SSFP

- Removing banding artifacts
  - Multi-acquisition bSSFP (phase cycled)
  - Image reconstruction (rSoS, MIP, etc.)





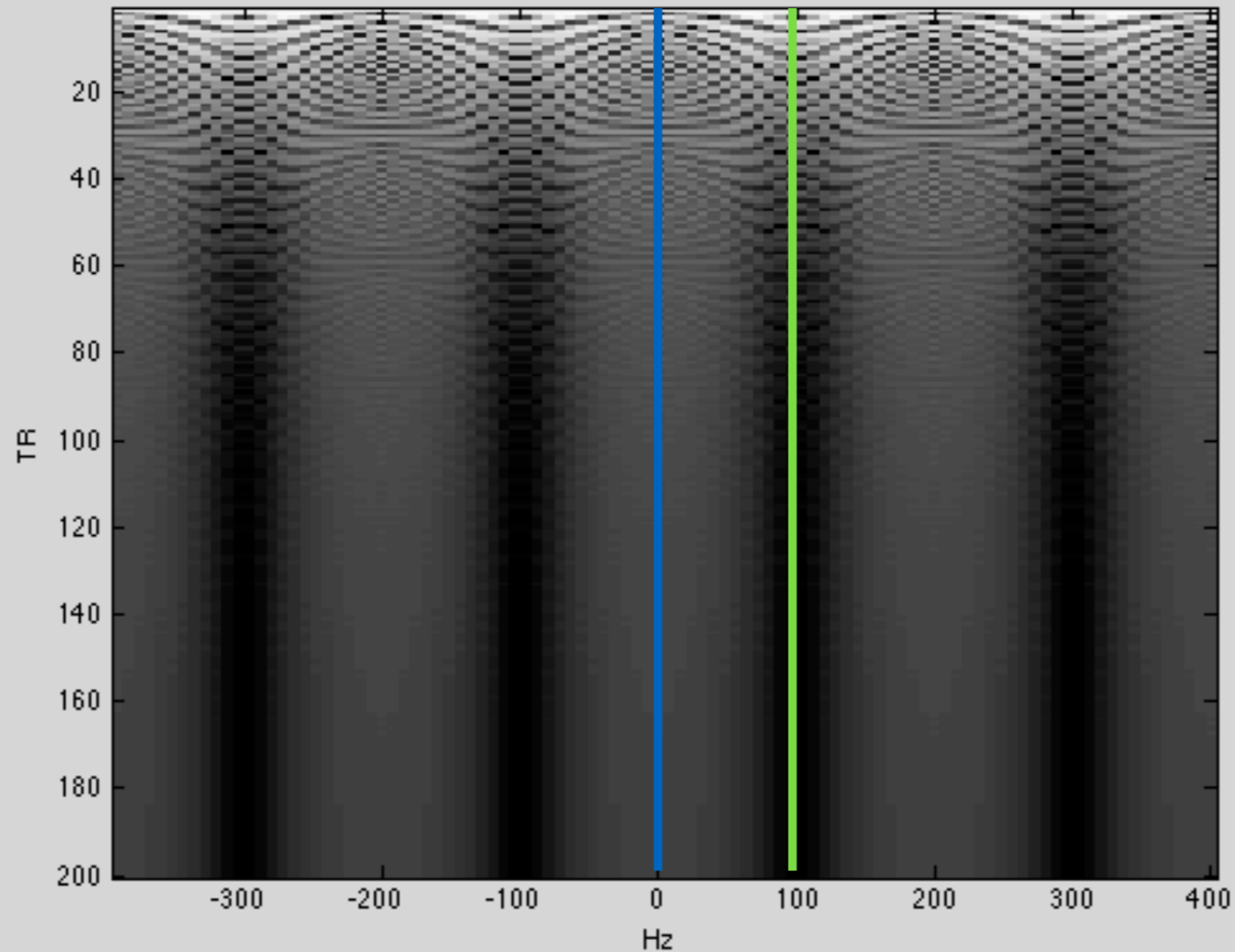
# Balanced SSFP

Transition to steady state:

$TR = 5 \text{ ms}$

$\Delta\phi = \pi$

$\theta = 60^\circ$



$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}$

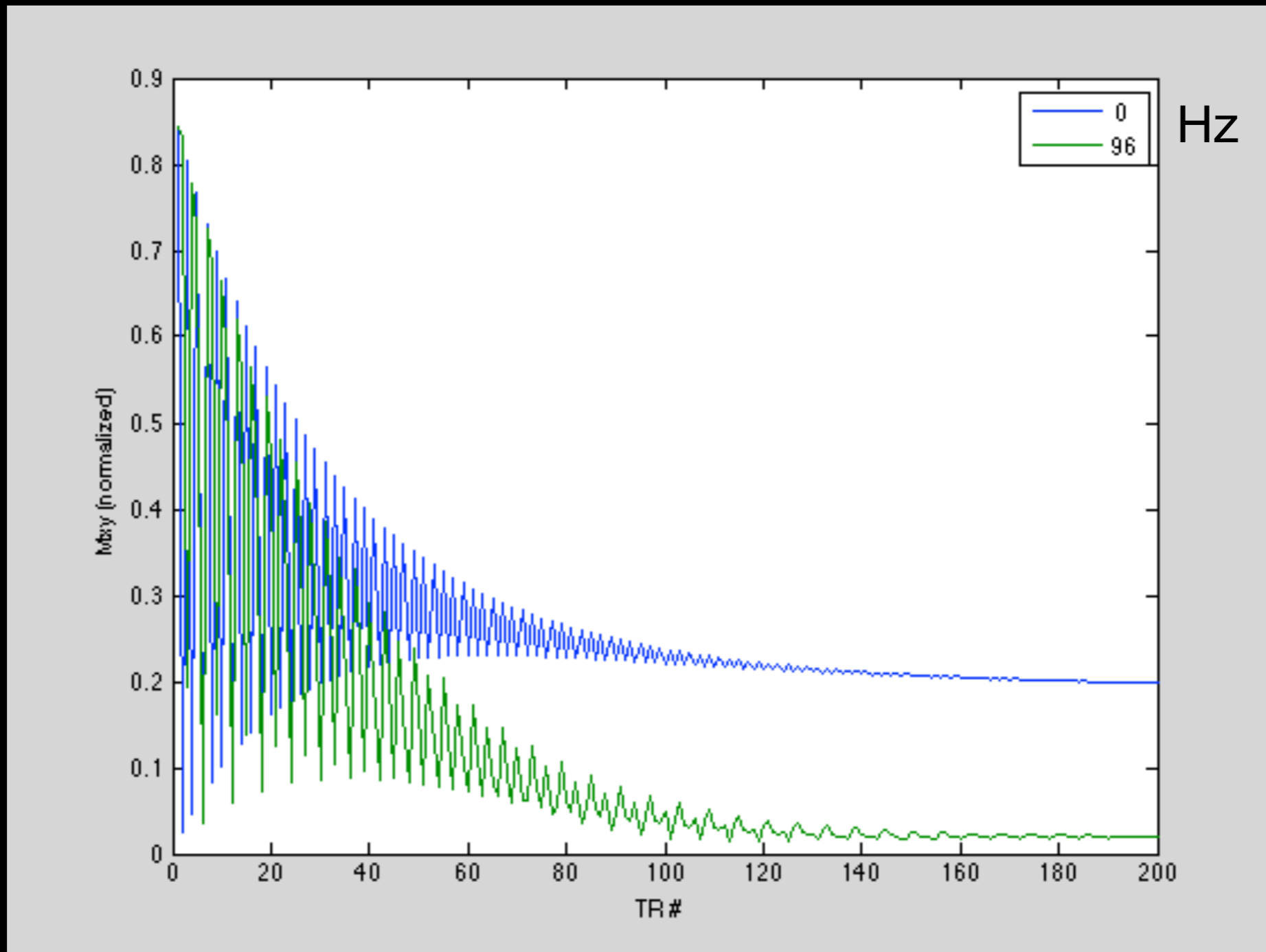
# Balanced SSFP

Transition to steady state:

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$



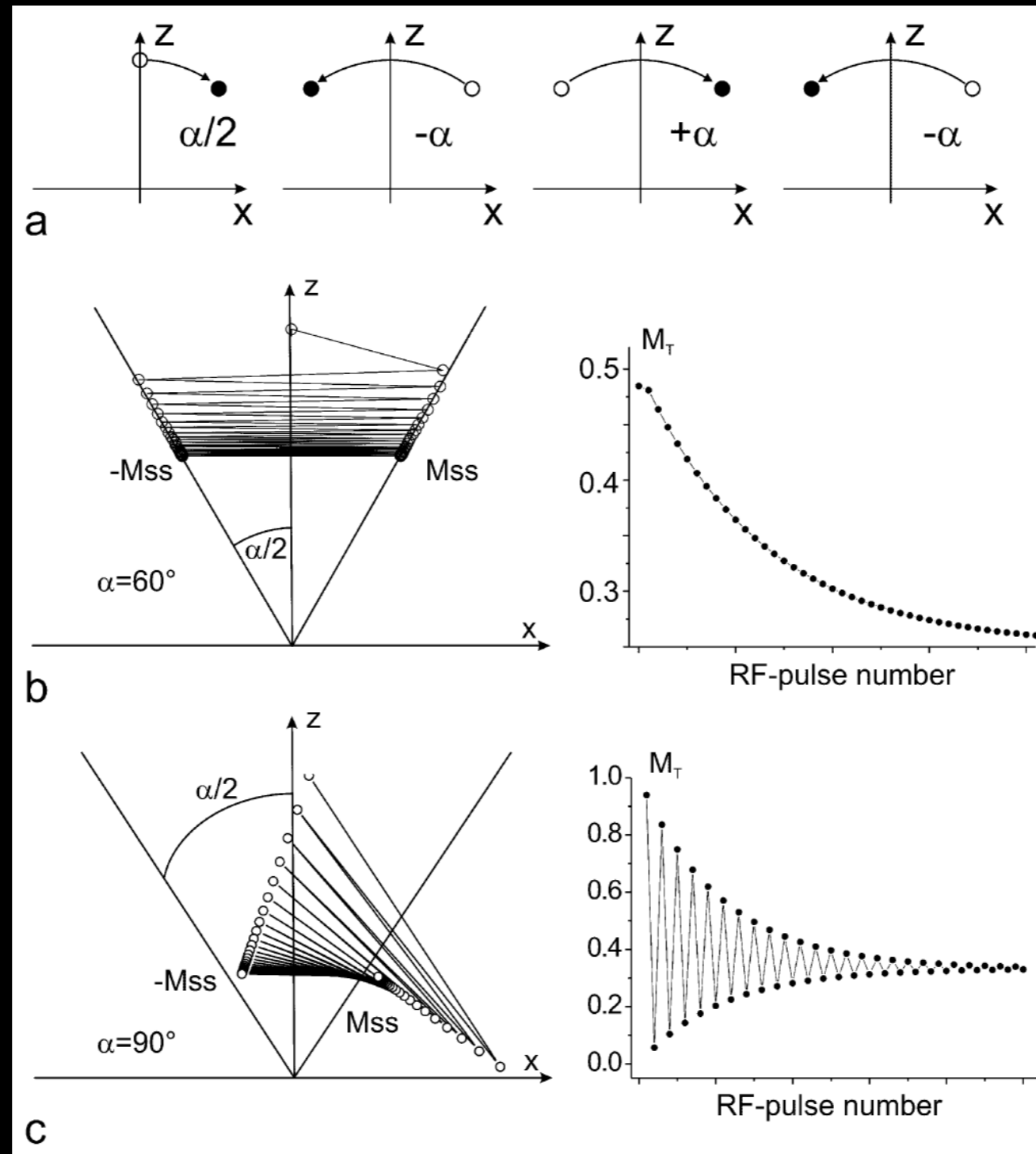
$T_1 = 600\text{ ms}, T_2 = 100\text{ ms}$

# Balanced SSFP

- Transient state
  - approach to steady state can take  $5 \cdot T_1$
  - depends on sequence and tissue params
  - longer transition for larger  $\theta$
  - artifacts and variable image contrast
- Catalyzation pulses
  - achieve smoother transition to steady state
  - simple approach:  $\theta/2$  - TR/2 preparation
  - other sophisticated designs

# Balanced SSFP

Transition to steady state ( $\theta/2$  -TR/2 prep):



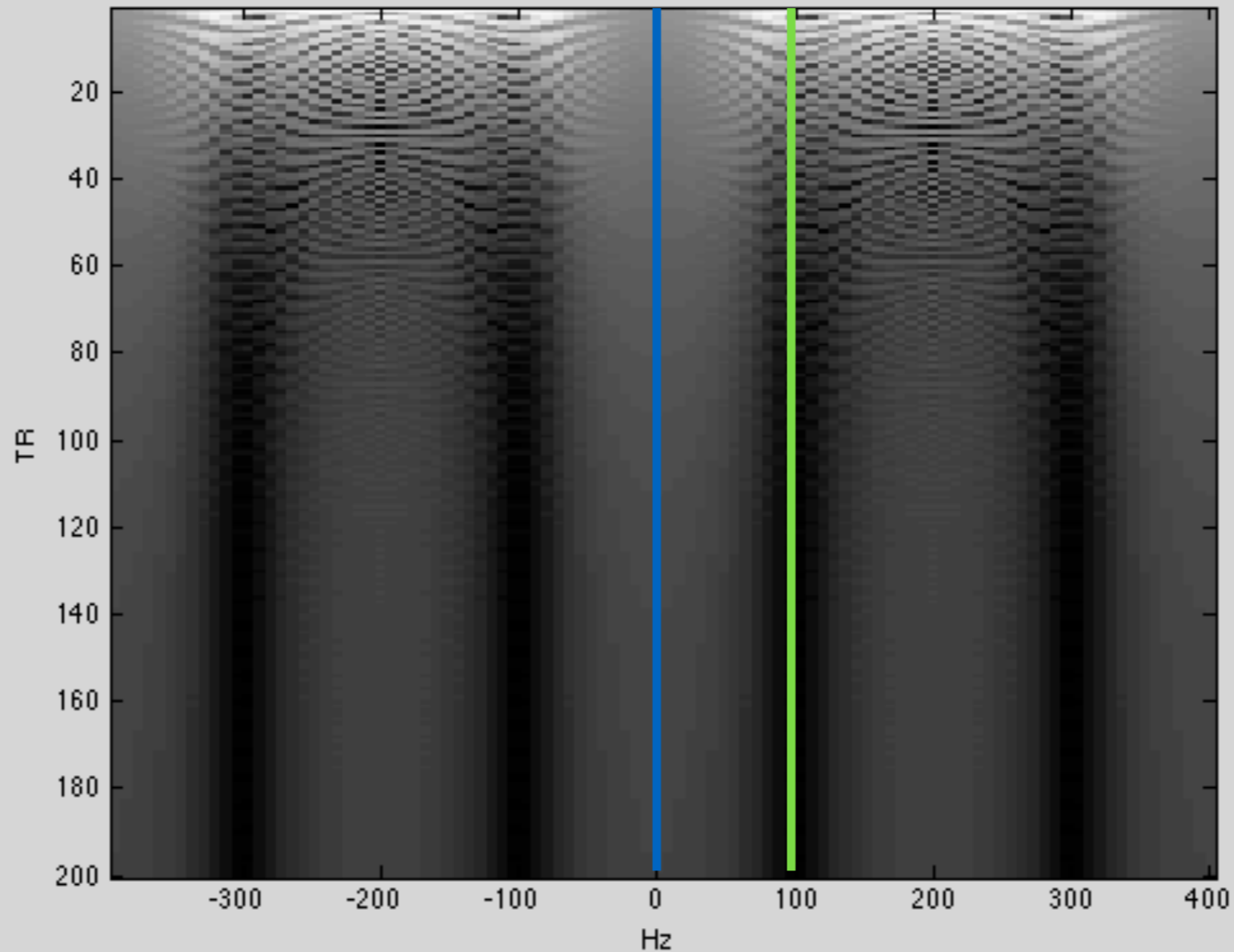
# Balanced SSFP

Transition to steady state ( $\theta/2$  -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$



$T_1 = 600$  ms,  $T_2 = 100$  ms

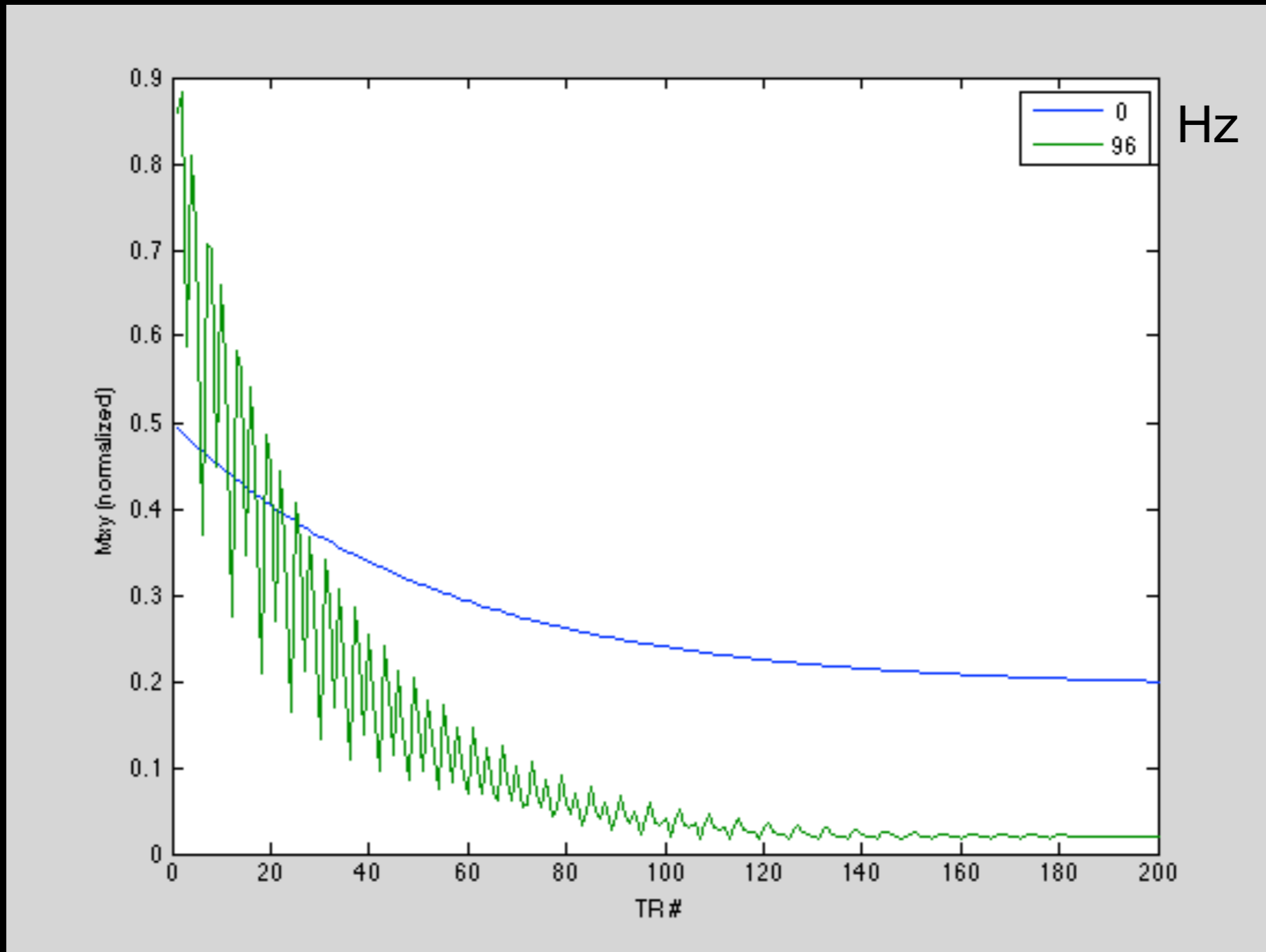
# Balanced SSFP

Transition to steady state ( $\theta/2$  -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$

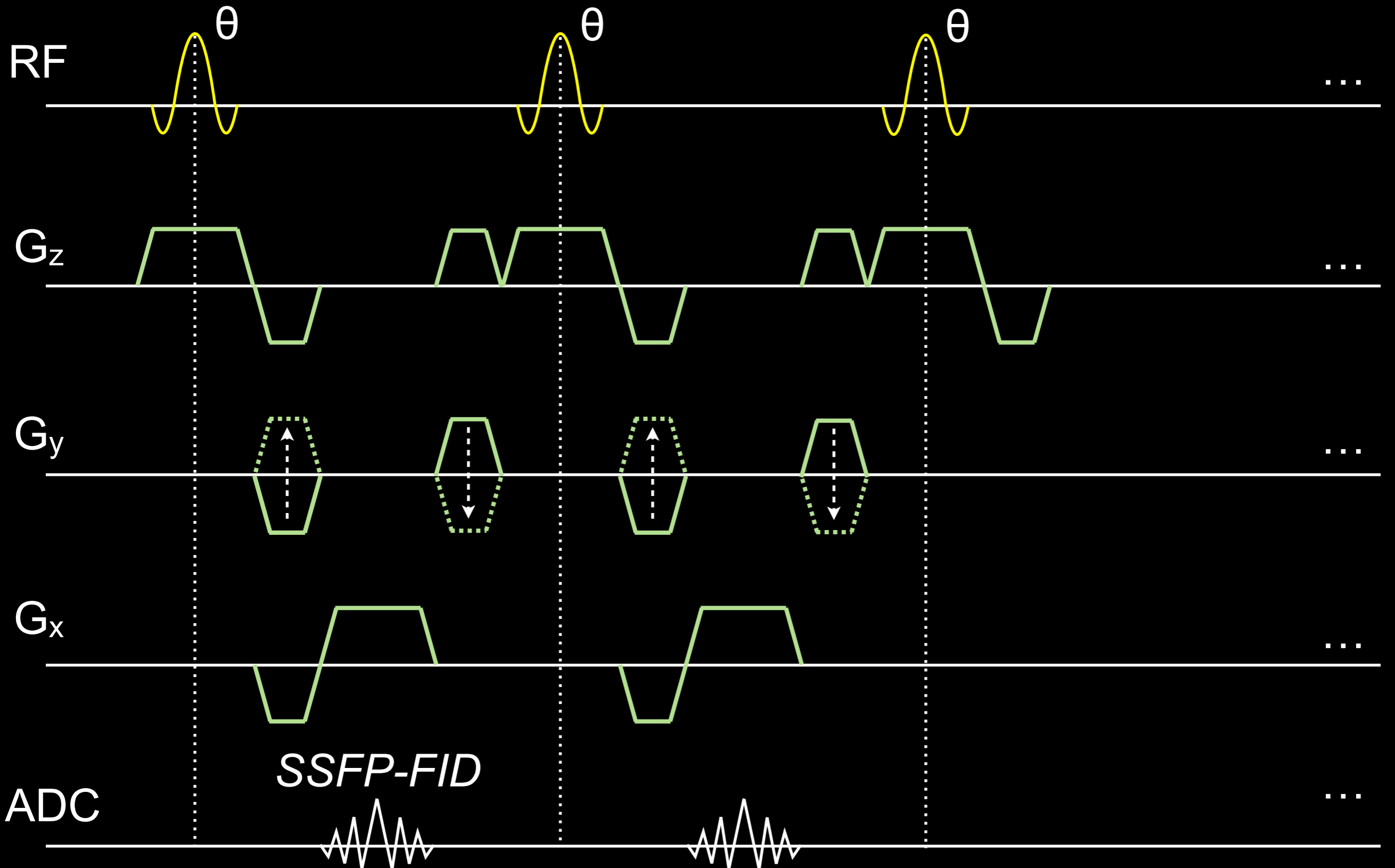


$T_1 = 600$  ms,  $T_2 = 100$  ms

# Balanced SSFP

- Advantages
  - High SNR efficiency
  - $G_x$  and  $G_z$  first moments nulled
- Challenges
  - Field homogeneity
  - TR
  - SAR
  - Catalyzation
  - Bright fat

# Gradient-spoiled GRE





# Gradient-spoiled GRE

- End-of-TR gradient spoiler
  - typically on  $G_x$  and/or  $G_z$
  - Range of  $\beta$  within each voxel
  - $M_{xy}$  is a complex sum of all spins
- Contrast depends on  $T_1$  and  $T_2$

# Gradient-spoiled GRE

Steady-state signal equation:

$$\text{SSFP}_{\text{FID}} = M_0 \frac{\sin \theta}{1 + \cos \theta} (1 - (E_1 - \cos \theta) f(E_1, E_2, \theta))$$

$$f(E_1, E_2, \theta) = \sqrt{\frac{1 - E_2^2}{(1 - E_1 \cos \theta)^2 - E_2^2 (E_1 - \cos \theta)^2}}$$

When  $\text{TR} \gg T_2$ :

$$\text{SSFP}_{\text{FID}} \rightarrow M_0 \sin \theta \frac{1 - E_1}{1 - E_1 \cos \theta}$$

*same as ideally spoiled GRE*

# Gradient-spoiled GRE

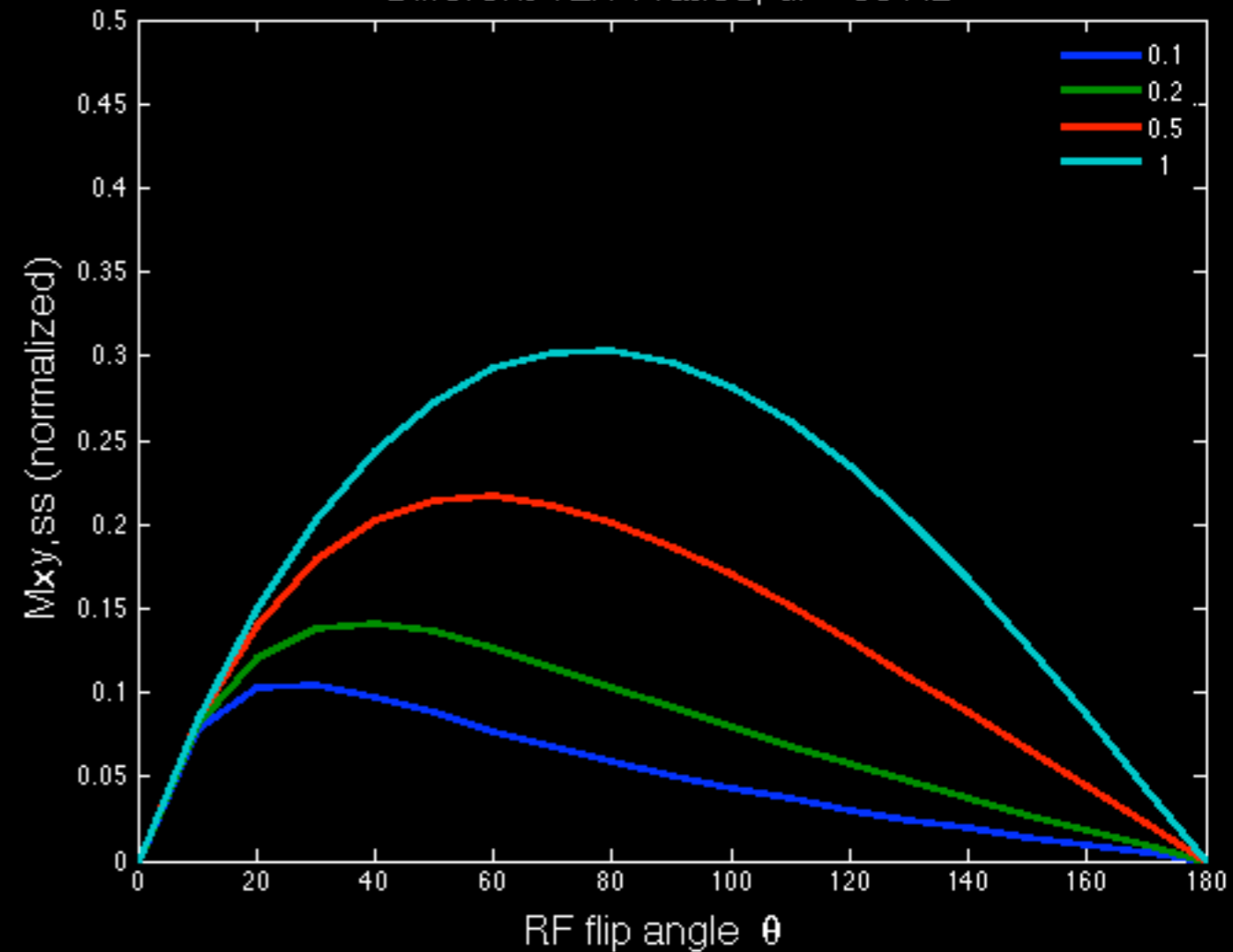
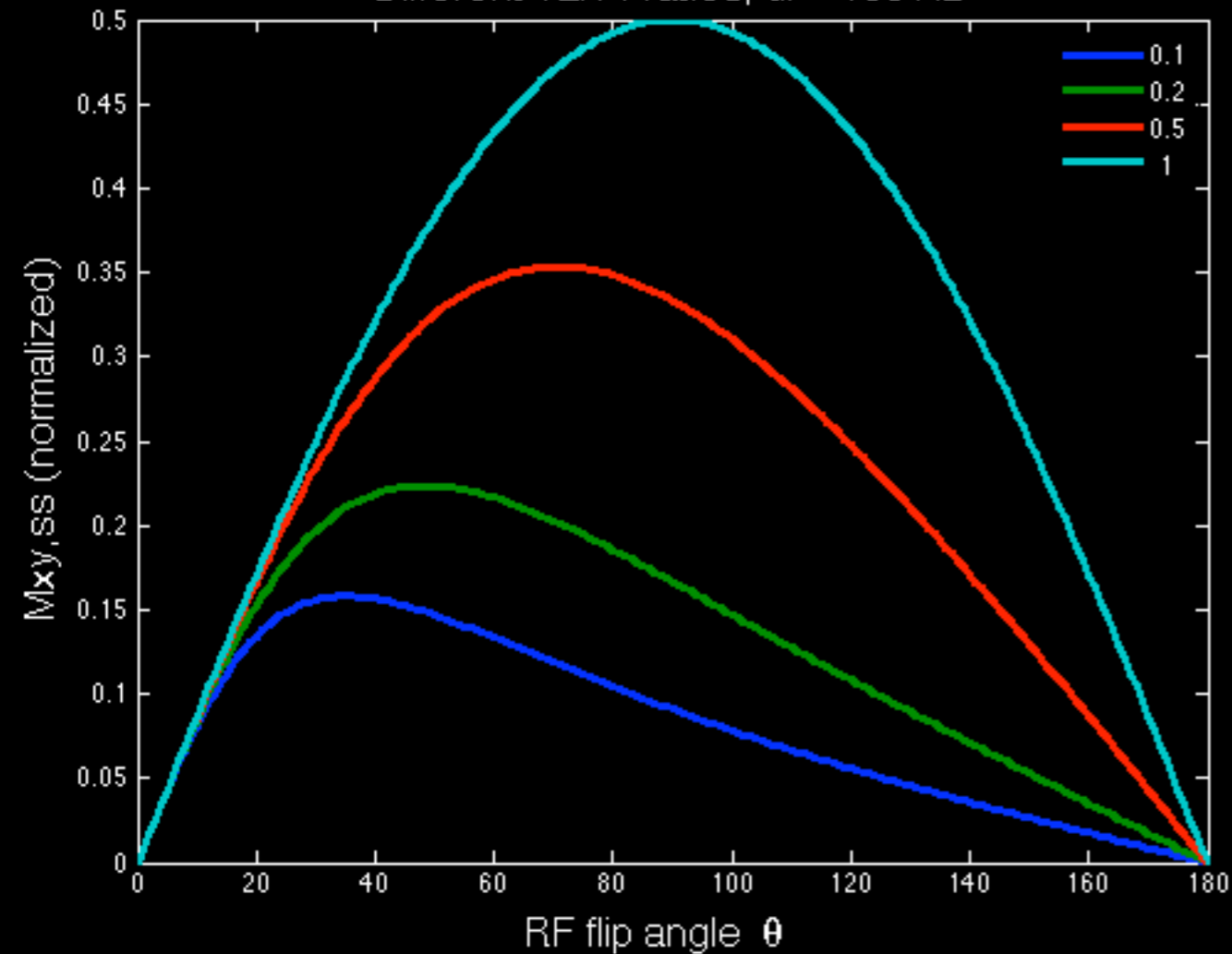
SS signal as a function of flip angle:

*bSSFP*

*GRE (SSFP-FID)*

Different T2/T1 ratios, df = 100 Hz

Different T2/T1 ratios, df = 50 Hz



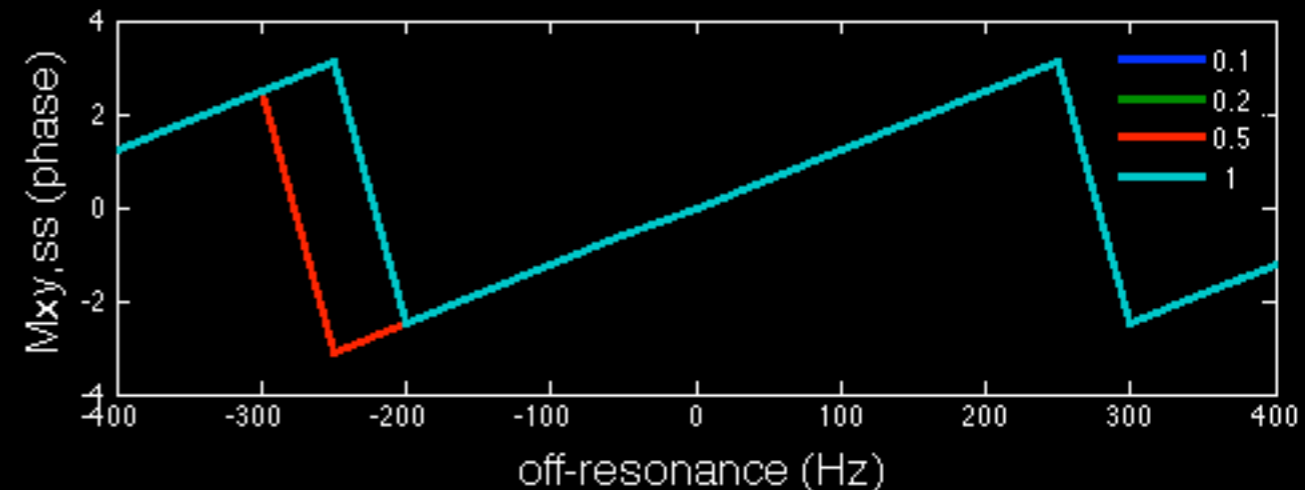
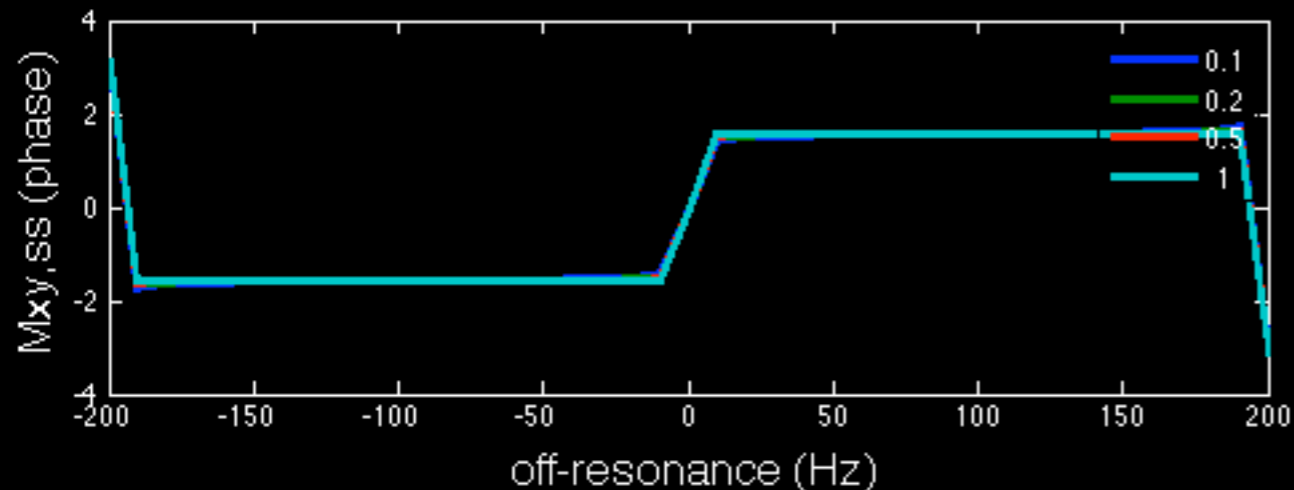
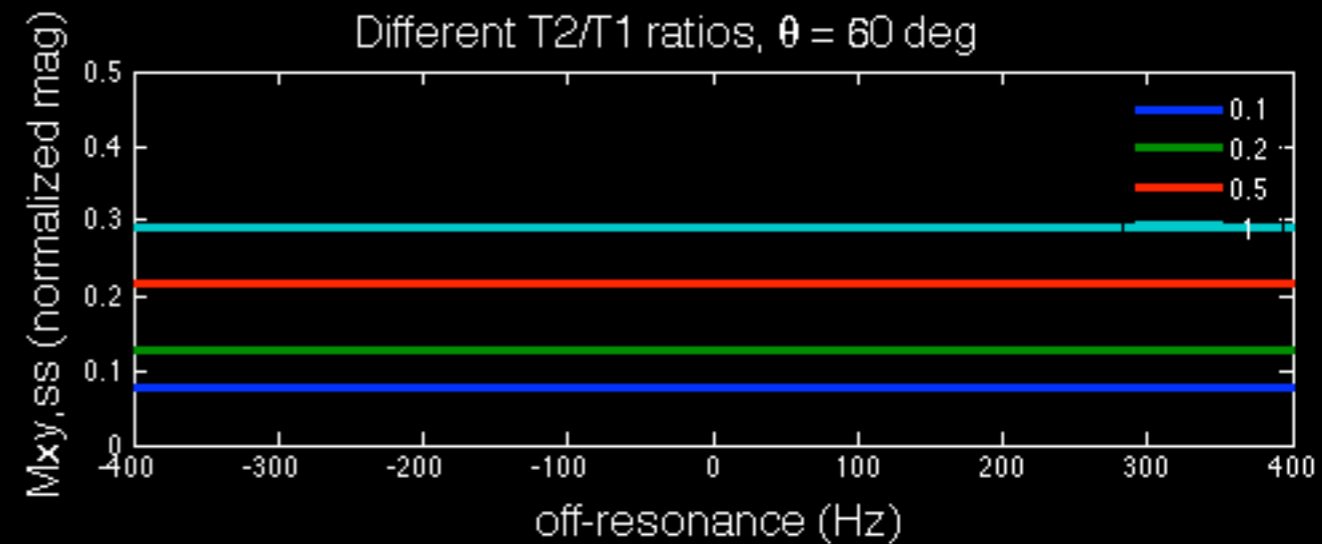
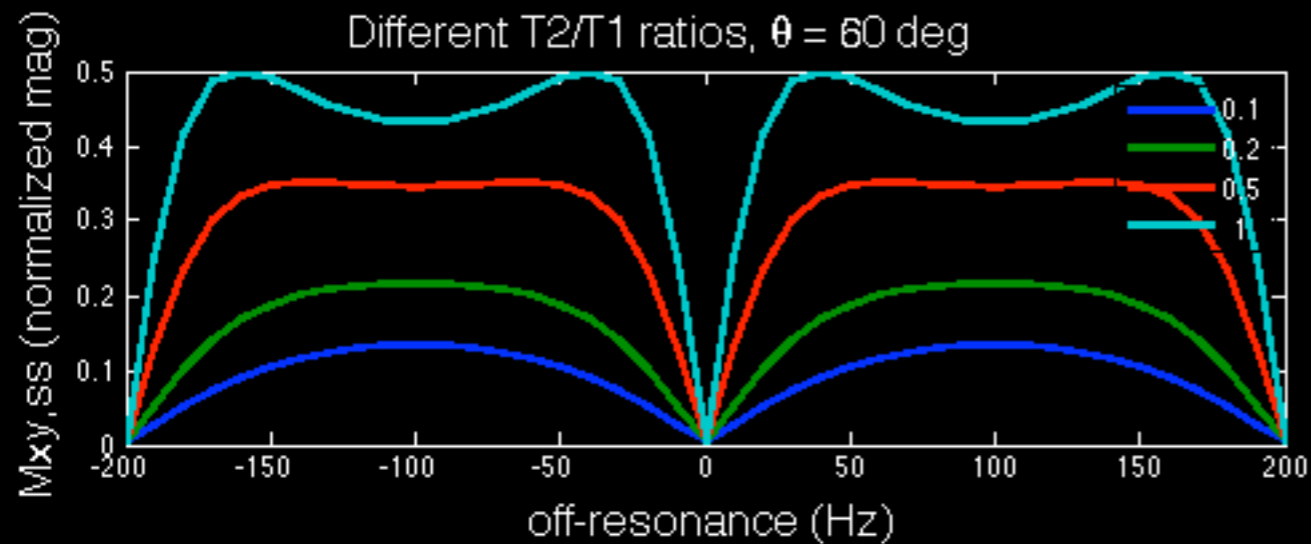
$T_1 = 1000$  ms,  $T_2 = 100, 200, 500, 1000$  ms

# Gradient-spoiled GRE

SS signal as a function of off-resonance:

*bSSFP*

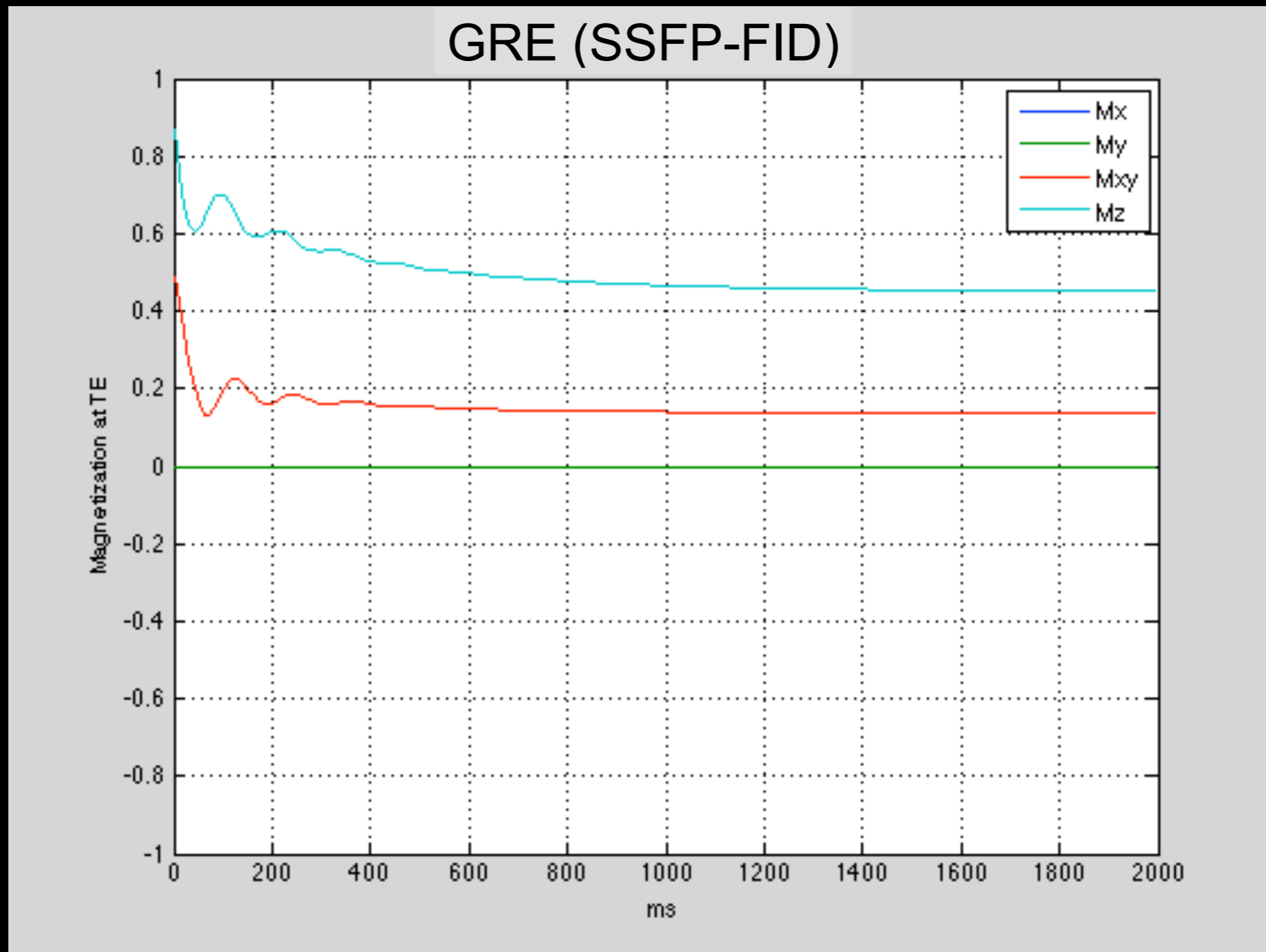
*GRE (SSFP-FID)*



$T_1 = 1000$  ms,  $T_2 = 100, 200, 500, 1000$  ms

# Gradient-spoiled GRE

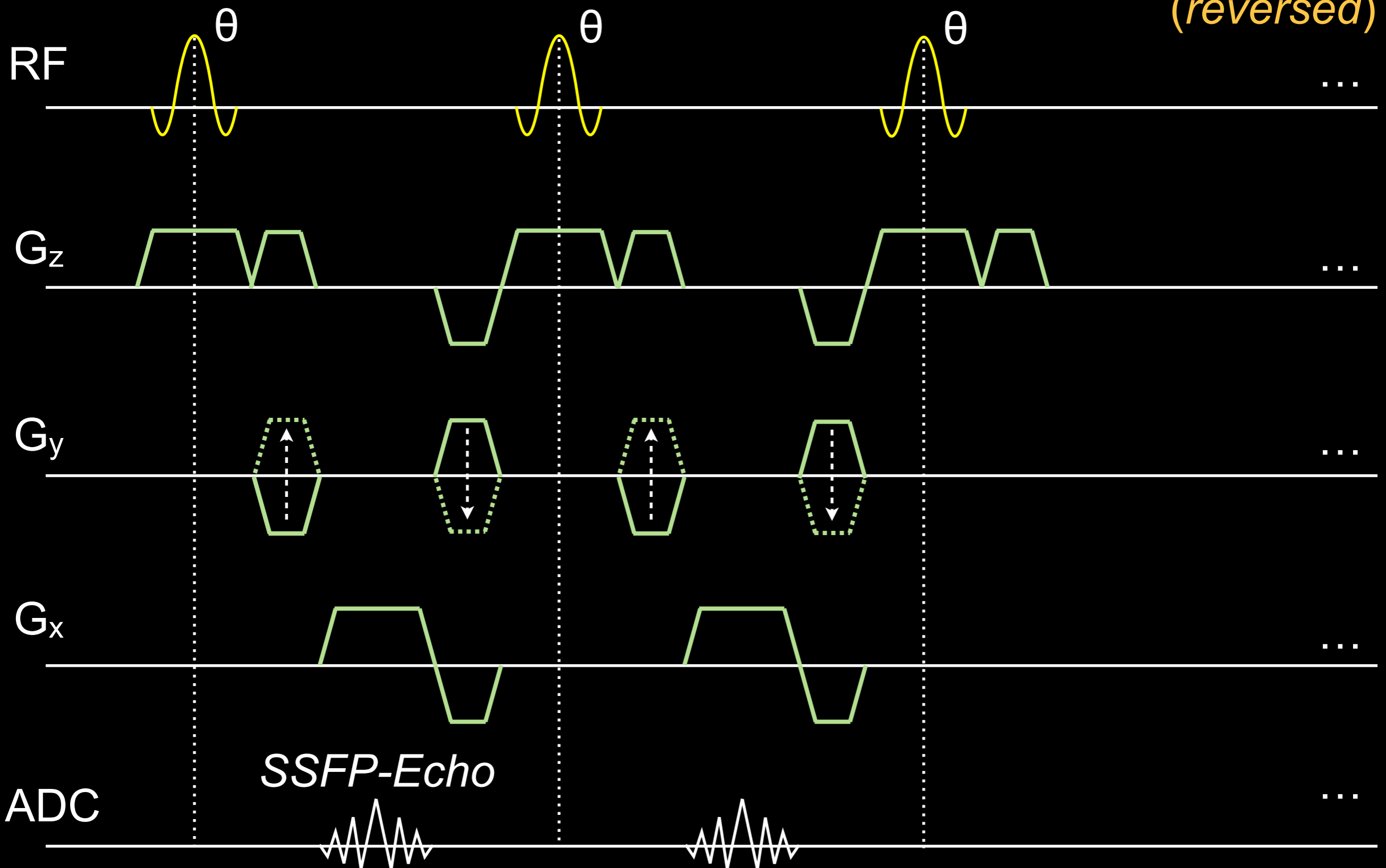
Transition to steady state:



$$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}, TE/TR = 2/10 \text{ ms}, \theta = 30^\circ$$

# Gradient-spoiled GRE

*(reversed)*



# Gradient-spoiled GRE

(reversed)

Steady-state signal equation:

$$\text{SSFP}_{\text{Echo}} = M_0 \frac{\sin \theta}{1 + \cos \theta} (1 - (1 - E_1 \cos \theta) f(E_1, E_2, \theta))$$

$$f(E_1, E_2, \theta) = \sqrt{\frac{1 - E_2^2}{(1 - E_1 \cos \theta)^2 - E_2^2 (E_1 - \cos \theta)^2}}$$

When  $\text{TR} \ll T_1$ :

$$\frac{\text{SSFP}_{\text{Echo}}}{\text{SSFP}_{\text{FID}}} \sim E_2^2 = e^{-2\text{TR}/T_2}$$

*higher  $T_2$  contrast weighting than  $\text{SSFP}_{\text{FID}}$*

# Gradient-spoiled GRE

(reversed)

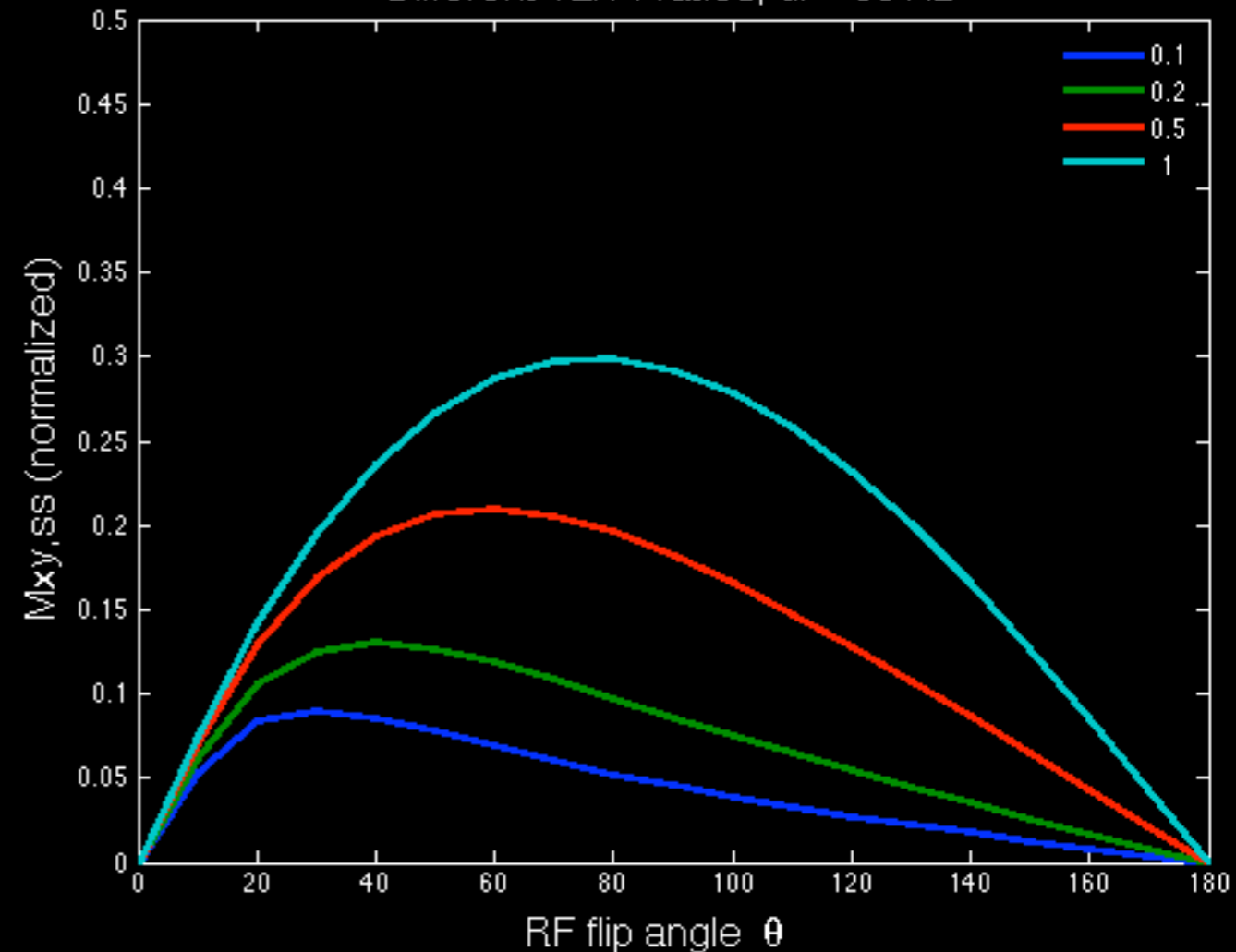
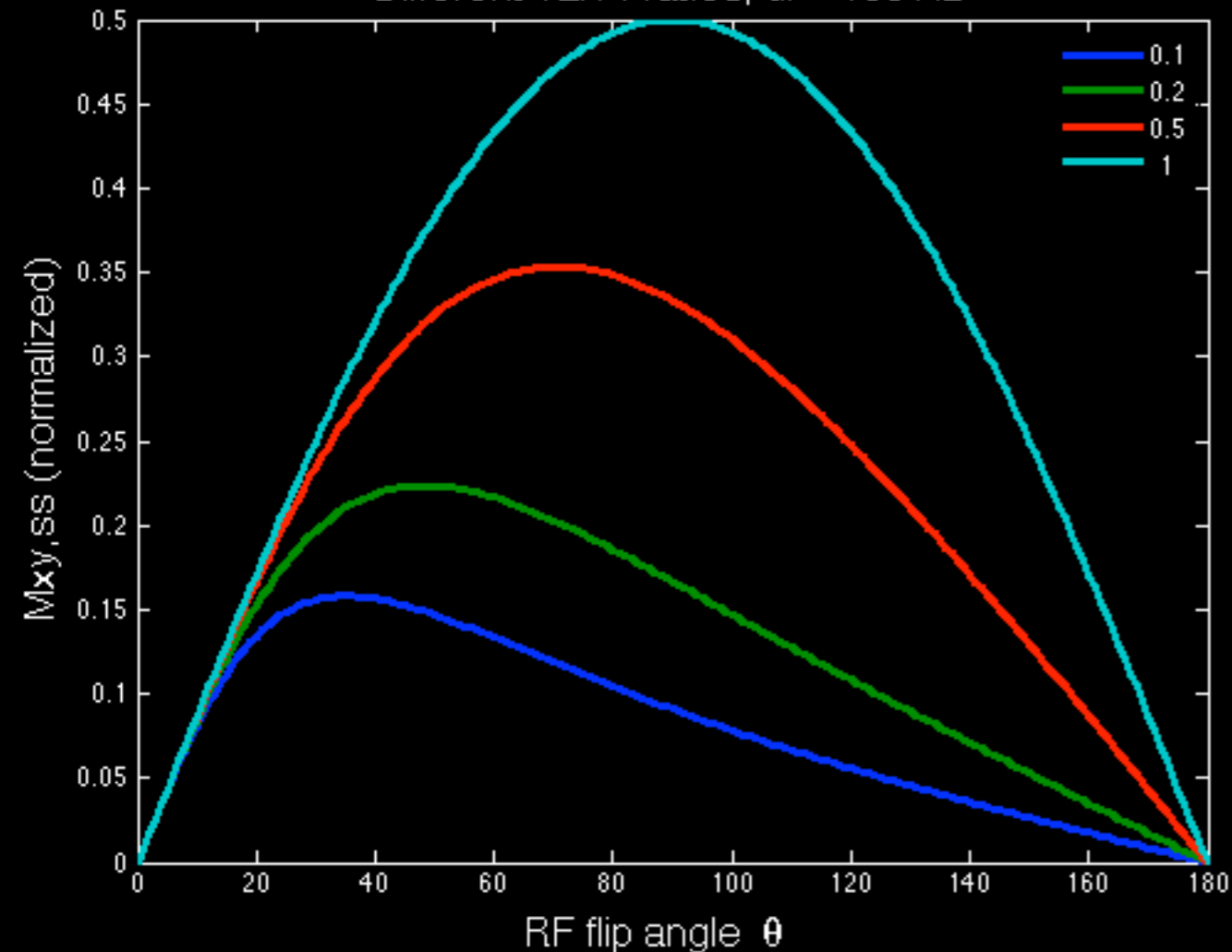
SS signal as a function of flip angle:

*bSSFP*

*GRE (SSFP-Echo)*

Different T2/T1 ratios, df = 100 Hz

Different T2/T1 ratios, df = 50 Hz



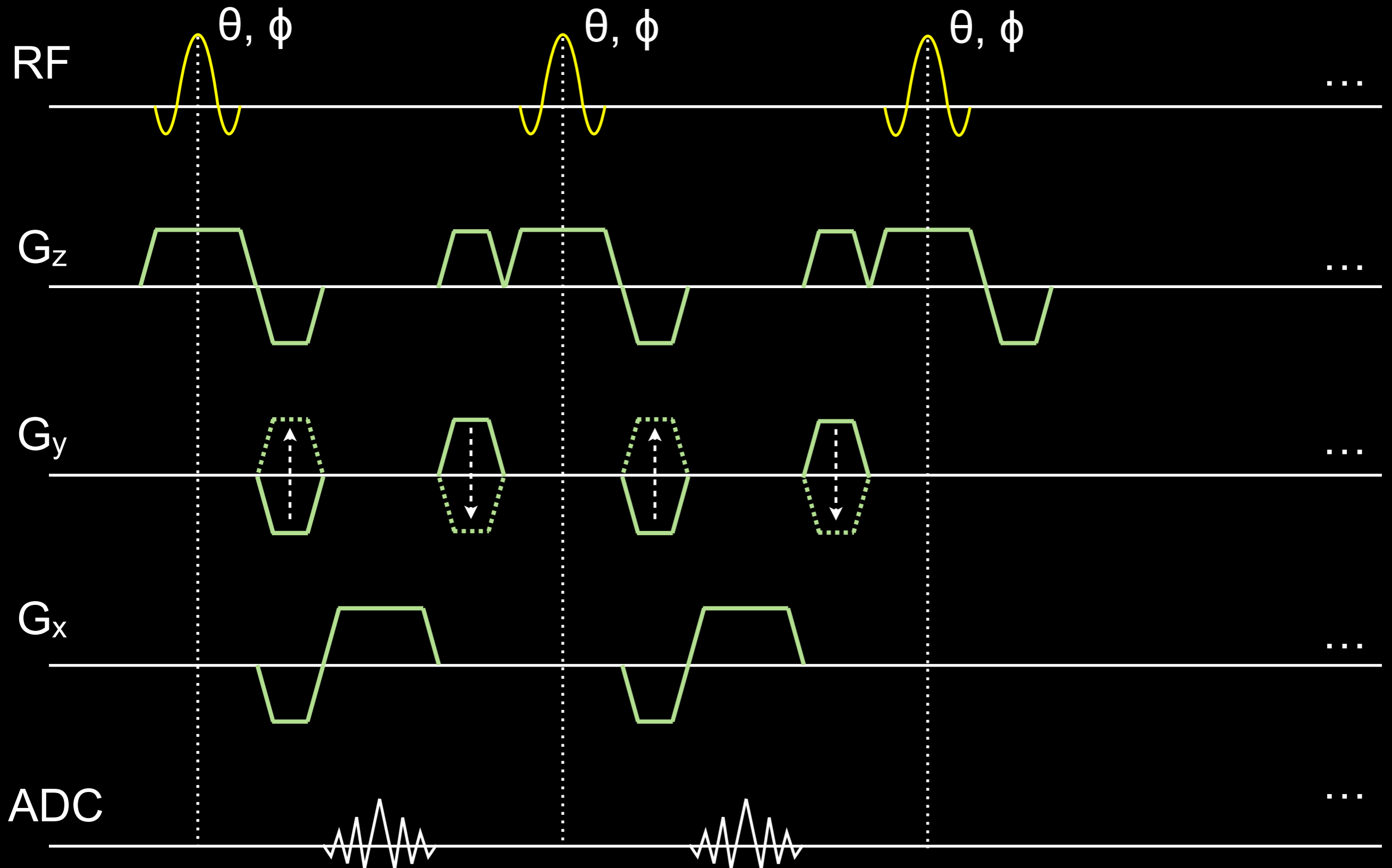
$T_1 = 1000$  ms,  $T_2 = 100, 200, 500, 1000$  ms



# Gradient-spoiled GRE

- Image characteristics
  - no banding (averaged in voxel)
  - SSFP-FID:  $T_2/T_1$  contrast
  - SSFP-Echo: more  $T_2$  contrast
  - sensitive to motion / flow / diffusion
- When all gradients are balanced
  - SSFP-FID and SSFP-Echo coalesce
  - $T_2$  instead of  $T_2^*$  weighting
  - Balanced SSFP!

# Gradient & RF-spoiled GRE

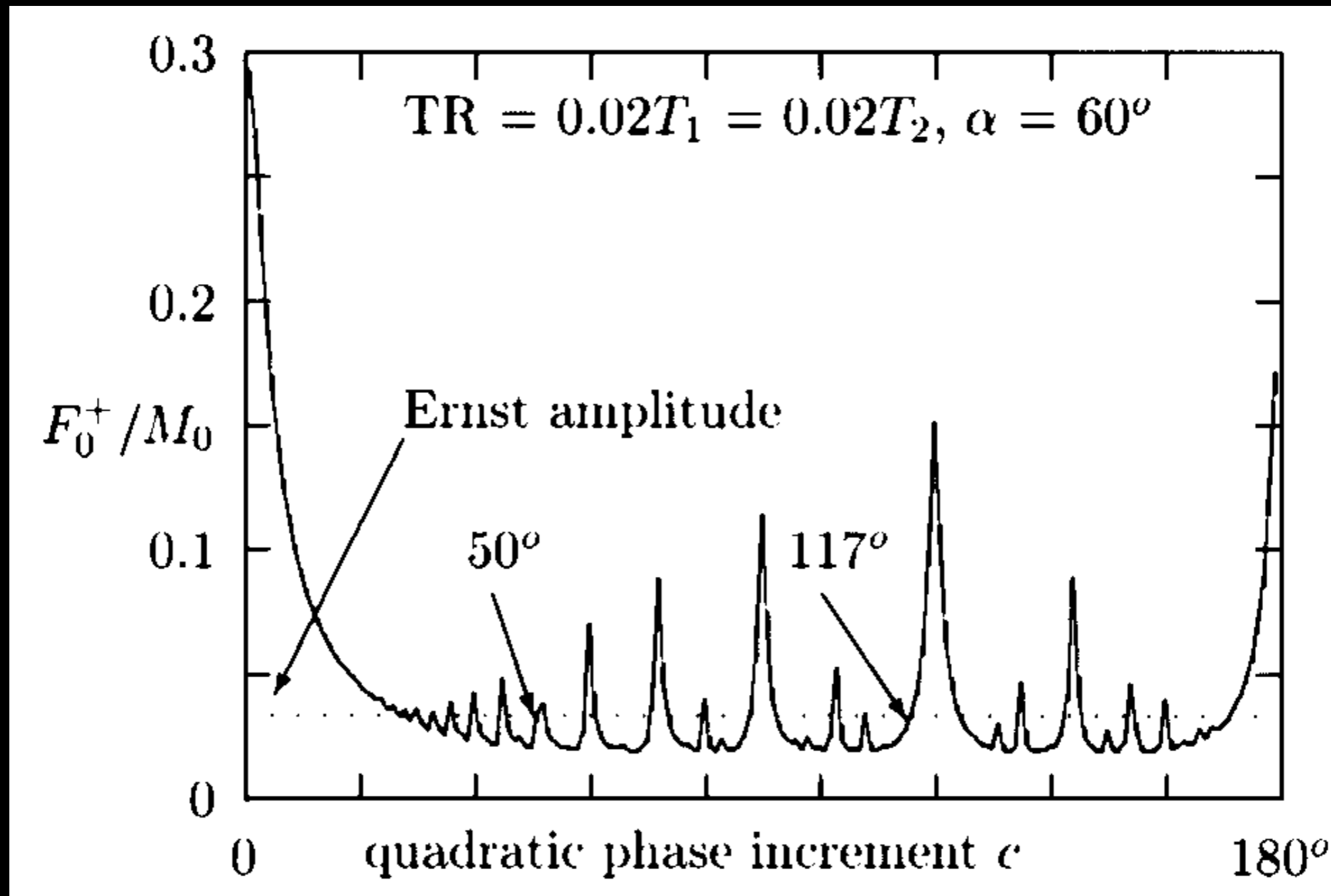


# Gradient and RF-spoiled GRE

- RF spoiling (quadratic)
  - $\phi_n = \phi_{n-1} + n\phi_0 = (1/2)\phi_0(n^2 + n + 2)$
  - typically  $\phi_0 = 50^\circ$  or  $117^\circ$
  - ADC phase each TR also needs to match  $\phi_n$
- $T_1$ -weighted contrast
  - approaches contrast of ideally spoiled GRE
  - at expense of reduced SNR  
(removes T2w contributions)

# Gradient and RF-spoiled GRE

Choice of RF phase increment:



# Gradient and RF-spoiled GRE

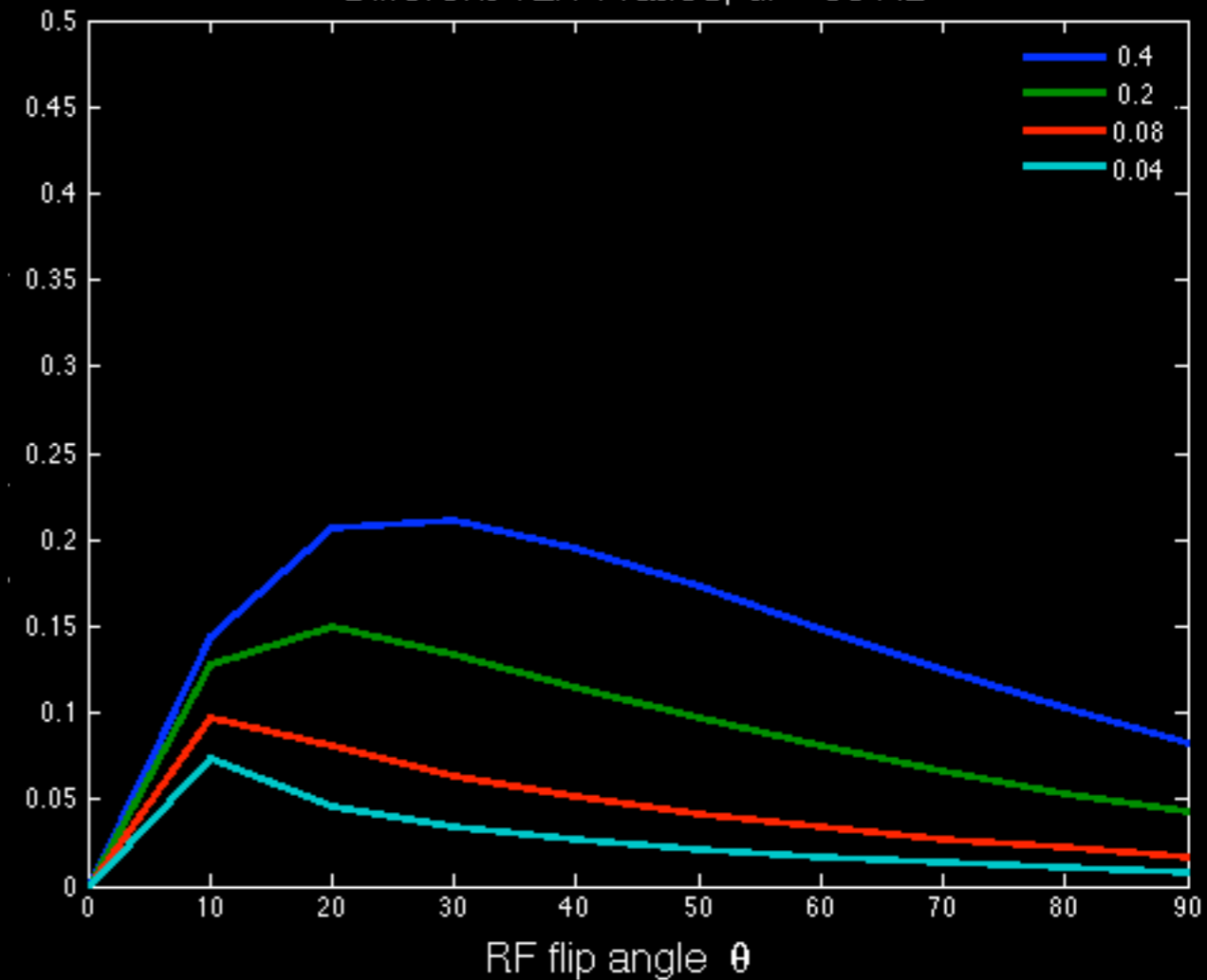
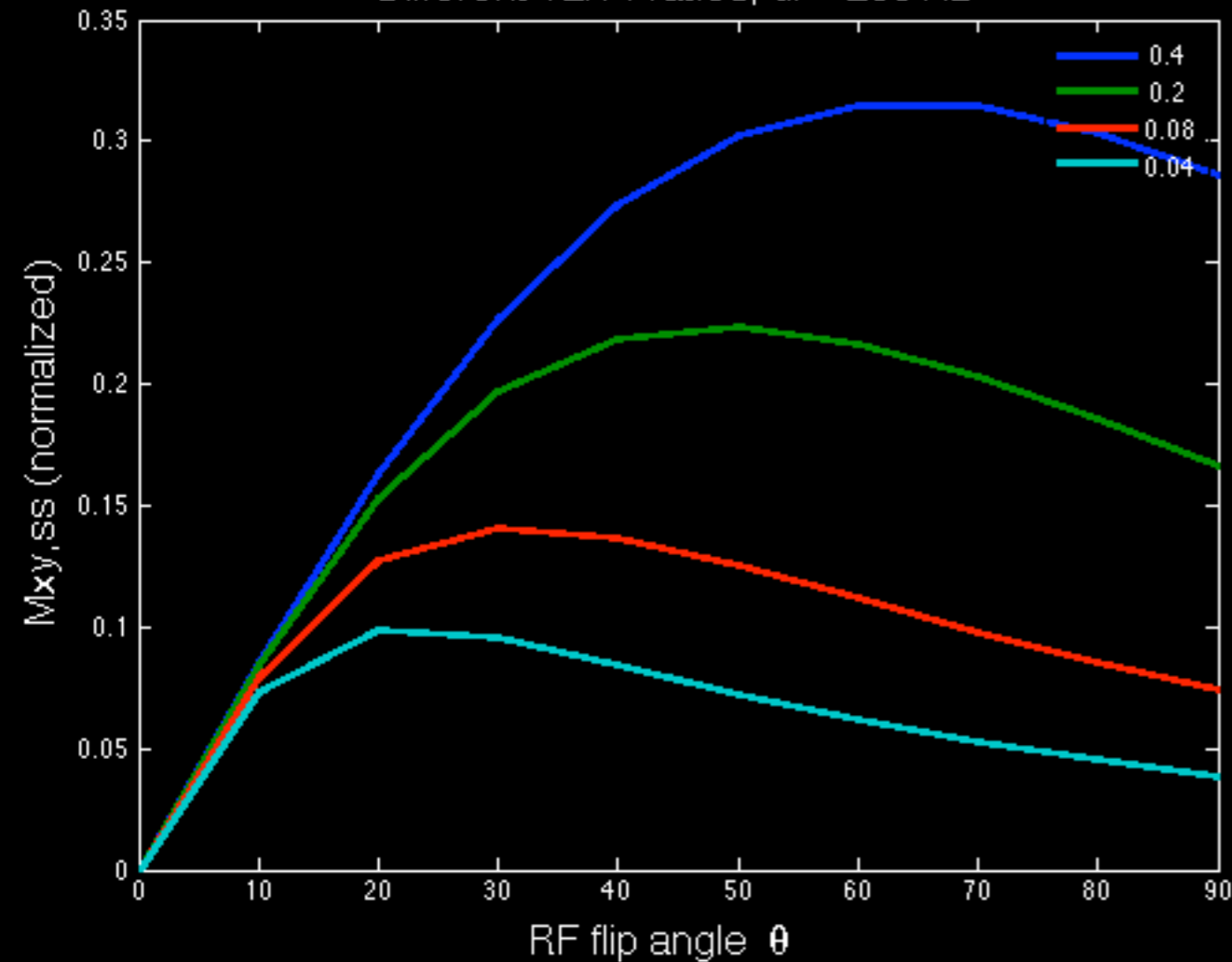
SS signal as a function of flip angle:

*bSSFP*

*Spoiled GRE*

Different T2/T1 ratios, df = 200 Hz

Different T2/T1 ratios, df = 50 Hz

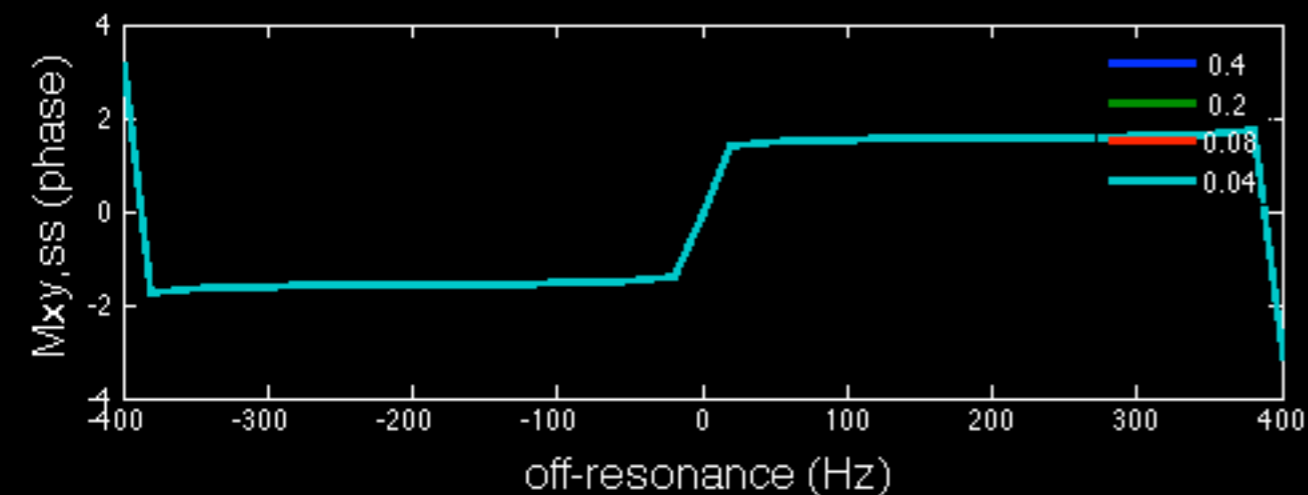
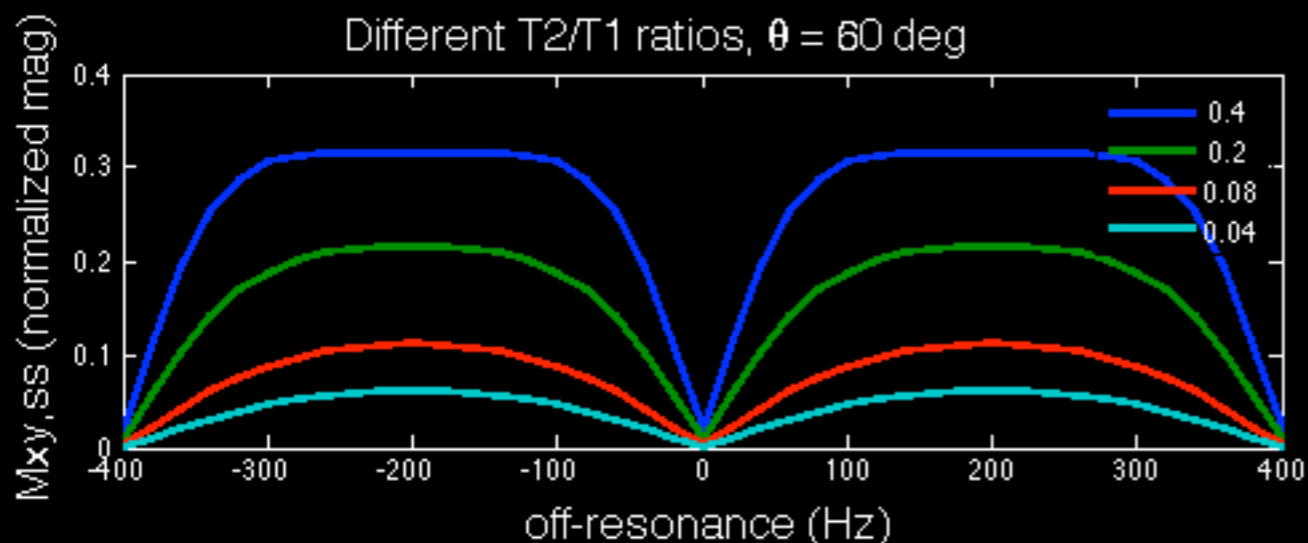


$T_1 = 100, 200, 500, 1000 \text{ ms}, T_2 = 40 \text{ ms}$

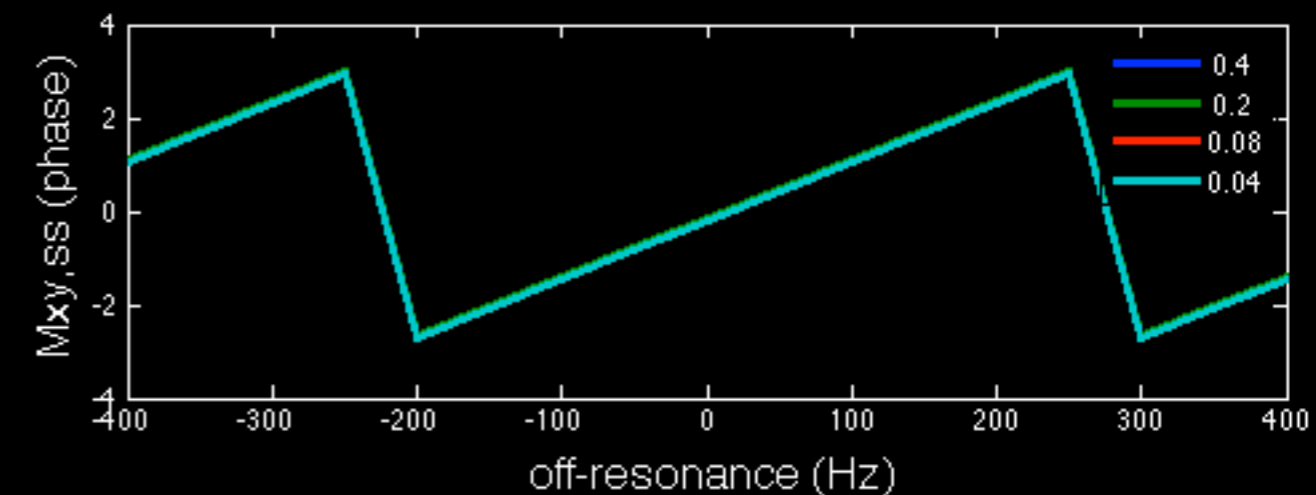
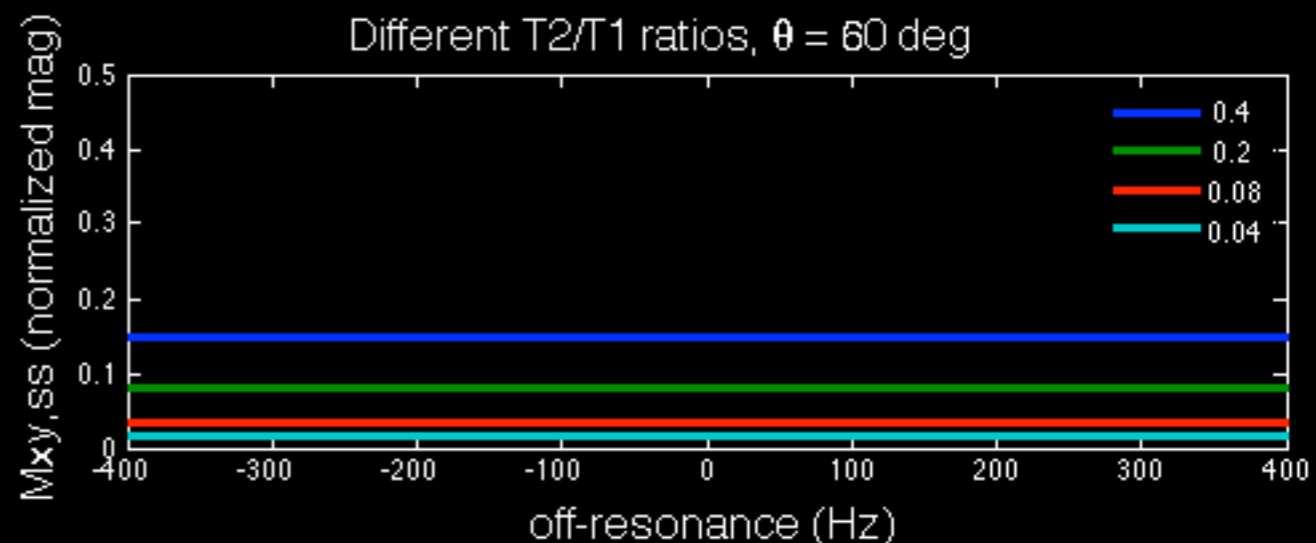
# Gradient and RF-spoiled GRE

SS signal as a function of off-resonance:

*bSSFP*



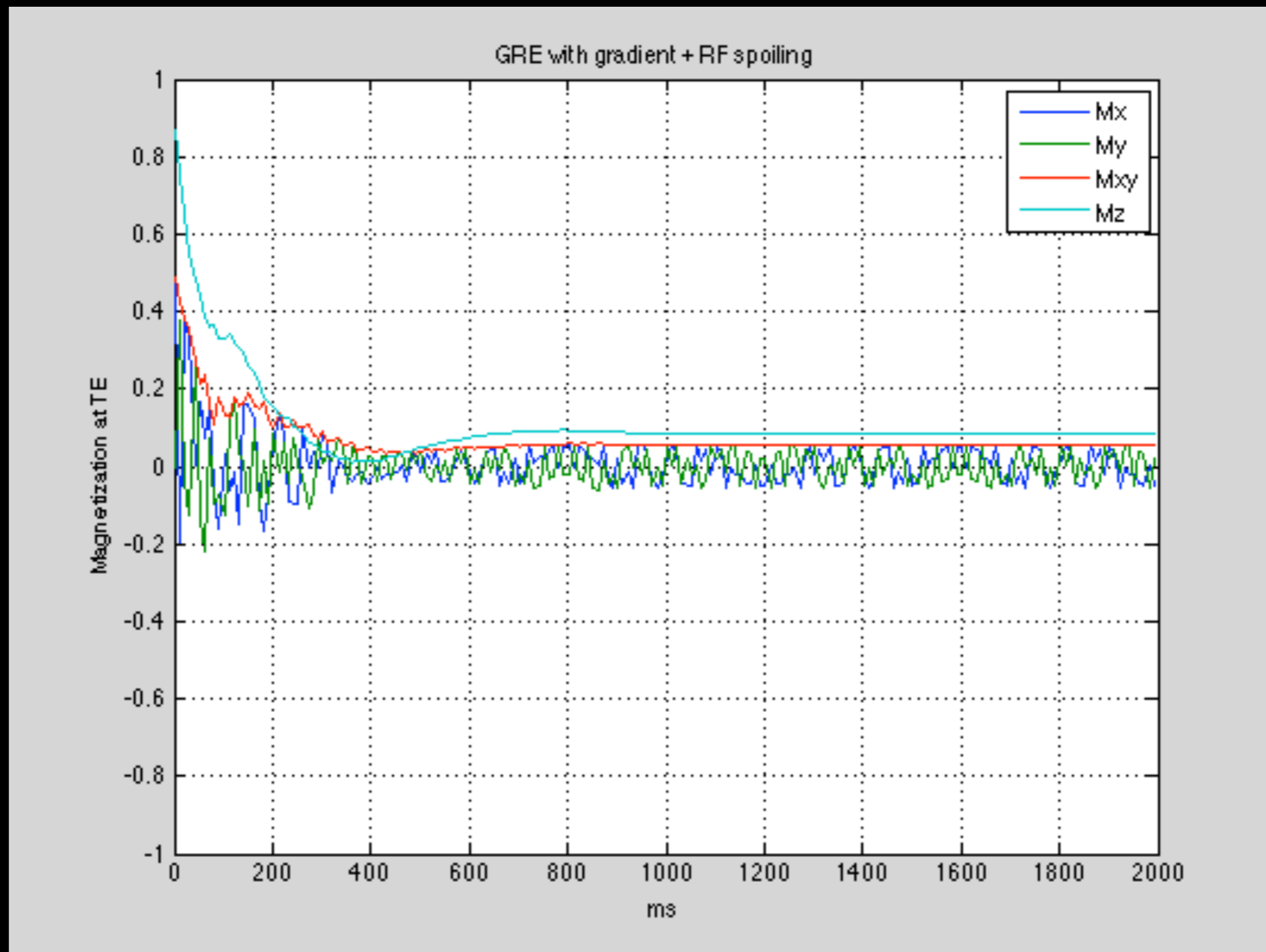
*Spoiled GRE*



$T_1 = 100, 200, 500, 1000$  ms,  $T_2 = 40$  ms

# Gradient and RF-spoiled GRE

Transition to steady state:



$$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}, TE/TR = 2/10 \text{ ms}, \theta = 30^\circ$$

# Gradient and RF-spoiled GRE

- Image characteristics
  - no banding
  - $M_{xy}$  spoiled before next TR
  - T1w contrast with short TR
  - $\theta$  controls degree of  $T_1$  contrast
  - TE controls degree of  $T_2^*$  contrast
  - robust to motion

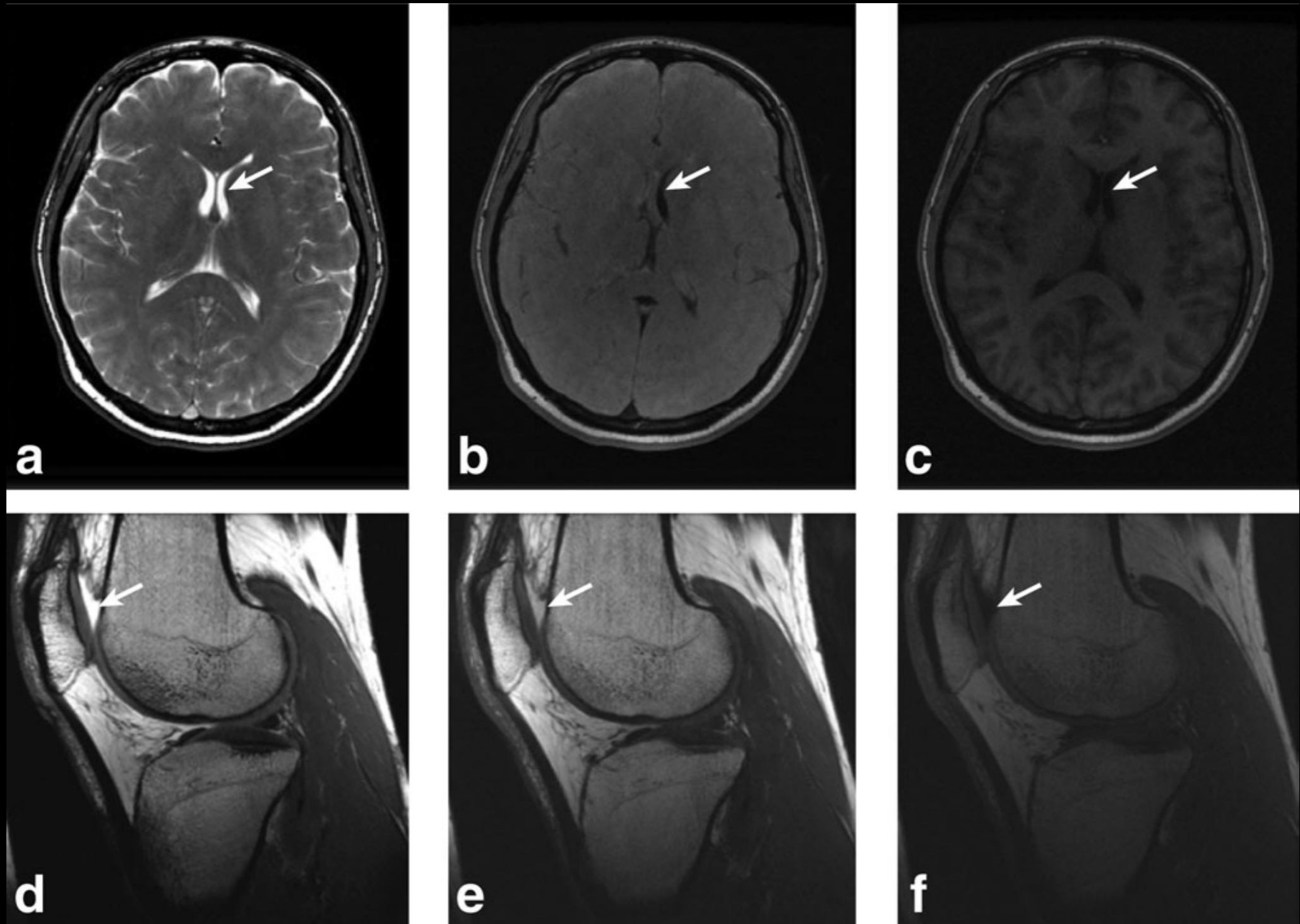


# Rapid GRE - Comparison

*bSSFP*

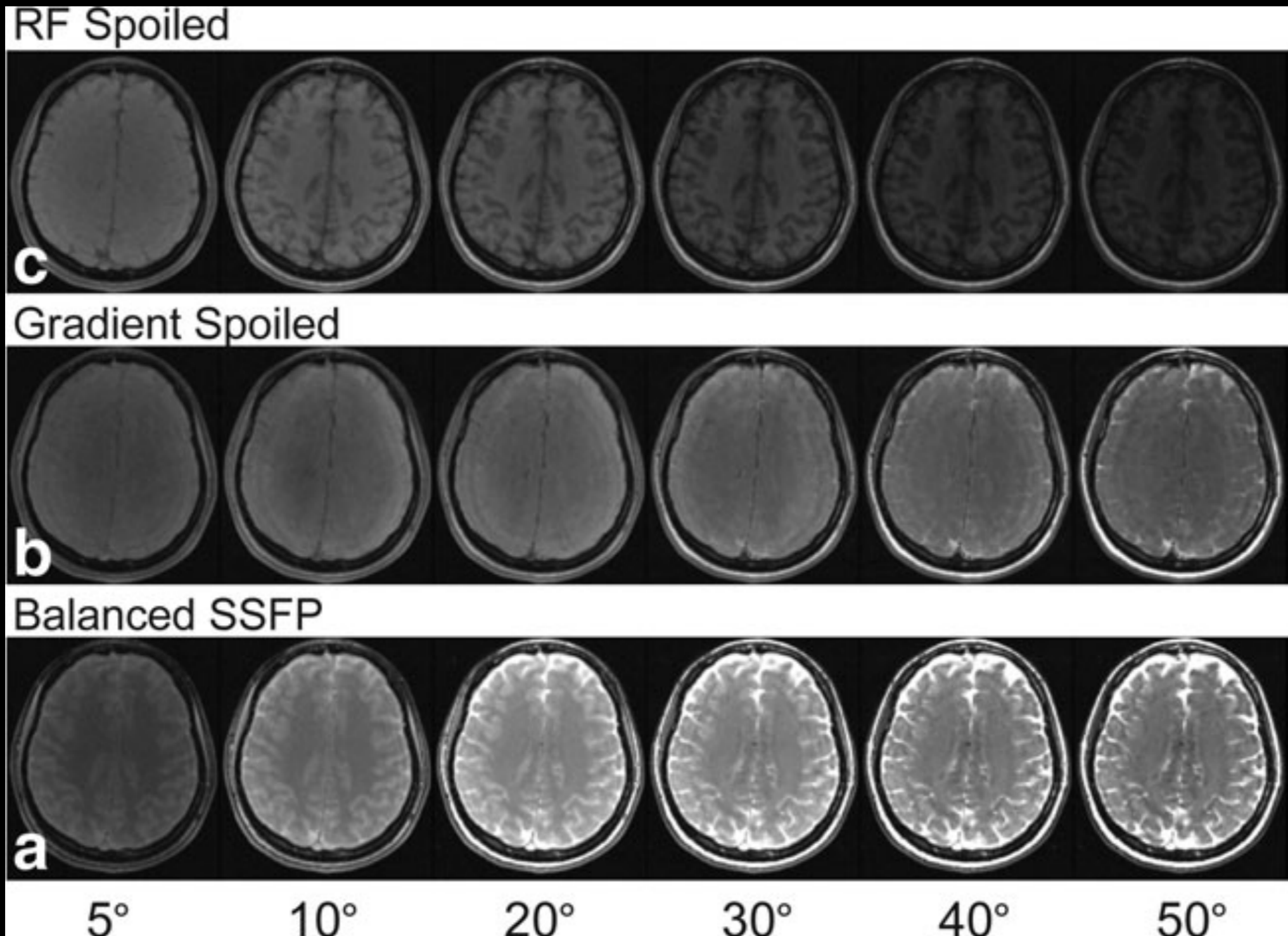
*Grad spoiled*

*RF spoiled*



# Rapid GRE - Comparison

*Flip angle*



# Rapid GRE - Comparison

Pulse Sequence		Mxy	Contrast	SNR	Artifacts
Balanced SSFP	bSSFP	retained	$T_2/T_1$	high	banding
Gradient-spoiled GRE	SSFP-FID	averaged	$T_2/T_1$	mid	motion
	SSFP-Echo	averaged	$T_2+T_2/T_1$	mid	motion
Gradient and RF-spoiled GRE	Spoiled GRE	cancelled	$T_1; T_2^*$	low	minimal

SS transition

# Considerations

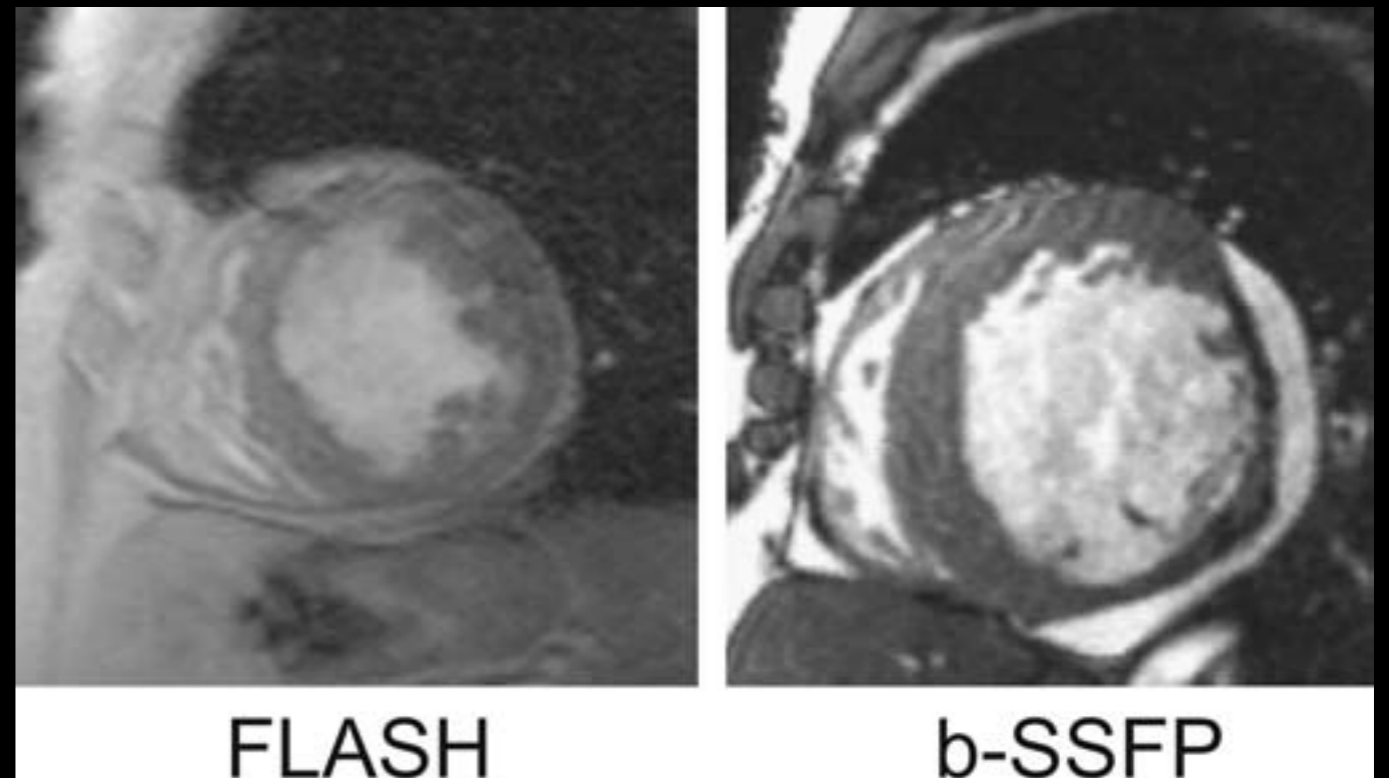
- Chemical shift
- Flow
- Diffusion

# Extensions and Variations

- Partial echo
- Multi-echo
- Ultra-short TE
- Magnetization preparation
- Multiple steady states

# Applications

- bSSFP
  - Cardiac
  - MRA
  - $T_2$ -like imaging
  - fMRI
  - phase contrast
  - Mag-prep



# Applications

- SSFP-FID / Echo
  - $T_2$ -like imaging (e.g., cartilage)
  - Bright fluid (bSSFP-like without banding)
  - Diffusion-weighted imaging (SSFP-Echo)

# Applications

- Spoiled GRE
  - T1w imaging
  - $T_2^*$  BOLD fMRI
  - Susceptibility-weighted imaging (SWI)
  - Phase contrast
  - Thermometry
  - Time-of-flight MRA
  - Contrast-enhanced imaging
  - Mag-prep imaging



# Thanks!

- Web resources
  - ISMRM 2010 Edu: Weigel, Bieri, Miller
  - ISMRM 2011 Edu: Weigel, Miller
  - ISMRM 2012 Edu: Miller, Bieri
- Further reading
  - Bernstein et al., Handbook of MRI Sequences
  - Haacke et al., Magnetic Resonance Imaging
  - Nishimura, Principles of MRI
  - [pubmed.org](http://pubmed.org)

# Thanks!

- Acknowledgments
  - Suba's slides from M219 (2014)
  - Brian Hargreaves's Bloch simulator

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