Compressed Sensing MRI

M229 Advanced Topics in MRI

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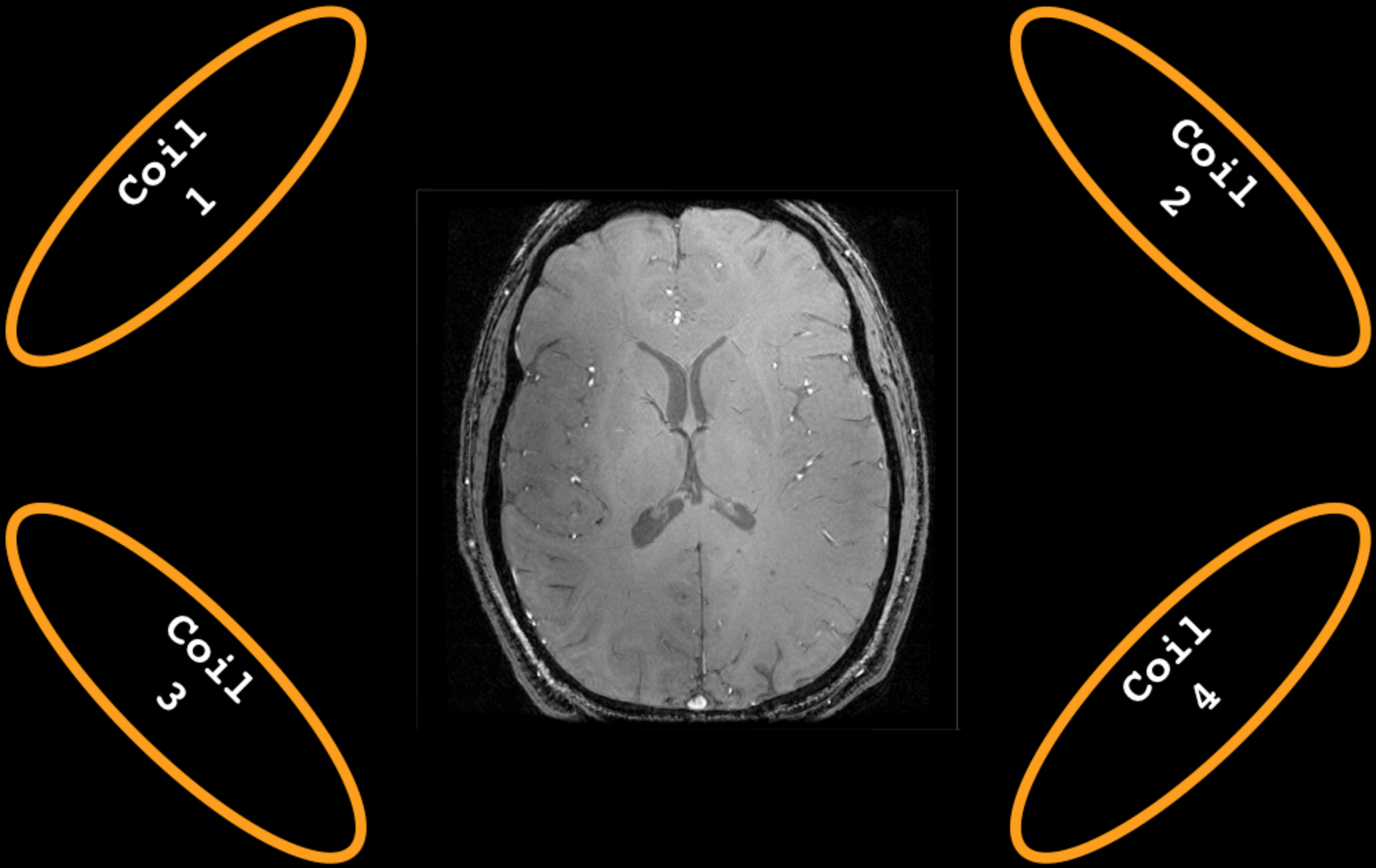
Today's topics

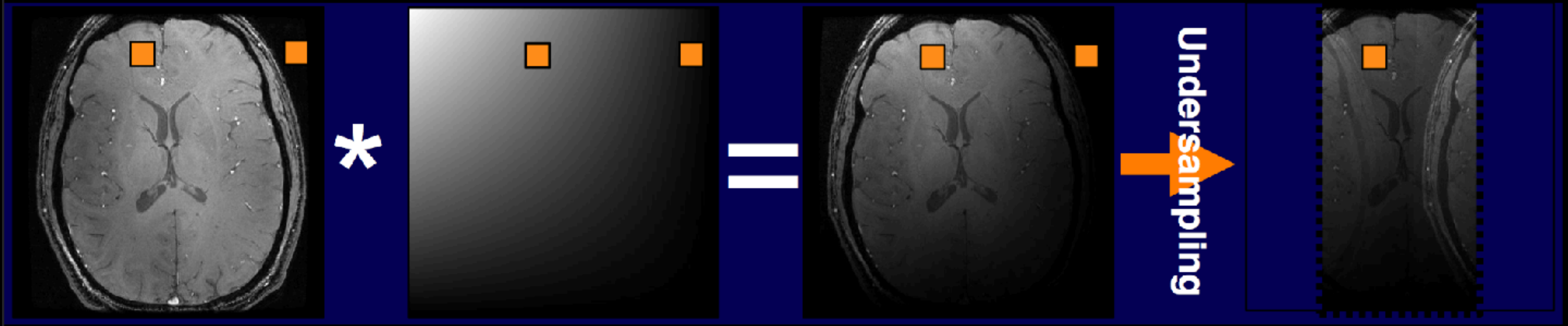
- k-Space properties review
- Compressed sensing MRI (with code examples)
  - Sparse representation
  - Incoherent artifacts
  - Nonlinear reconstruction
- Compressed sensing MRI applications

Multi-coil Arrays

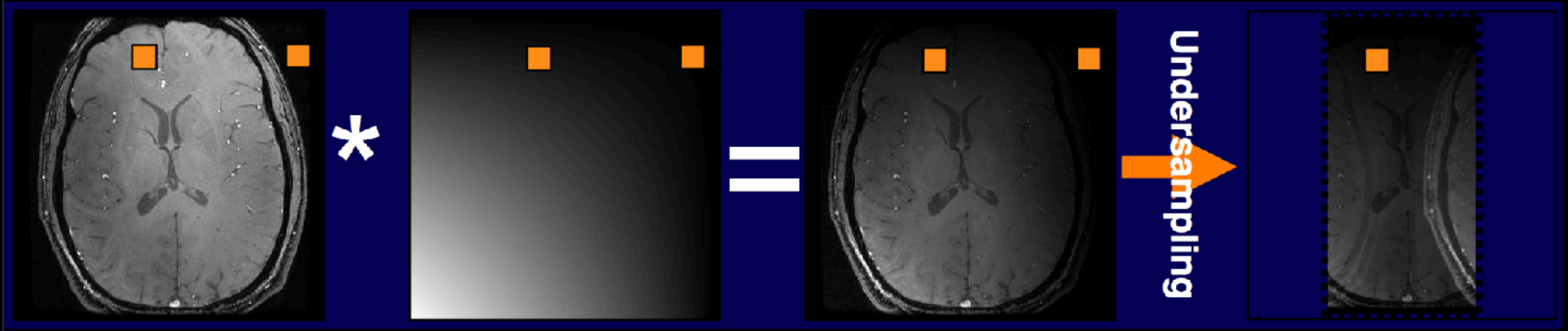


Cartesian SENSE

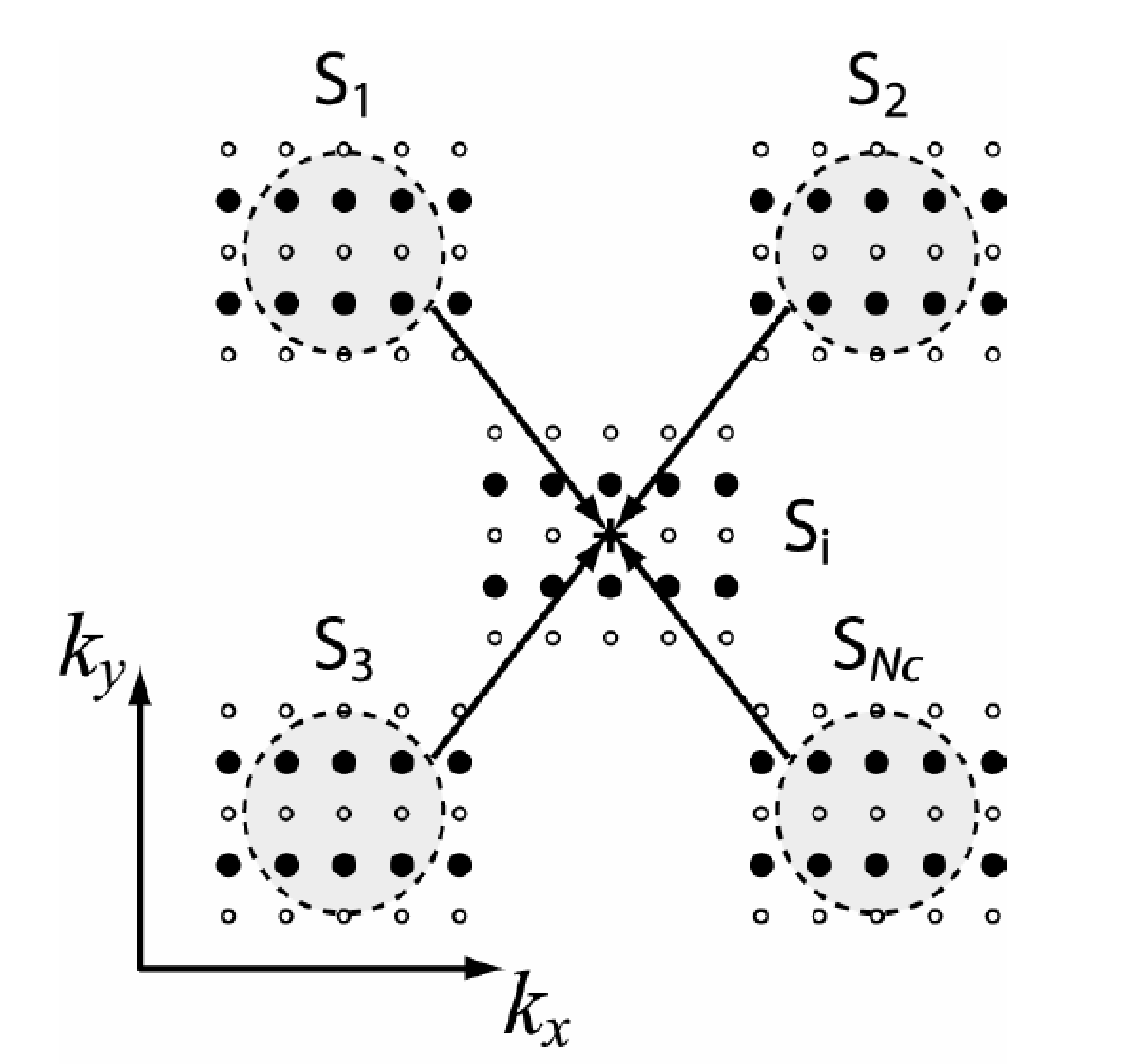
$$m_1(\vec{x_1}) = C_1(\vec{x_1})m(\vec{x_1}) + C_1(\vec{x_2})m(\vec{x_2})$$



$$m_2(\vec{x_1}) = C_2(\vec{x_1})m(\vec{x_1}) + C_2(\vec{x_2})m(\vec{x_2})$$



GRAPPA - Synthesis



Review: Parallel imaging reconstruction

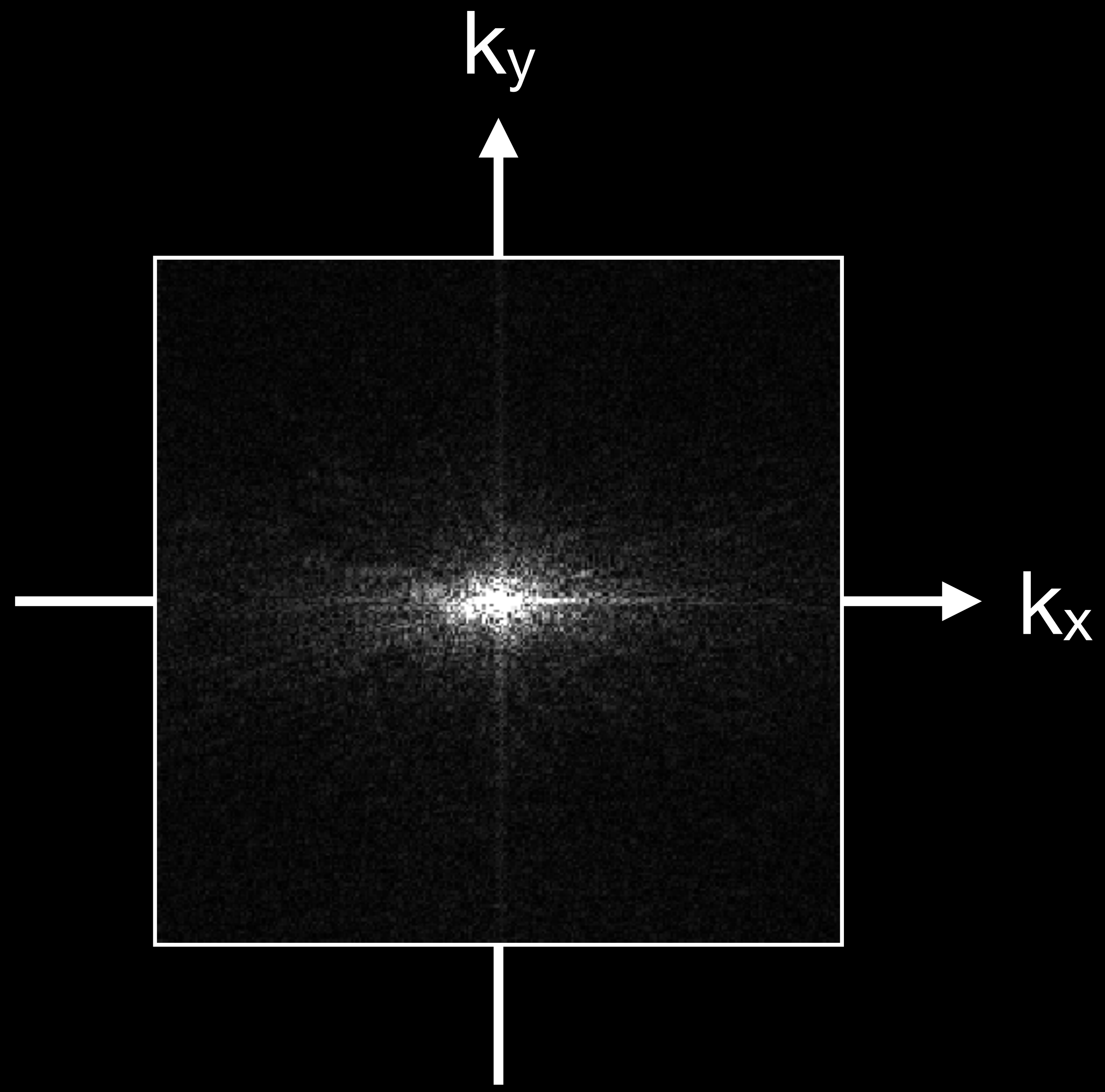
- Parallel imaging utilizes information from multiple coils to accelerate MRI
- Different parallel imaging approaches:
  - SENSE (image-based)
  - GRAPPA (k-space-based)
  - …

Accelerated MRI

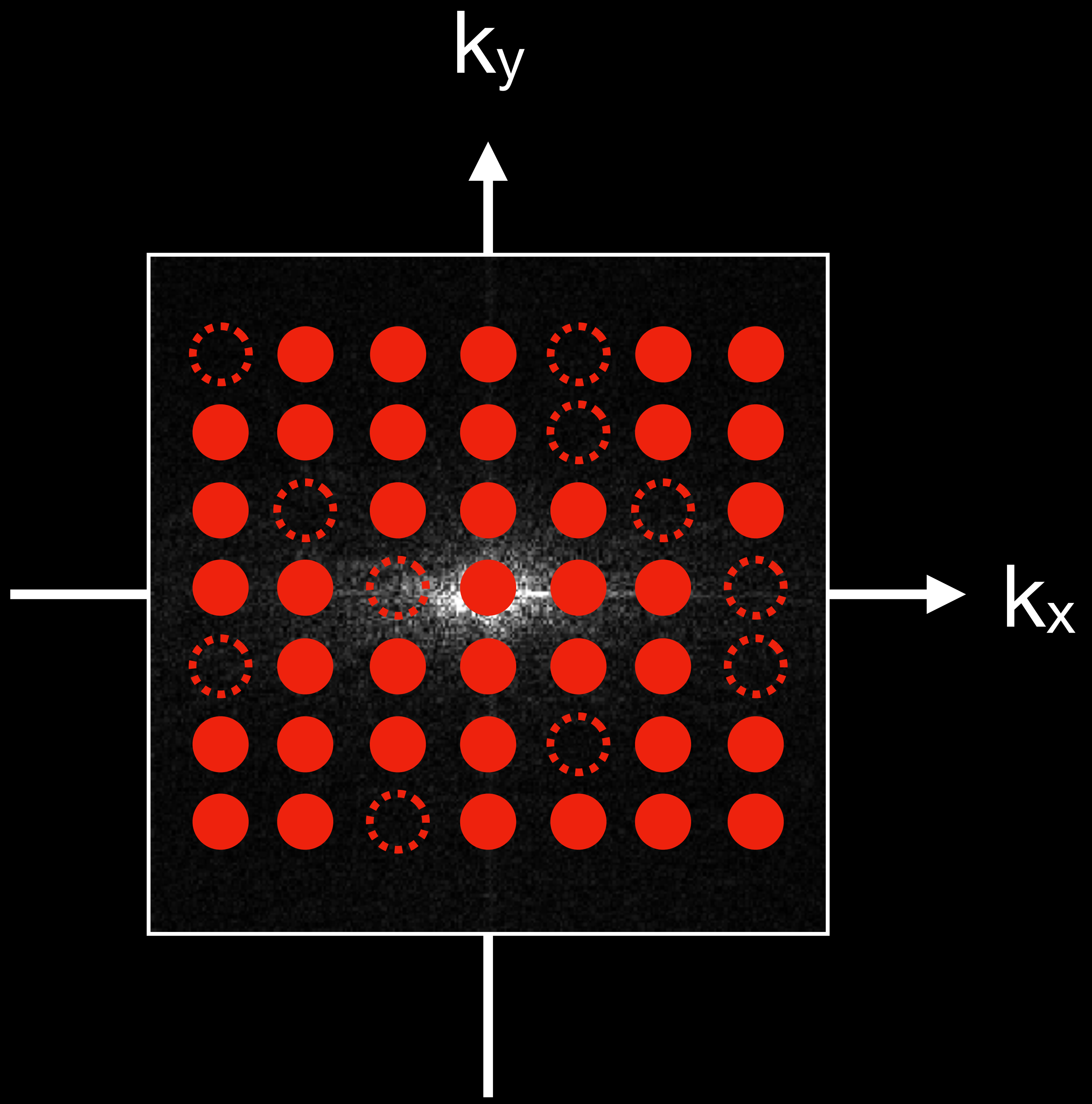
- MRI acquisition time is limited by physics and hardware constraints
- MRI scans can be accelerated by acquiring undersampled k-space data followed by advanced reconstruction
- Accelerated MRI approaches
  - (1) Parallel imaging
    - Use information from multiple coils
  - (2) Compressed sensing
    - Use sparsity constraints as prior information
  - (3) Deep learning
    - Use a nonlinear neural network trained with a large dataset
  - … and more

Underdetermined system

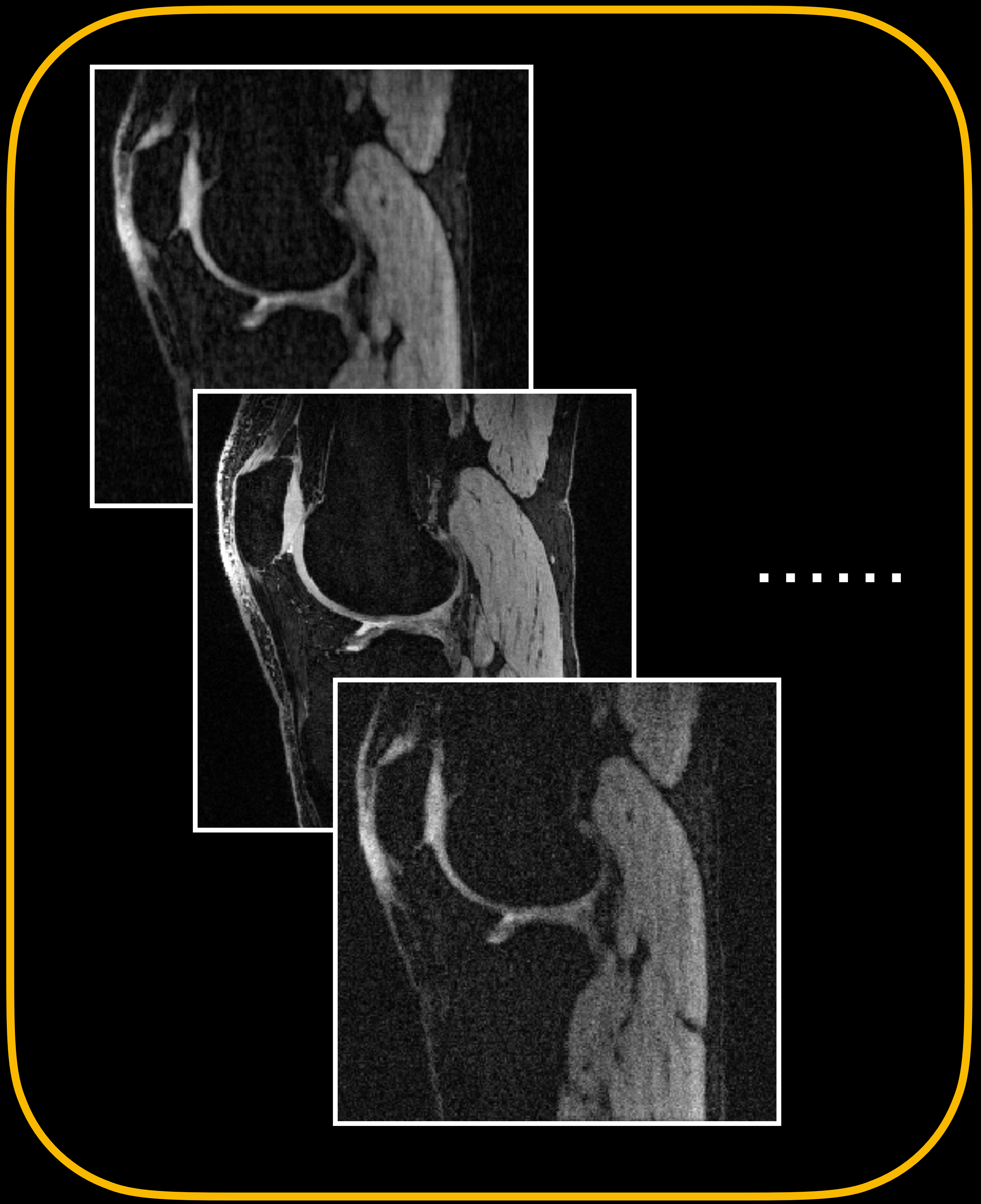
Fully sampled



Undersampled



Images with the same undersampled k-space data



Use prior information about the images to help us solve the underdetermined problem

Compressed sensing MRI

- Compressed sensing MRI can reconstruct an image with high fidelity from undersampled k-space data given
  - (1) the image has transform sparsity (or a sparse representation in some transform domain)
  - (2) the k-space sampling pattern generates incoherent artifacts in the sparse transform domain
- Compressed sensing MRI usually involves a nonlinear reconstruction method to recover the image

Sparse representation

- Many images have a sparse representation in some transform domain
- Example 1: Discrete cosine transform (DCT)
  - JPEG uses DCT for image compression

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right] \qquad for\ k = 0, \ldots N-1$$

Original image

2D DCT coefficients

Compressed image (3.7-fold)
by preserving large DCT coefficients

See code example 01

Sparse representation

- Example 2: Wavelet transform
  - JPEG 2000 uses Wavelet transform for image compression

Original image

2D Wavelet coefficients

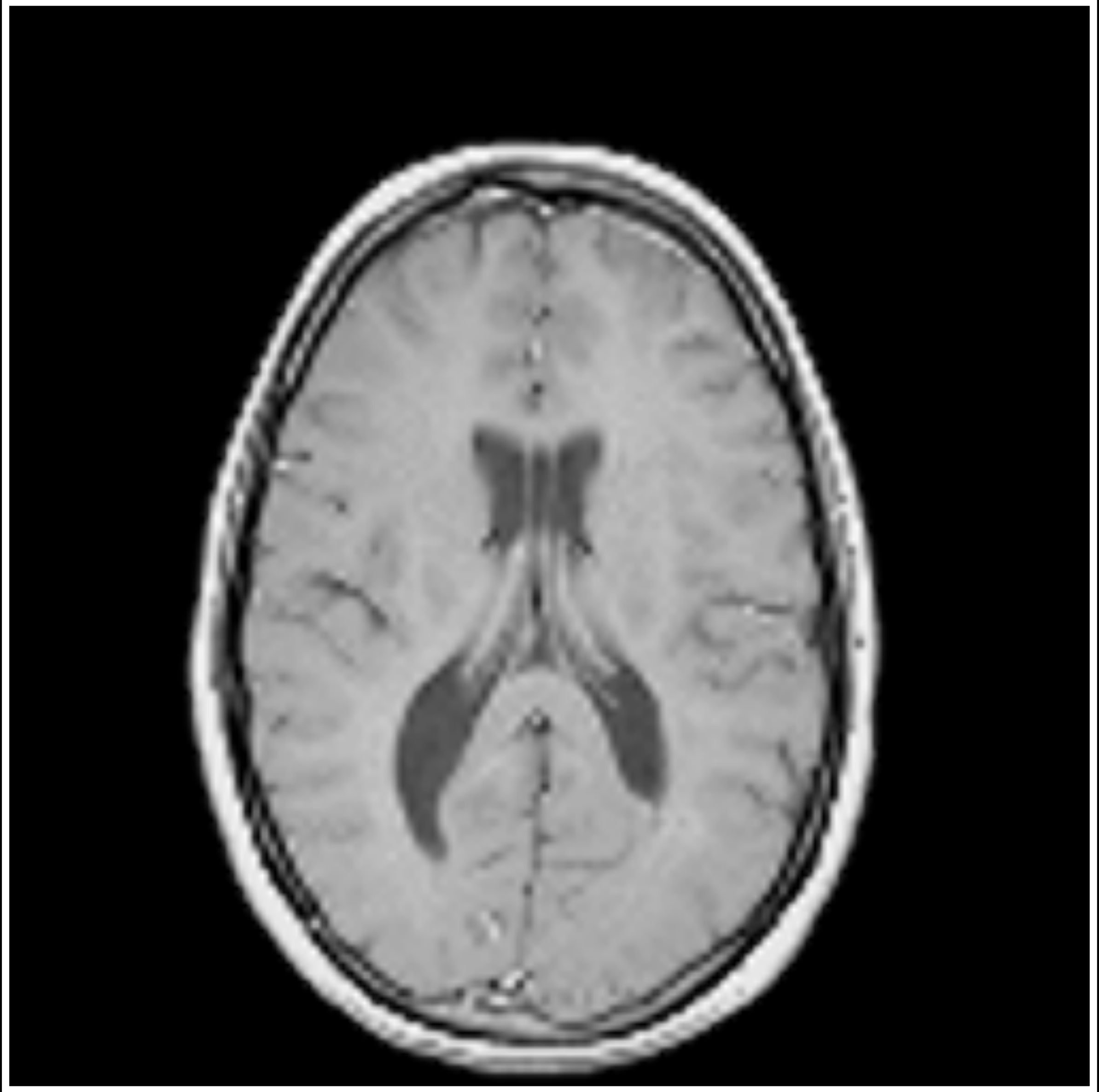
Compressed image (5.3-fold)
by preserving large Wavelet coefficients

See code example 02

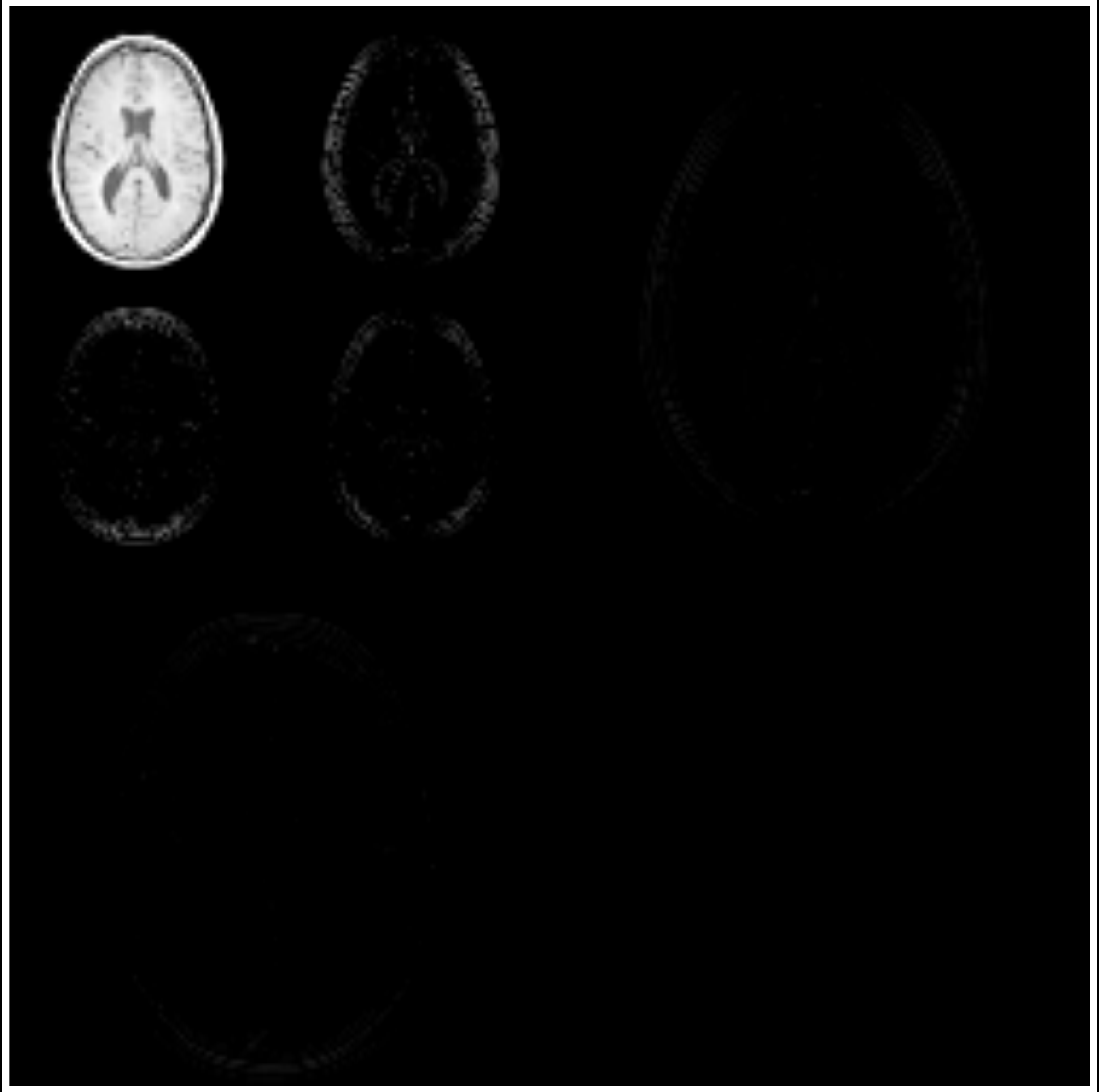
Sparse representation

- Example 3: Wavelet transform for a brain image

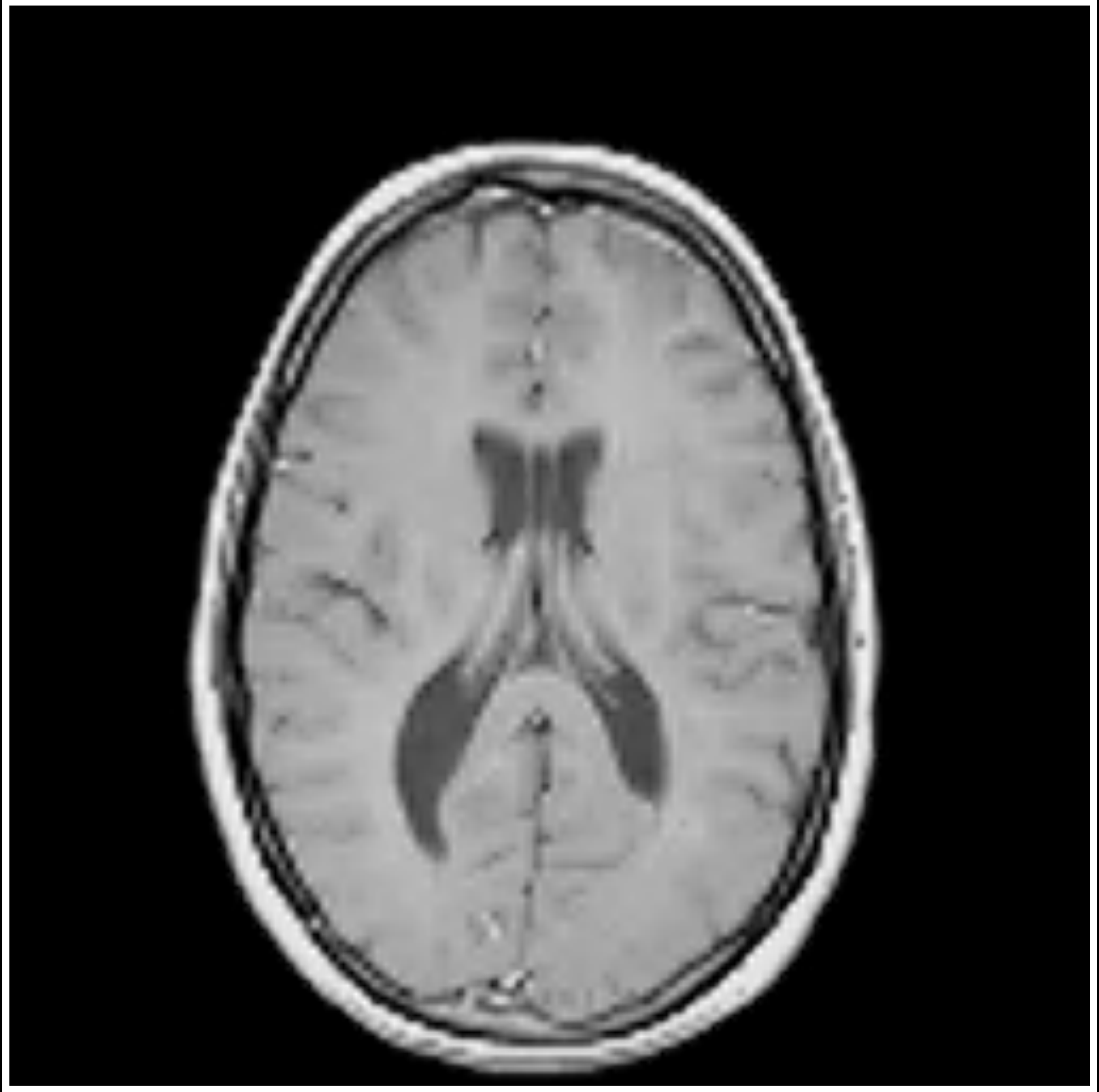
Original image



2D Wavelet coefficients



Compressed image (4.8-fold)
by preserving large Wavelet coefficients

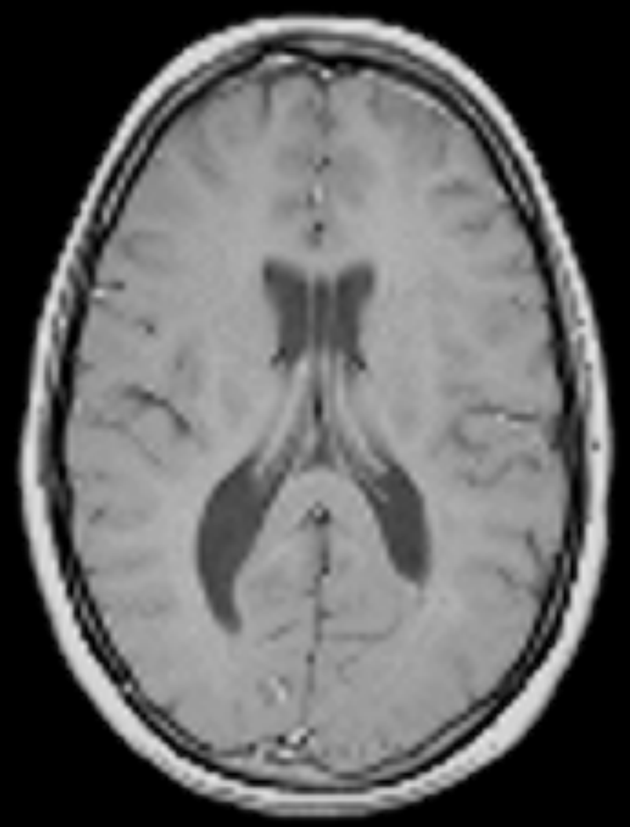


See code example 03

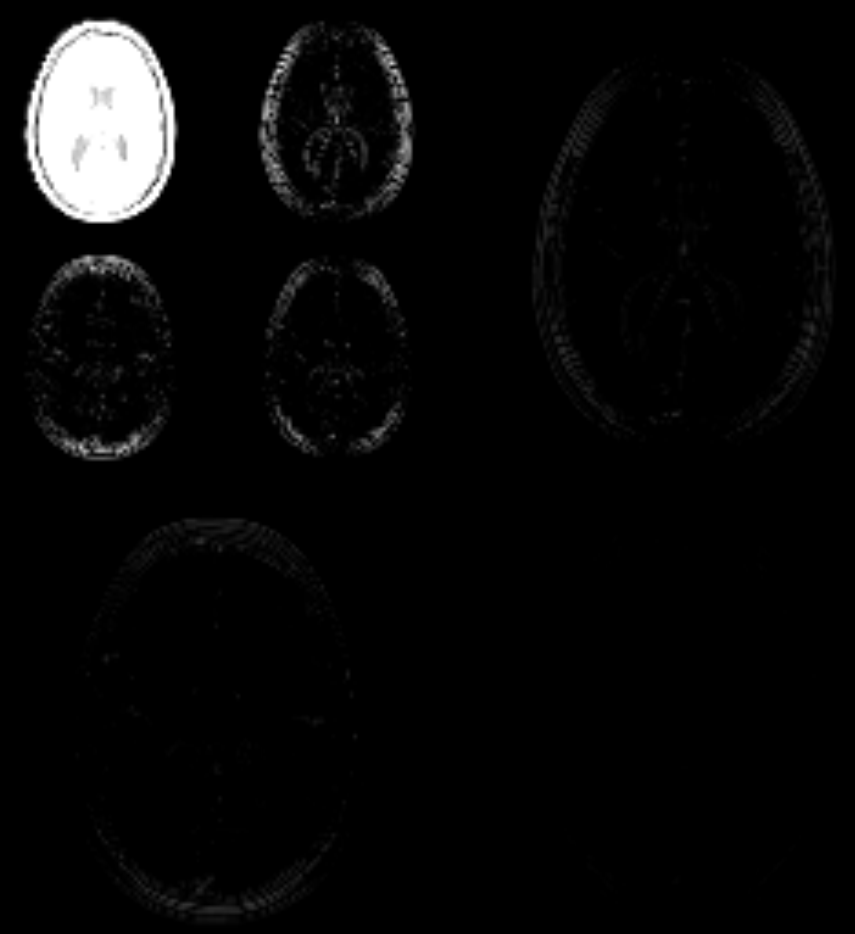
Sparse representation

- Many images have a sparse representation in some transform domain

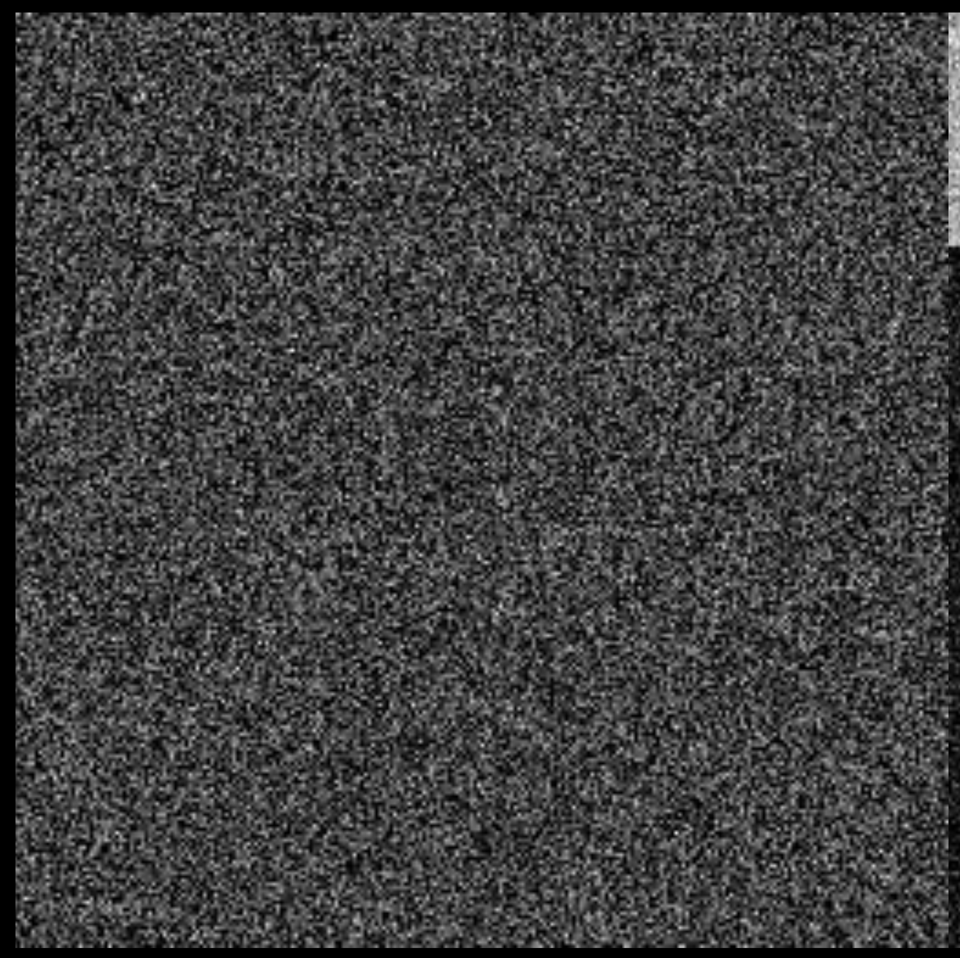
Brain image



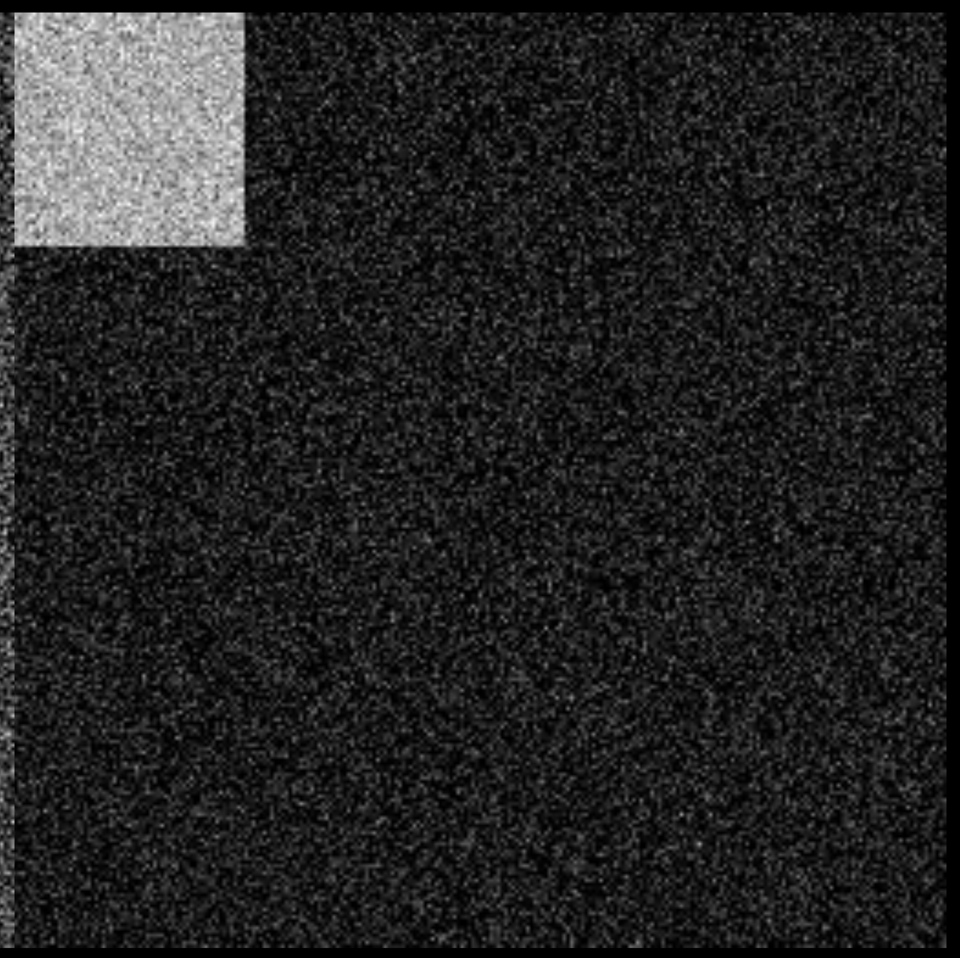
2D Wavelet coefficients of a brain image



Noisy image



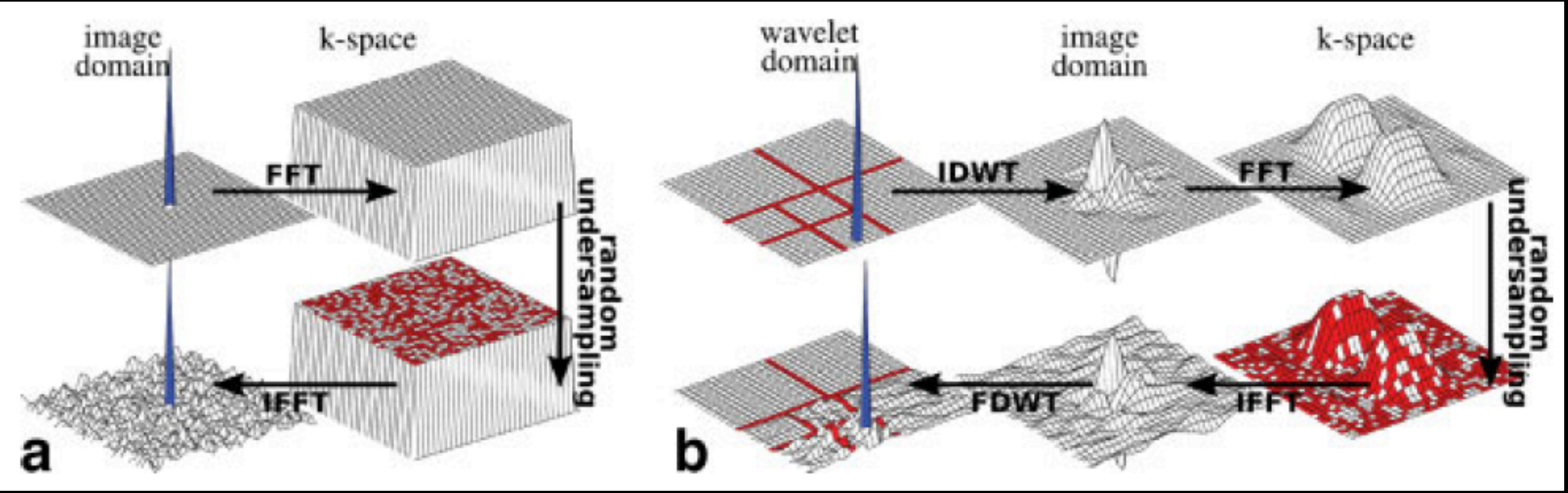
2D Wavelet coefficients of a noisy image



See code example 04

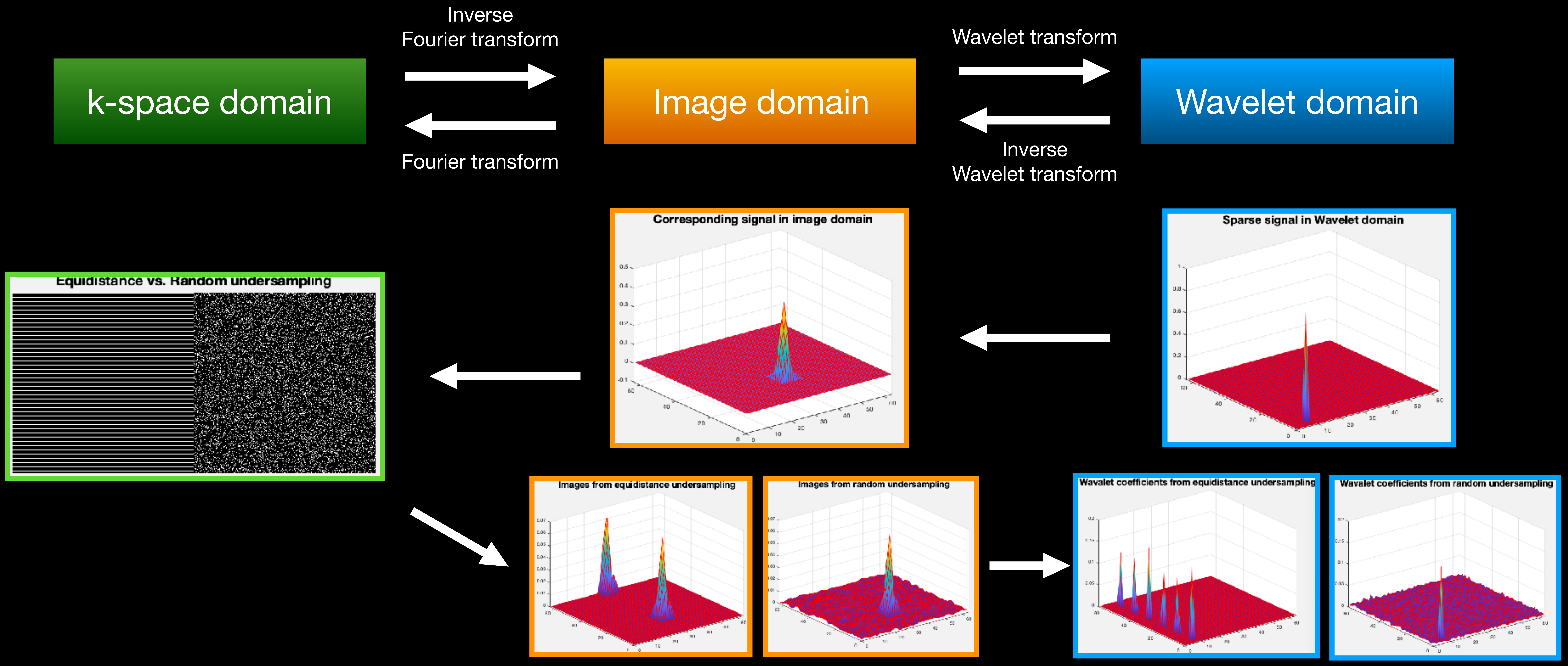
Incoherent artifacts

- The second requirement for CS MRI is that the undersampling pattern should generate incoherent artifacts in the sparse transform domain



(Figure from: Lustig et al., MRM 2007)

Incoherent artifacts



See code example 05

L0, L1 and L2 norm

- Vector norm: a method to measure the length of a vector

- L_0 norm ($\| x \|_0$): number of non-zero entries

- L_1 norm ($\| x \|_1$): sum of absolute values of the entries

$$\| x \|_1 = |x_1| + |x_2| + \ldots + |x_n|$$

- L_2 norm ($\| x \|_2$): square root of sum of squared values of the entries

$$\| x \|_2 = \sqrt{|x_1|^2 + |x_2|^2 + \ldots + |x_n|^2}$$

L0, L1 and L2 norm

$$v_1 = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 2 \\ -3 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 2 \\ -4 \\ 2 \end{bmatrix}$$

- $\| v_1 \|_2 = \sqrt{38}$
- $\| v_1 \|_1 = 10$
- $\| v_1 \|_0 = 3$

- $\| v_2 \|_2 = \sqrt{38}$
- $\| v_2 \|_1 = 14$
- $\| v_2 \|_0 = 6$

- Two vectors with similar energy (L_2 norm) can have different levels of sparsity (L_1 norm)

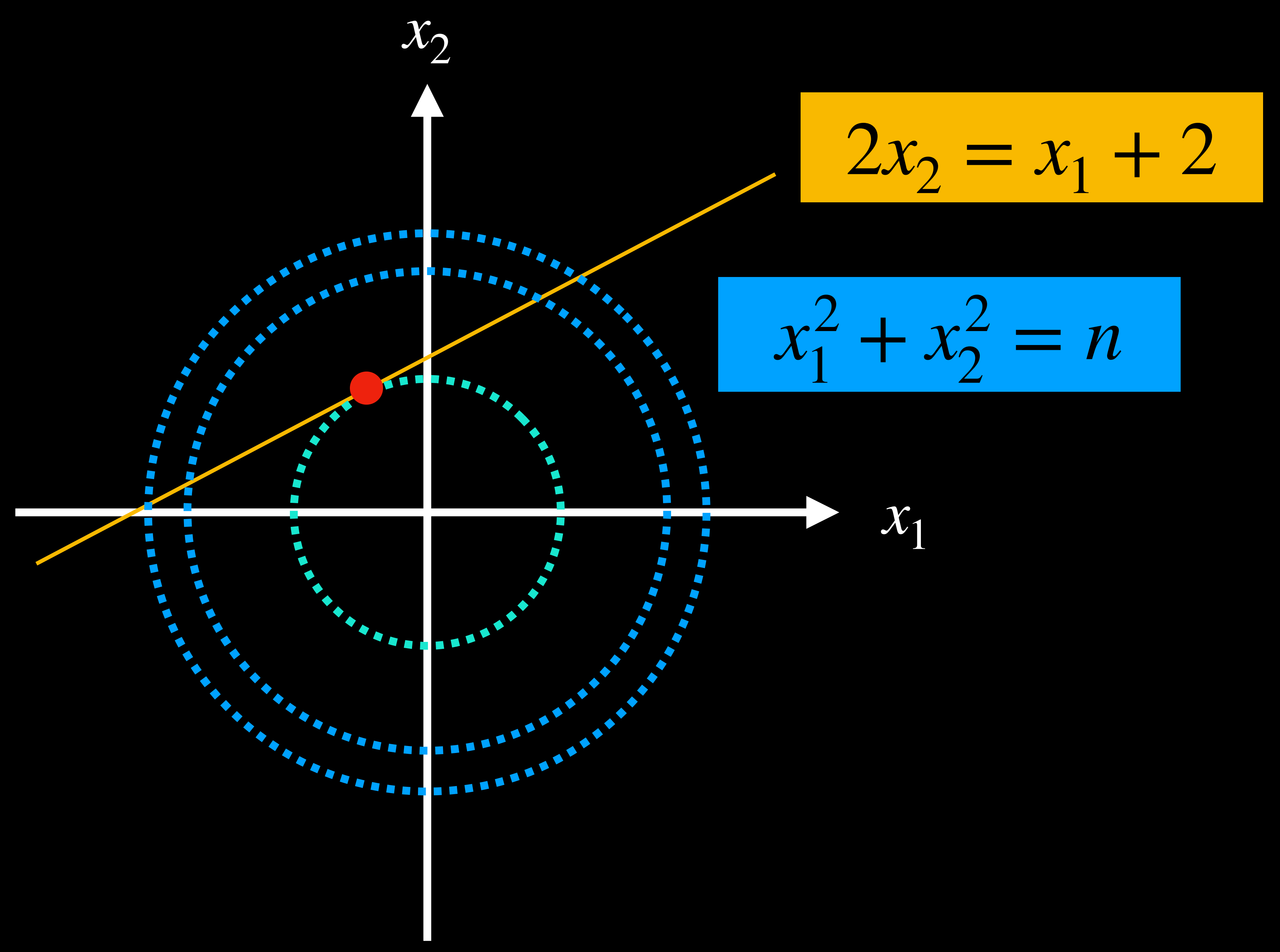
Exercises

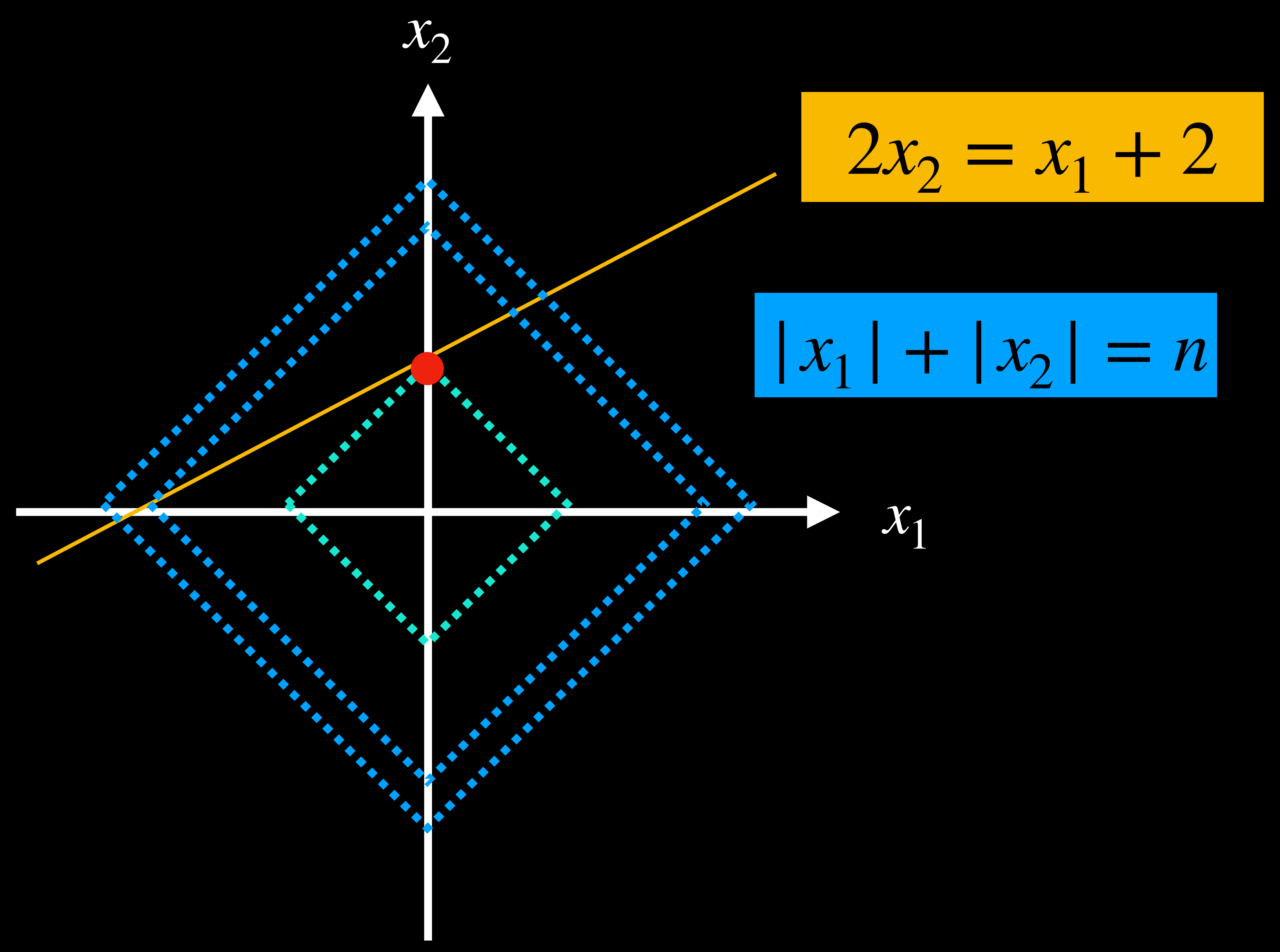
- Suppose we have a 2D vector $x = [x_1, x_2]$

- Exercise 1: $$argmin_x \quad ||x||_2$$ $$s.t. \quad 2x_2 = x_1 + 2$$

- Exercise 2: $$argmin_x \quad ||x||_1$$ $$s.t. \quad 2x_2 = x_1 + 2$$

- Example 3: $$argmin_x \quad ||x||_0$$ $$s.t. \quad 2x_2 = x_1 + 2$$





Exercises

- L_2 norm minimization: Find a solution with smallest energy
- L_1 and L_0 norm minimization: Find a sparse solution

Mathematical formulation

Our goal: Find an image that has the sparsest coefficients in the Wavelet domain and the image is consistent with the undersampled k-space data

Turn into an optimization problem →

$$\operatorname{argmin}_x \quad \| Wx \|_0$$
$$\text{subject to} \quad \| Fx - y \|_2 < \epsilon$$

Convex relaxation using L1 norm →

$$\operatorname{argmin}_x \quad \| Wx \|_1$$
$$\text{subject to} \quad \| Fx - y \|_2 < \epsilon$$

W: Wavelet transform operator
x: reconstructed image
F: Fourier transform operator
y: acquired undersampled k-space data

Mathematical formulation

W: Wavelet transform operator
x: reconstructed image
F: Fourier transform operator
y: acquired undersampled k-space data
λ: regularization parameter
U: k-space sampling pattern

$$argmin_x \quad \| Wx \|_1$$
$$subject\ \ to \quad \| Fx - y \|_2 < \epsilon$$

Use Lagrangian form

$$\rightarrow \quad argmin_x \quad \| Fx - y \|_2^2 + \lambda \| Wx \|_1$$

Explicitly include an sampling operator

$$\rightarrow \quad argmin_x \quad \| UFx - y \|_2^2 + \lambda \| Wx \|_1$$

Mathematical formulation

W: Wavelet transform operator
x: reconstructed image
F: Fourier transform operator
y: acquired undersampled k-space data
λ: regularization parameter
U: k-space sampling pattern

$$argmin_x \quad \| Wx \|_1$$

$$subject\ to \quad \| Fx - y \|_2 < \epsilon$$

Use Lagrangian form

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Explicitly include an sampling operator

$$\rightarrow \quad argmin_x \quad \| UFx - y \|_2^2 + \lambda \| Wx \|_1$$

Cost function

Optimization algorithm

- Solving $\min \| UFx - y \|_2^2 + \lambda \| Wx \|_1$ is non-trivial since the cost function is not smoothed at Wx=0

- Different approaches have been used to solve $\min \| UFx - y \|_2^2 + \lambda \| Wx \|_1$
  - Conjugate gradient descent[1]
  - ADMM[2,3]
  - Primal-dual algorithm[4]
  - …

[1] Lustig et al., Magn Reson Med. 2007;58(6):1182-95
[2] Wang et al., SIAM J Imag Sci. 2008;1(3):248-72
[3] Ramani et al., IEEE Trans Med Imaging. 2011;30(3):694-706
[4] Chambolle et al., J Math Imaging Vision. 2011;40(1):120-45

Optimization algorithm

- Conjugate gradient descent

$$argmin_m \quad f(m) = \| UFm - y \|_2^2 + \lambda \| Wx \|_1$$

% Initialization

$k = 0;\ m = 0;\ g_0 = \nabla f(m_0);\ \Delta m_0 = -g_0$

% Iterations

while $(||g_k||_2 < \text{TolGrad and } k > \text{maxIter})$ {

% Backtracking line-search

$t = 1;$ *while* $(f(m_k + t\Delta m_k) > f(m_k) + \alpha t \cdot Real(g_k^* \Delta m_k))$

$\{t = \beta t\}$

$m_{k+1} = m_k + t\Delta m_k$

$g_{k+1} = \nabla f(m_{k+1})$

$\gamma = \frac{||g_{k+1}||_2^2}{||g_k||_2^2}$

$\Delta m_{k+1} = -g_{k+1} + \gamma \Delta m_k$

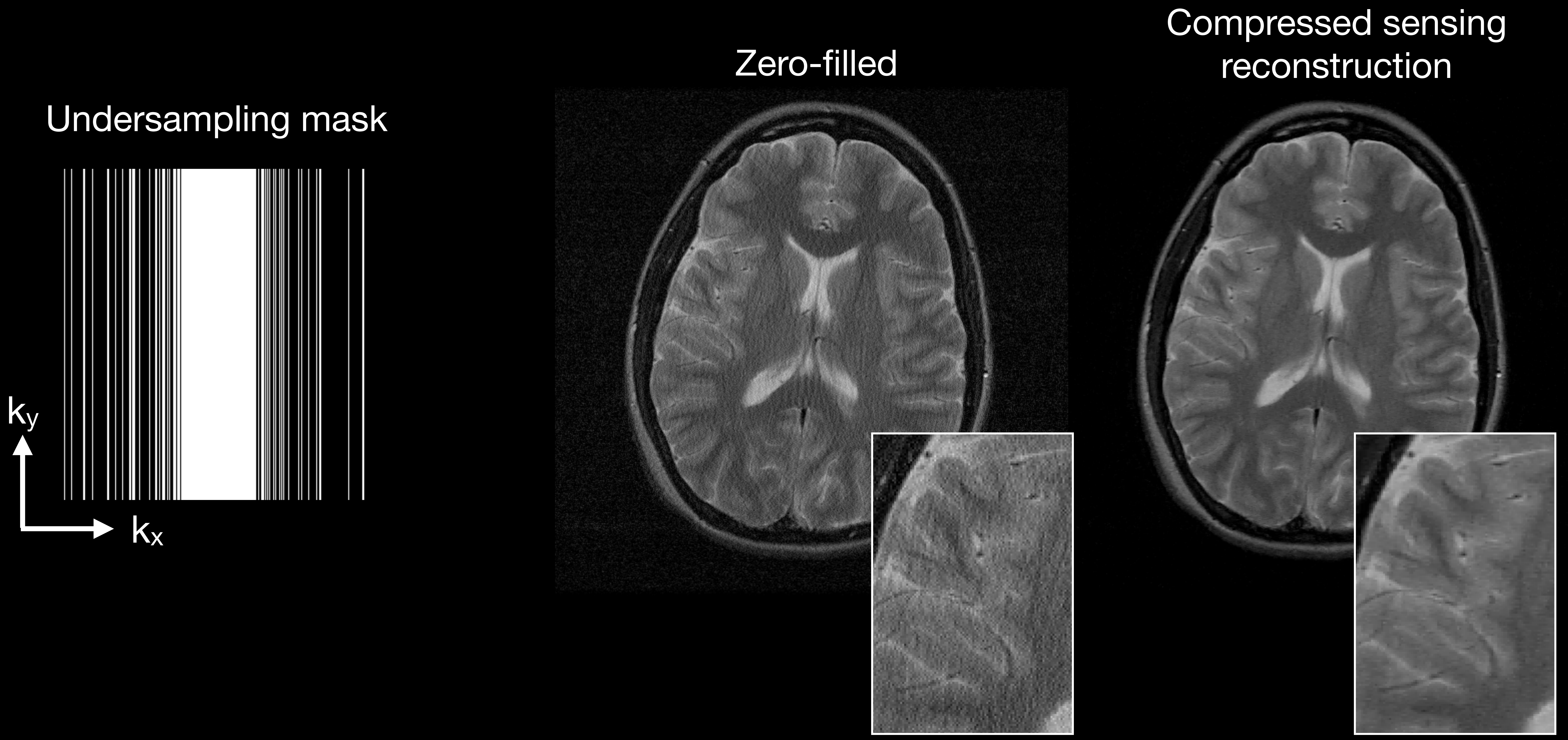
$k = k + 1$ }

g_k: gradient at k[th] iteration
m_k: updated image result at k[th] iteration
TolGrad: stopping criteria
MaxIter: stopping criteria on iterations
α, β: line search parameters

From: Lustig et al., MRM 2007

Compressed sensing MRI

- Let's run codes to reconstruct images using compressed sensing MRI… (see *code example 06*)



Compressed sensing MRI

- Compressed sensing MRI can reconstruct an image with high fidelity from undersampled k-space data given
  - (1) the image has transform sparsity (or a sparse representation in some transform domain)
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- Compressed sensing MRI usually involves a nonlinear reconstruction method to recover the image

Choice of regularization parameters

$$argmin_x \quad \| UFx - y \|_2^2 + \lambda \| Wx \|_1$$

- Many compressed sensing methods require manually tuning of regularization parameters.
- Larger weights on the sparsity term (larger λ):
  - Better suppression on noise or artifacts / Improved perceived SNR
  - Features more likely to be over-smoothed / Resulting in images with artificial appearance
- The regularization parameter is dataset-dependent
- Methods for automatic regularization parameters selection have been investigated

Compressed sensing + Parallel imaging

- Parallel imaging: Use information from multiple coils (e.g., coil sensitivity in SENSE reconstruction)
- Compressed sensing: Use sparsity constraints

- Combination of these two techniques:

Coil sensitivity maps

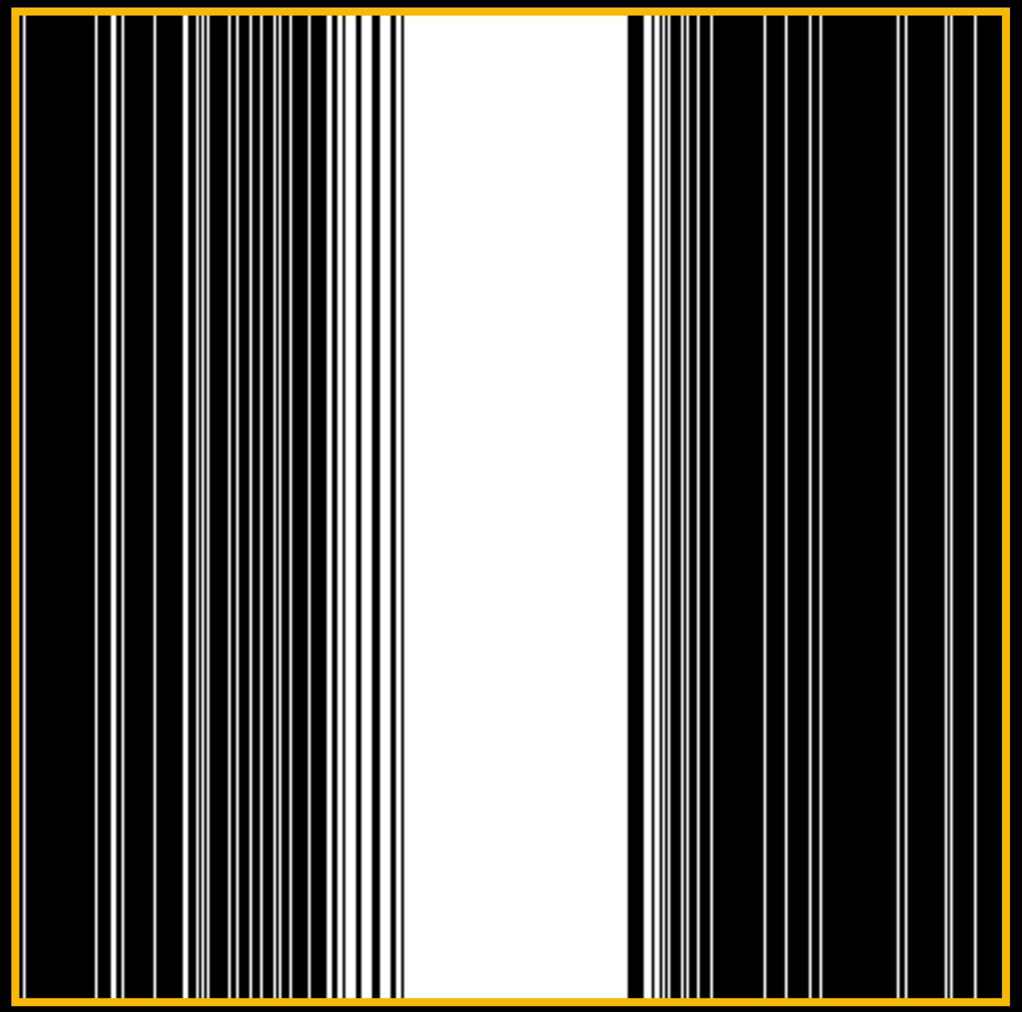
$$argmin_x \quad \| UFSx - y \|_2^2 + \lambda \| Wx \|_1$$

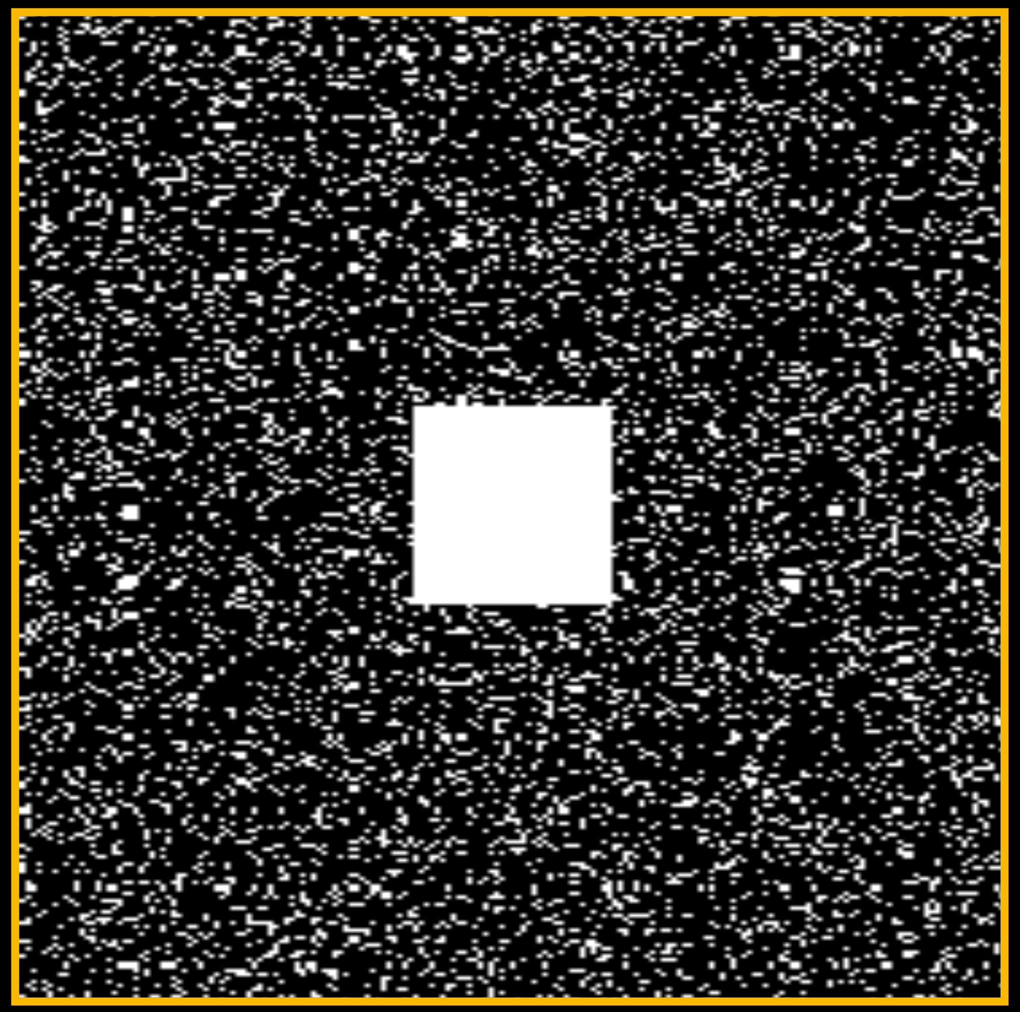
Coil combined image

Multi-coil k-sapce data

Compressed sensing + Parallel imaging

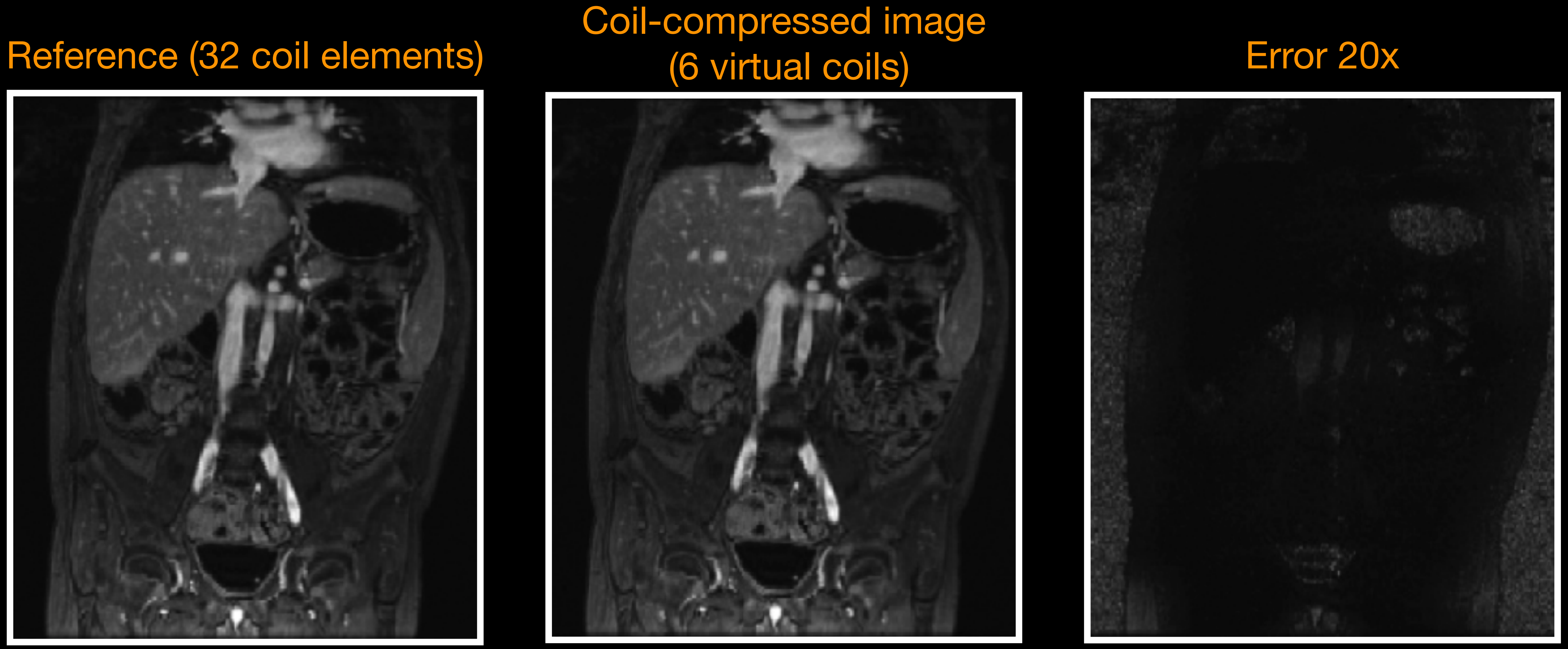
- Sampling trajectory:
  - The fully sampled region can be used to estimate coil sensitivity maps
  - The overall sampling scheme needs to generate incoherent artifacts





Coil compression

- A problem in applying compressed sensing reconstruction in some applications is the increased memory requirement and computational complexity due to a large number of coils.
- Coil compression (e.g., singular value decomposition-based technique) has been developed to reduce the number of coils before compressed sensing reconstruction.

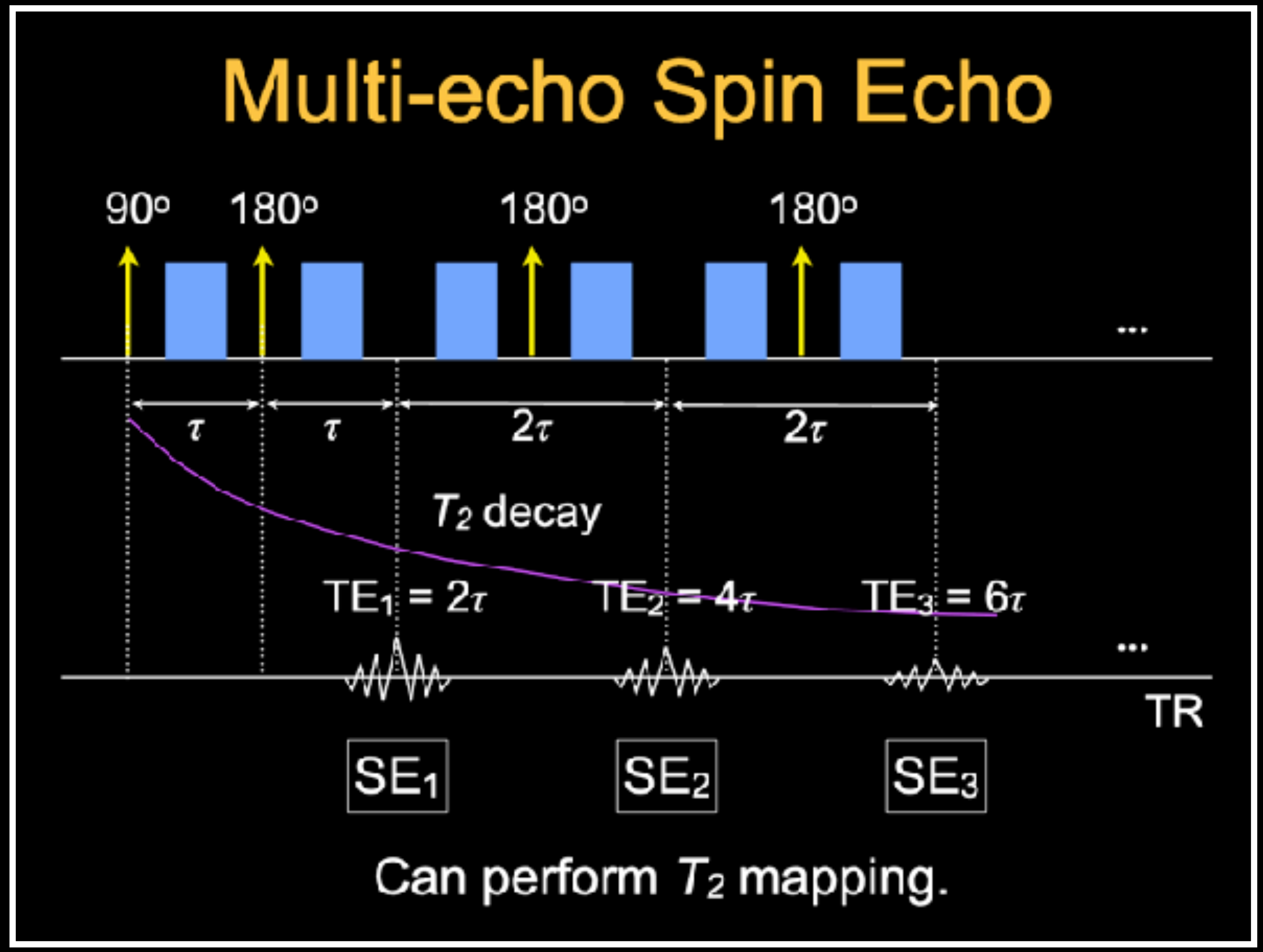


(Figures from: Zhang et al., MRM 2013)

Example (1): Knee T_2 mapping

- T_2 values in the knee cartilage have been used to detect disease- and treatment changes in articular cartilage.
- T_2 quantification in the knee cartilage can help depict early cartilage degeneration.
- Challenges: Conventional multi-echo spin echo-based sequences are slow

Multi-echo spin-echo images

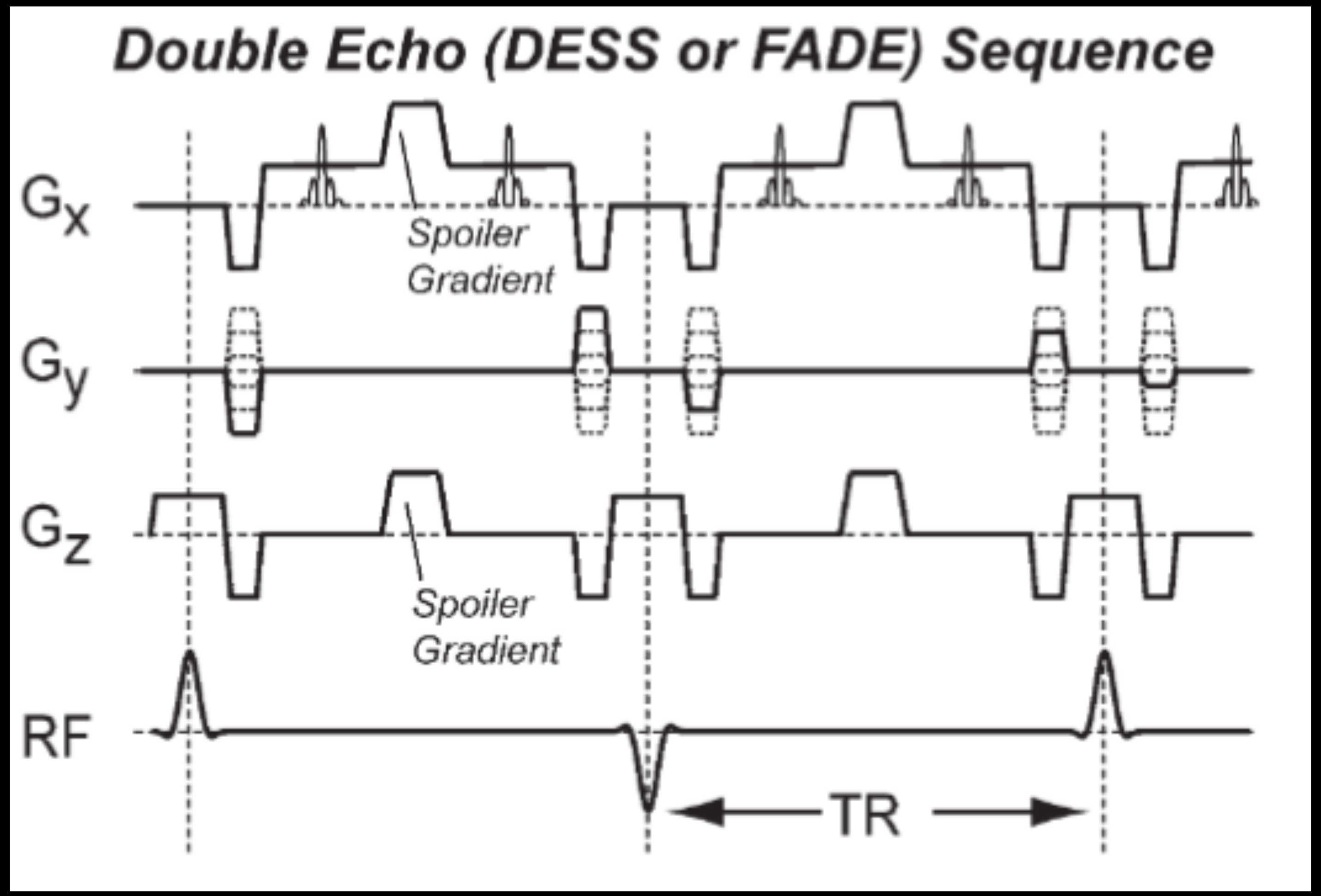


From previous lecture slide

Example (1): Knee T_2 mapping

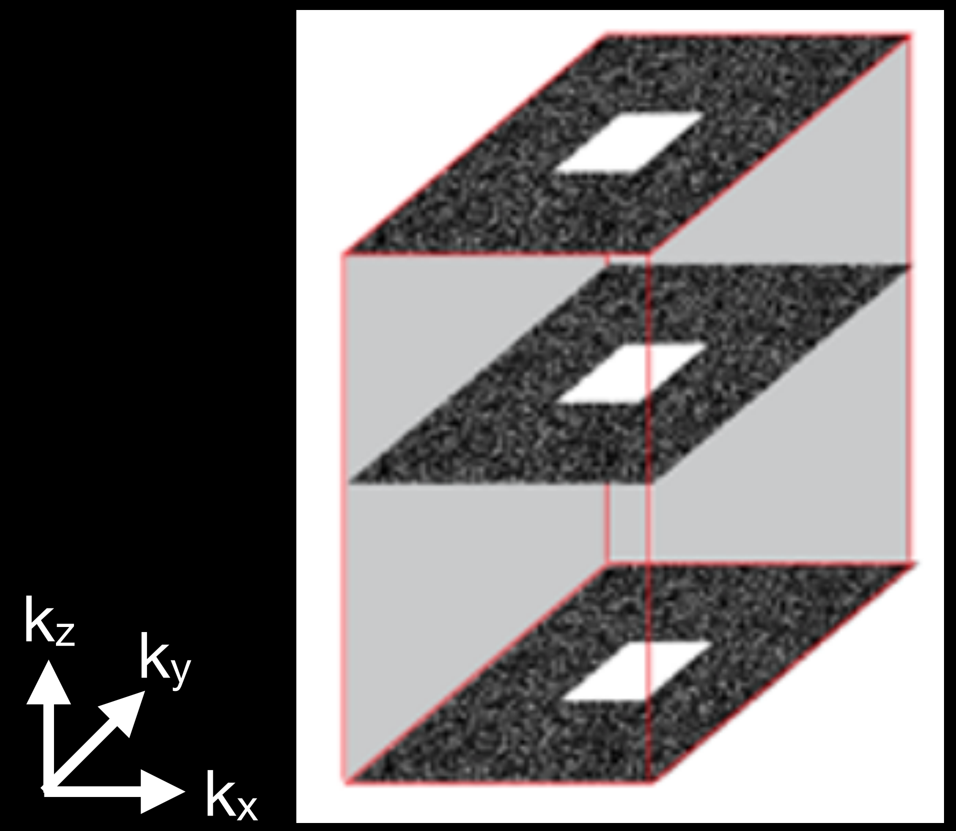
- Acceleration strategy
  - (1) Use a faster sequence: DESS (double/dual echo steady state)
  - (2) Use compressed sensing to accelerate

An extension to the gradient-spoiled GRE which acquires both SSFP-FID and SSFP-Echo



The difference between the two contrasts can be used to quantify T_2

Variable density sampling

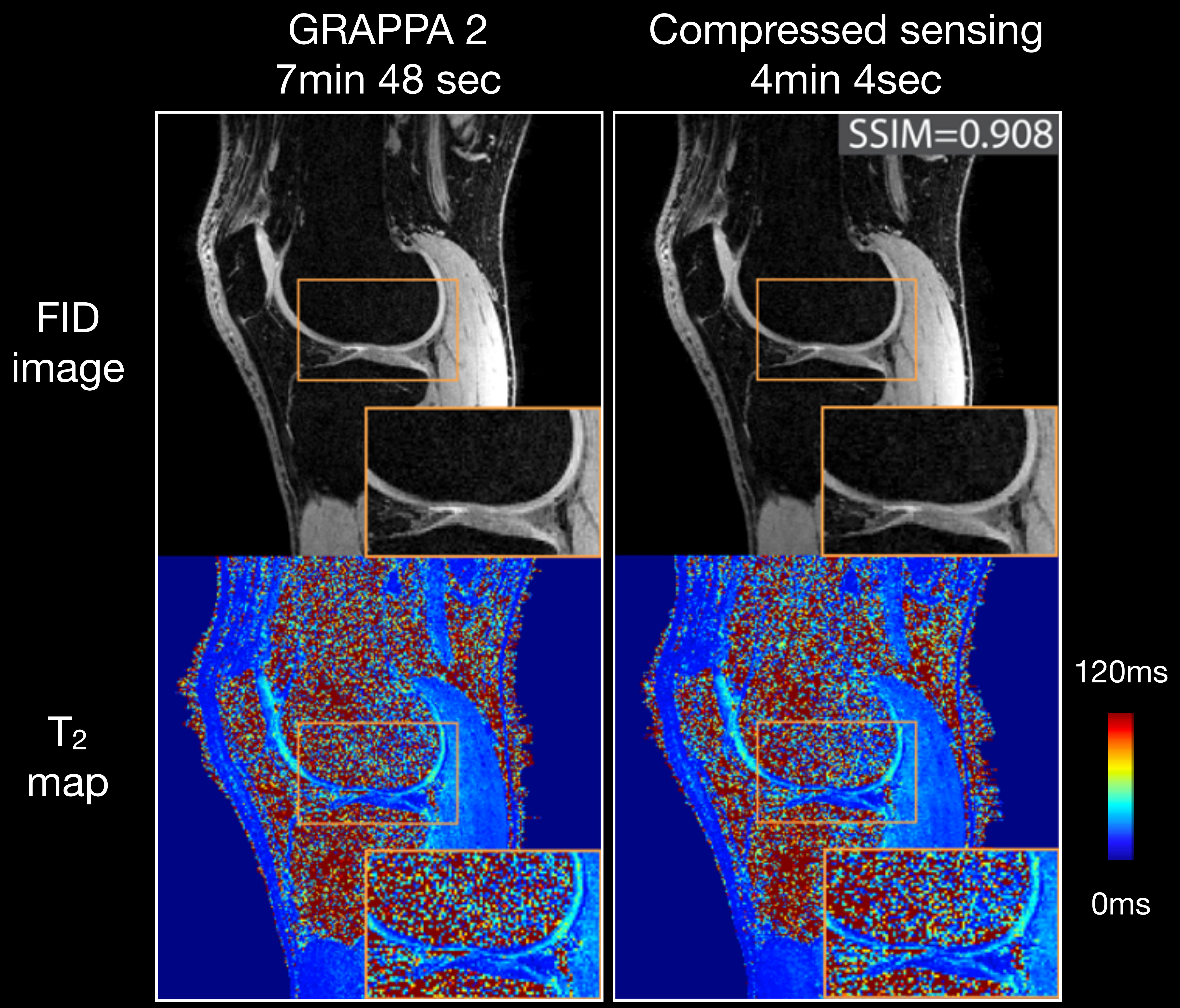


U: k-space sampling pattern
F: Fourier transform operator
S: coil sensitivity maps
x: reconstructed image
y: acquired undersampled k-space data
W: Wavelet transform operator
D: total variation operator
λ_1, λ_2: regularization parameters

Cost function

$$argmin_x \| UFSx - y \|_2^2 + \lambda_1(\| Wx_{fid} \|_1 + r \| Wx_{echo} \|_1) + \lambda_2(\| Dx_{fid} \|_1 + r \| Dx_{echo} \|_1)$$

Example (1): Knee T_2 mapping



(Figures from: Shih et al., ISMRM 2023)

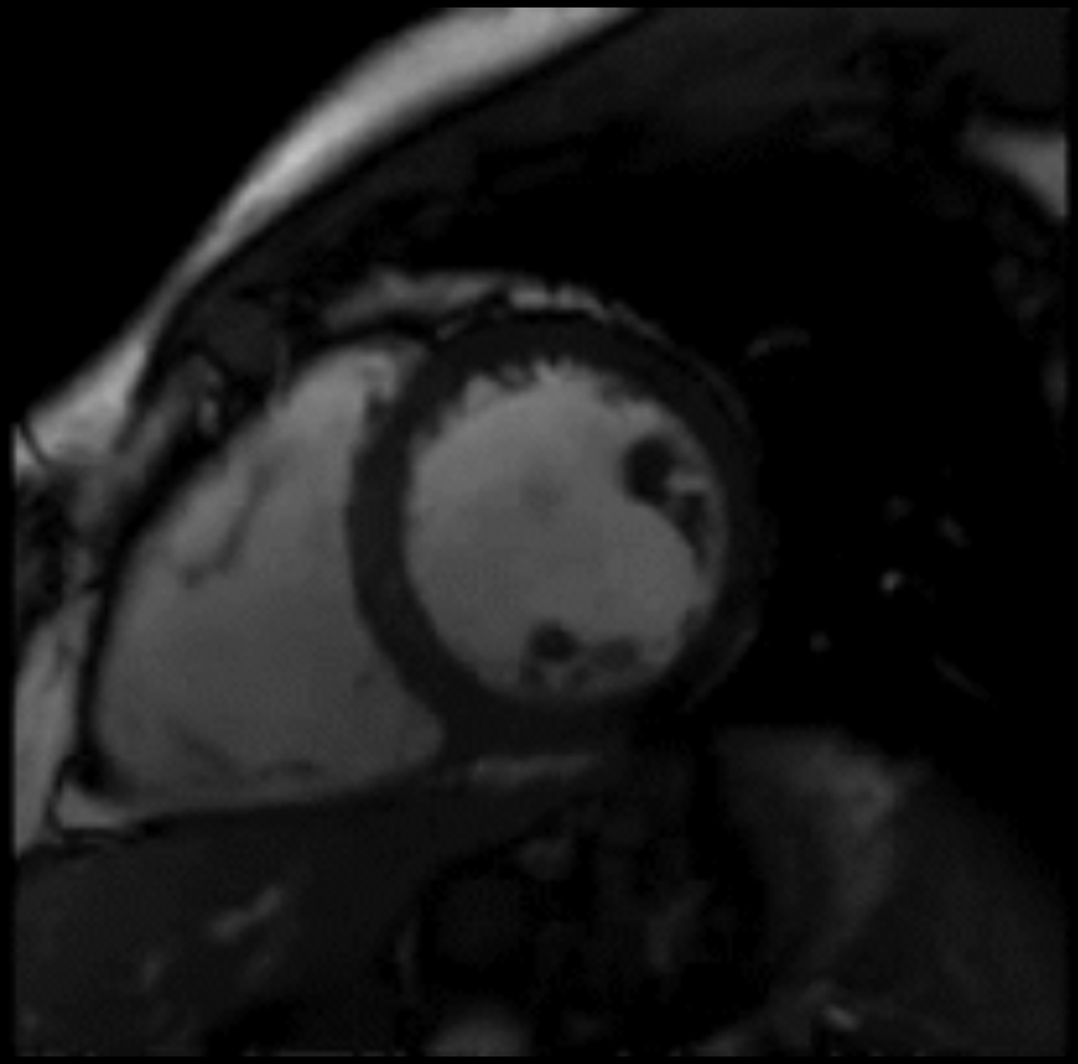
Example (1): Knee T_2 mapping

- Rapid knee cartilage T_2 mapping
  - *Constraint*: Sparsity in Wavelet transform and sparsity in total variation
  - *Incoherent measurement*: variable density random sampling
  - *Optimization function*: $argmin_x \| UFSx - y \|_2^2 + \lambda_1(\| Wx_{fid} \|_1 + r \| Wx_{echo} \|_1) + \lambda_2(\| Dx_{fid} \|_1 + r \| Dx_{echo} \|_1)$
  - *Reconstruction*: non-linear conjugate gradient method

U: k-space sampling pattern
F: Fourier transform operator
S: coil sensitivity maps
x: reconstructed image
y: acquired undersampled k-space data
W: Wavelet transform operator
D: total variation operator
λ_1, λ_2: regularization parameters

Example (2): Cardiac cine imaging

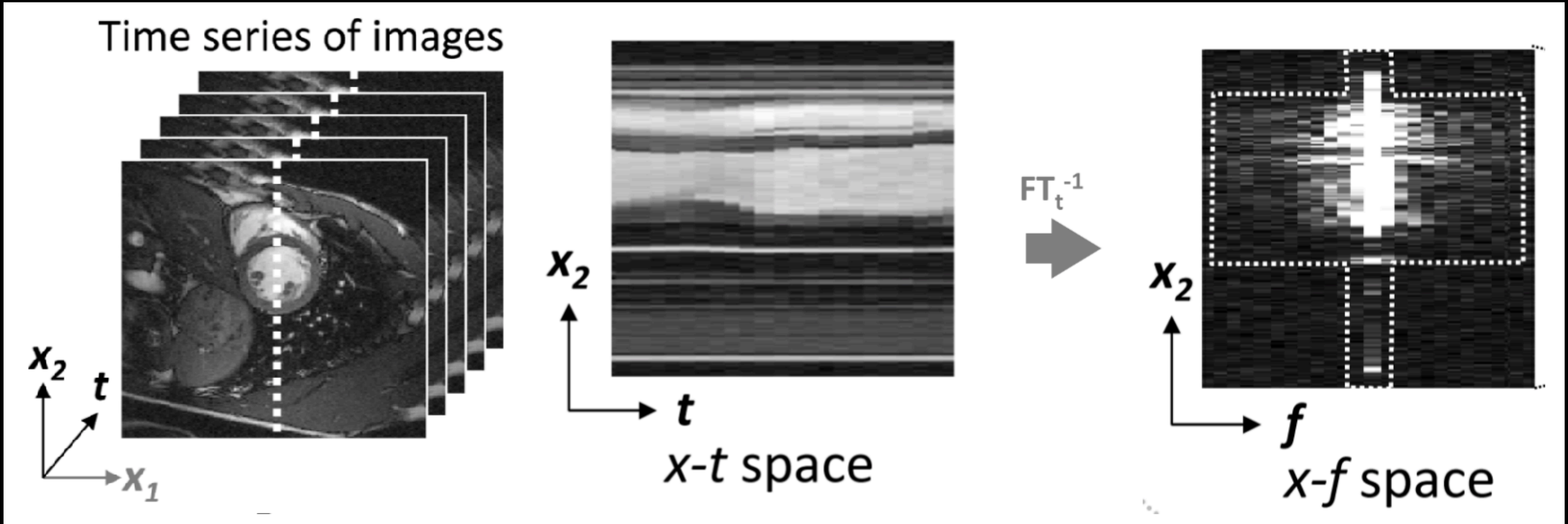
- Cardiac cine imaging for information of the heart function throughout the cardiac cycle
- Challenges: accelerating data acquisition without compromising the high resolution and image quality requirements



(Figure from: Otazo et al., MRM 2015)

Example (2): Cardiac cine imaging

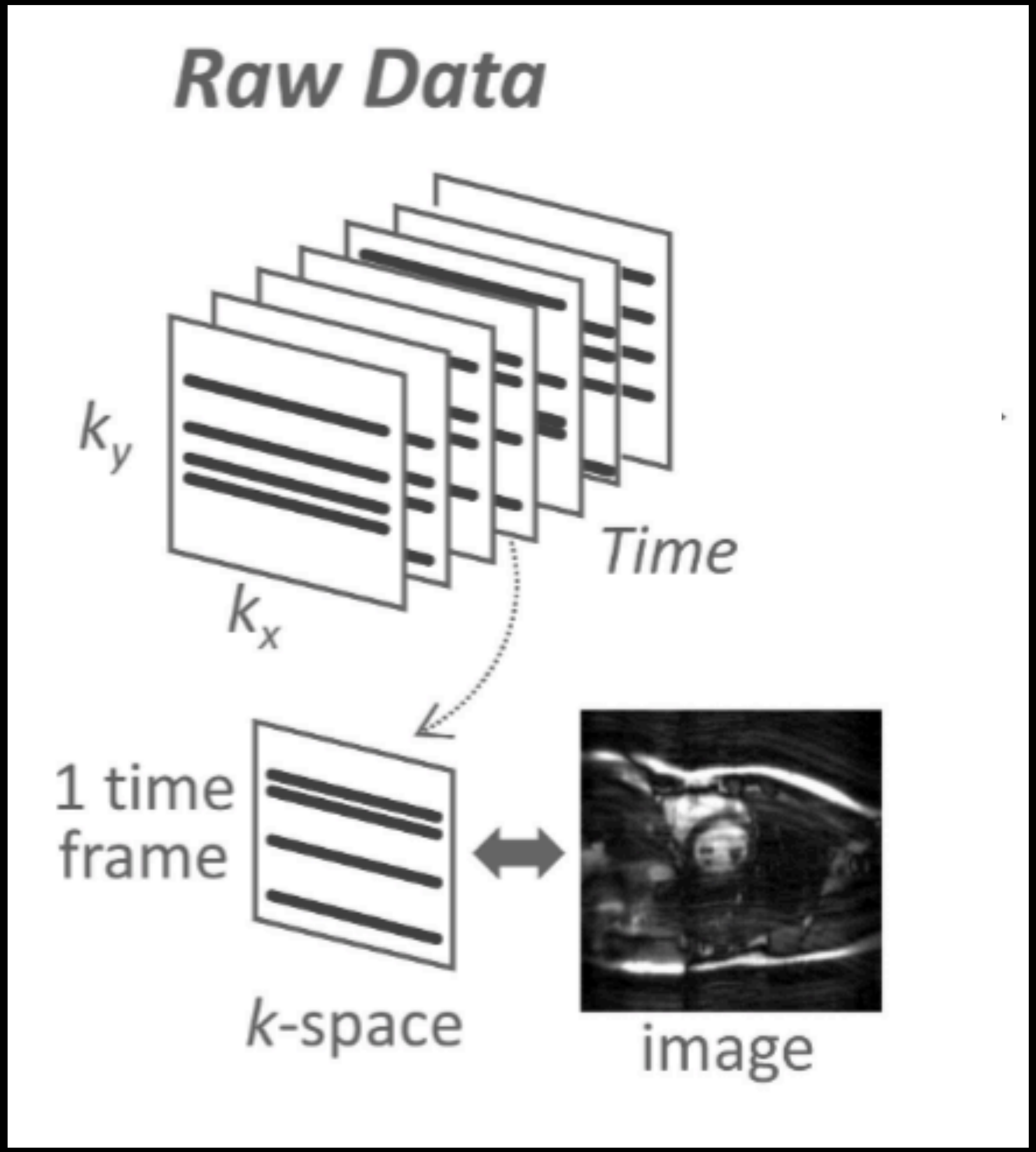
- Sparsity in the x-f space

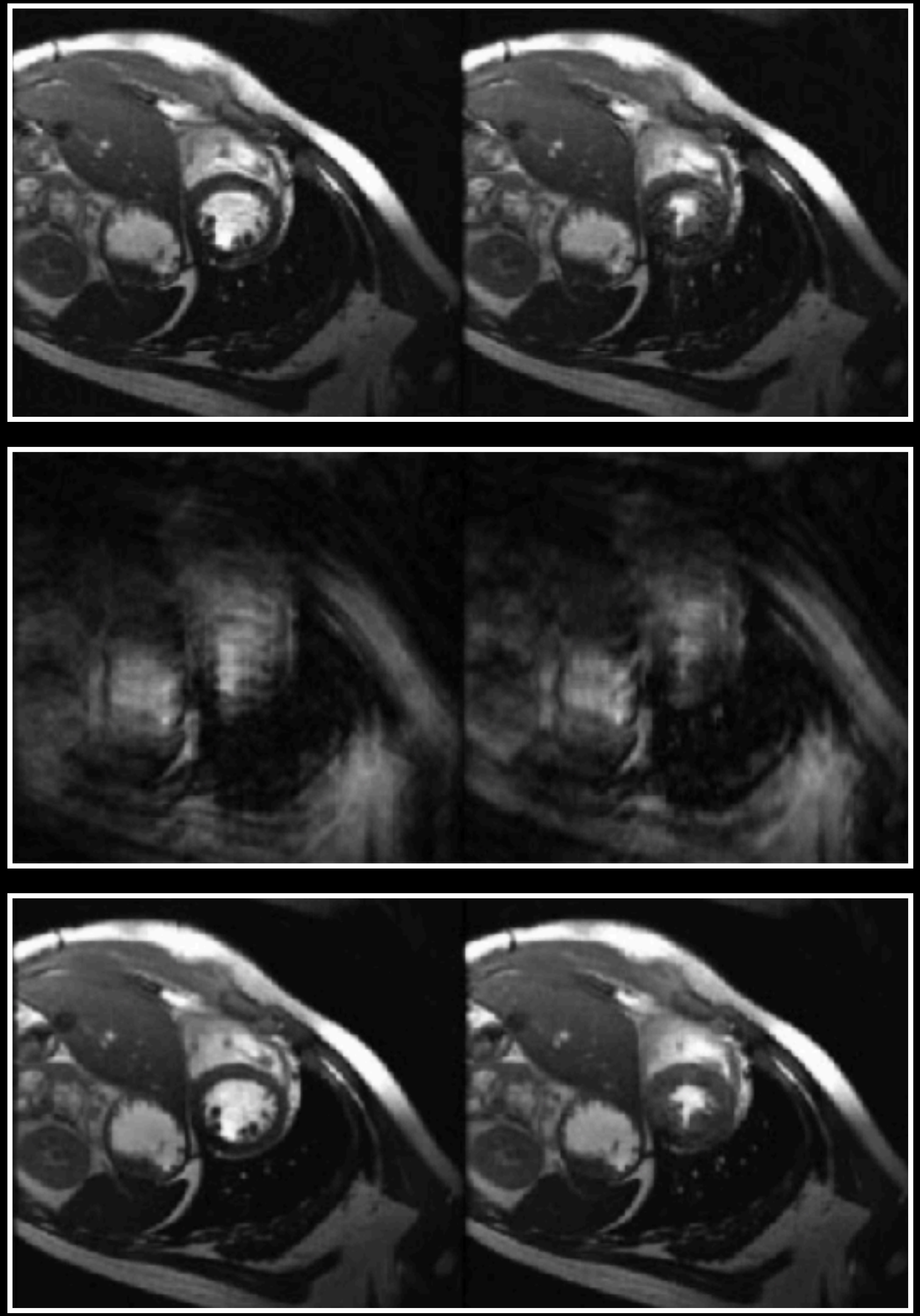


(Figures from: Tsao et al., JMRI 2012)

Example (2): Cardiac cine imaging

k-t sampling pattern





(Figures from: Tsao et al., JMRI 2012 and Jung et al., MRM 2009)

Example (2): Cardiac cine imaging

- k-t FOCUSS[1] (*k-t FOCal Underdetermined System Solver*)
  - *Application*: cardiac cine imaging
  - *Constraint*: sparsity in the x-f space
  - *Incoherent measurement*: k-t undersampling
  - *Optimization function*: $min_{\rho} \| y - DFS\rho \|_2^2 + \lambda \| \rho \|_1$
  - *Reconstruction*: reweighted quadratic optimization

Let $\rho = \rho_0 + \Delta\rho$

$$min_{\rho} \| y - DFS(\rho_0 + \Delta\rho) \|_2^2 + \lambda \| \Delta\rho \|_1$$

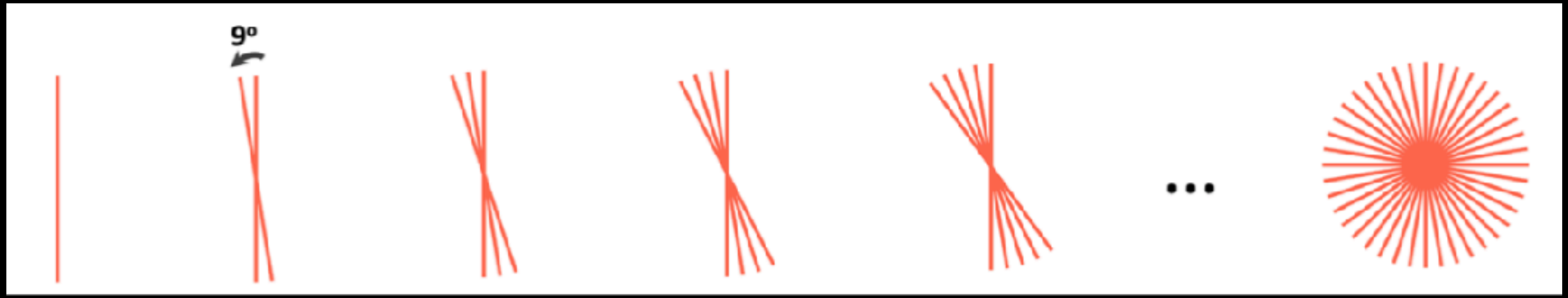
y: acquired k-space data
D: k-t sampling pattern
F: Transform operator between k-space and x-f space
S: coil sensitivity maps
ρ: reconstructed x-f space
λ: regularization parameter

[1] Jung et al., Magn Reson Med. 2009;61(1):103-16

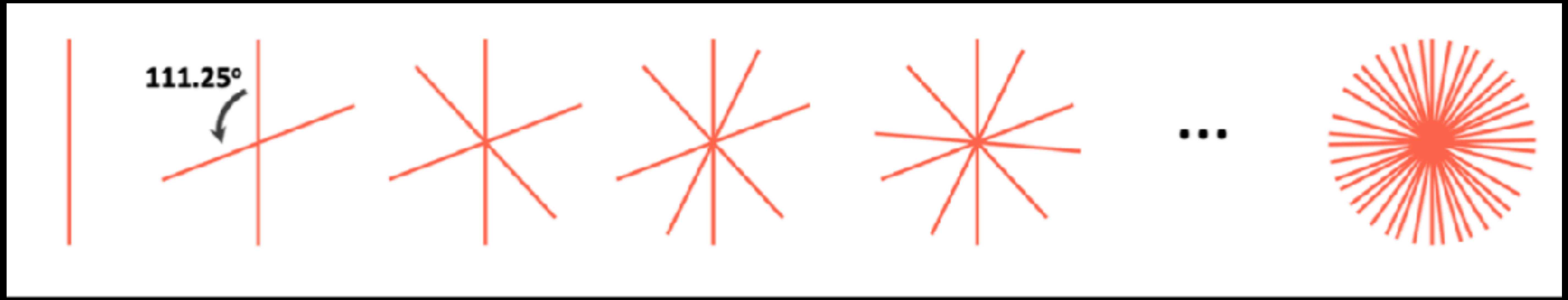
Example (3): Free-breathing radial MRI

- Radial MRI with inherent motion robustness can be used for free-breathing MRI
- Radial undersampling results in incoherent artifacts

Linear radial MRI



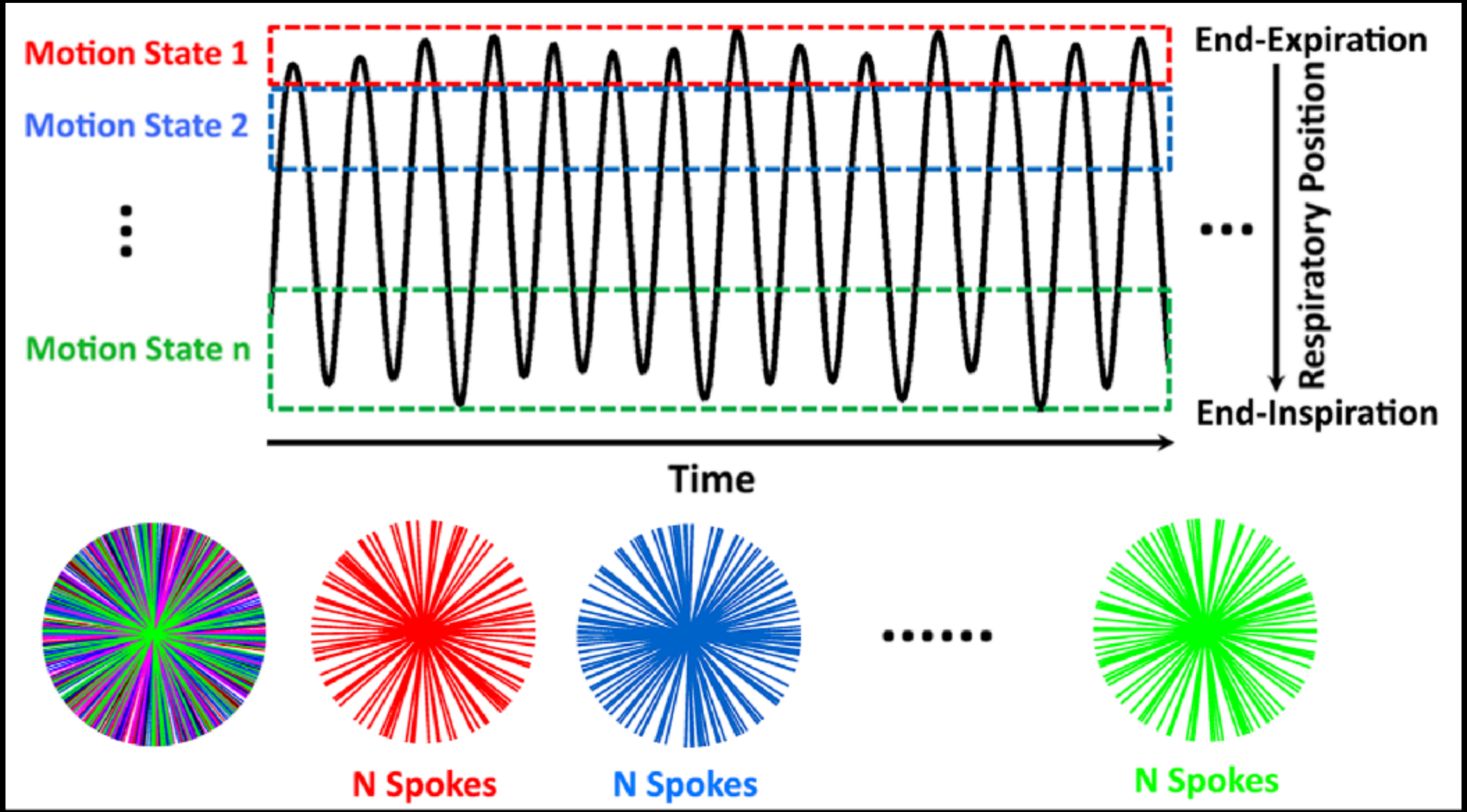
Golden-angle radial MRI



(Figure from: Feng et al., JMRI 2022)

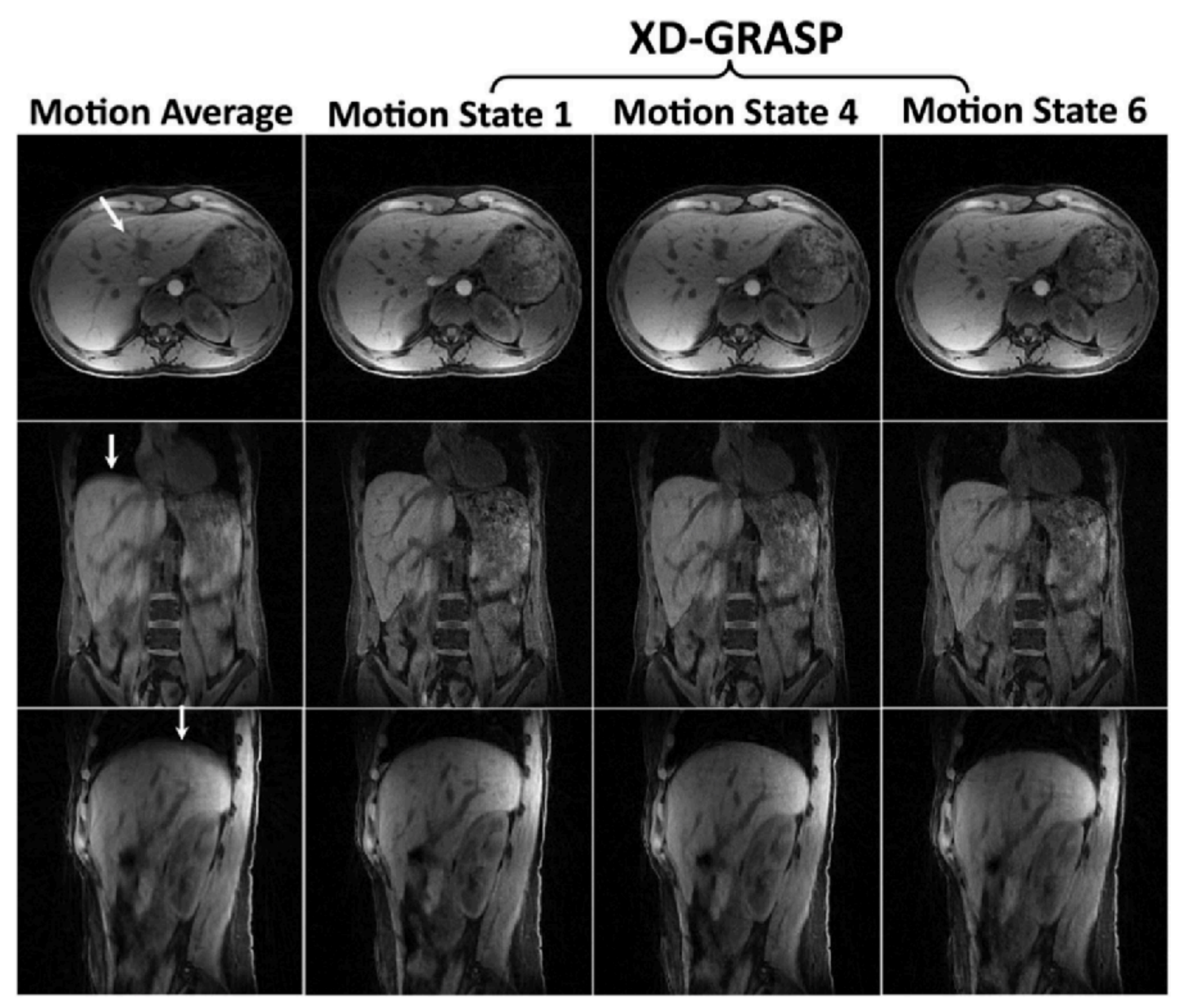
Example (3): Free-breathing radial MRI

- Stack-of-radial MRI provides self-navigation to track breathing motion
- We can group the k-space data into different motion states



(Figure from: Feng et al., MRM 2016)

Example (3): Free-breathing radial MRI



(Figure from: Feng et al., MRM 2016)

Example (3): Free-breathing radial MRI

- XD-GRASP[1] (*Golden-angle radial MRI with reconstruction of extra motion-state dimensions using compressed sensing*)
  - *Application*: free-breathing abdominal imaging
  - *Constraint*: temporal finite differences (or total variation) in dynamic dimension
  - *Incoherent measurement*: undersampled golden-angle radial MRI
  - *Optimization function*: $min_x \quad \| FCx - y \|_2^2 + \lambda_1 \| S_1 x \|_1 + \lambda_2 \| S_2 x \|_1$
  - *Reconstruction*: non-linear conjugate gradient

[1] Feng et al., Magn Reson Med. 2016;75(2):775-88

Compressed sensing MRI

- Limitations:
  - Requiring high computational complexity to solve the nonlinear reconstruction problem
  - Reconstruction result is dependent on the choice of regularization parameters

- Other related constrained reconstruction methods
  - Dictionary-based compressed sensing MRI
  - MRI reconstruction using low-rank constraints
  - …

Take home message

- 3 main components in compressed sensing MRI
  - The image has a sparse representation in some transform domain
  - The k-space sampling trajectory generates incoherent artifacts in the sparse transform domain
  - It involves a nonlinear reconstruction method

Take home message

- If we want apply compressed sensing to accelerate an application, check:
- (1) Can the images be sparsified in a certain (transform) domain?
  - Wavelet transform
  - Spatial total variation in images
  - Total variation in temporal frames
  - x-f space
  - …
- (2) Can the sampling pattern generate incoherent artifacts?
  - Variable density sampling pattern
  - Radial acquisition
  - Spiral acquisition
  - …

Thanks!

- Next time
  - Deep learning MRI reconstruction

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